

10-2: Congruent Chords

Objective

After studying this section, you will be able to

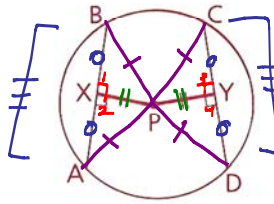
- Apply the relationship between congruent chords of a circle

Theorem 77 *If two chords of a circle are equidistant from the center, then they are congruent.*

chds eqdist $\Rightarrow \cong$ chds

Given: $\odot P$, $\overline{PX} \perp \overline{AB}$, $\overline{PY} \perp \overline{CD}$, $\overline{PX} \cong \overline{PY}$

Prove: $\overline{AB} \cong \overline{CD}$



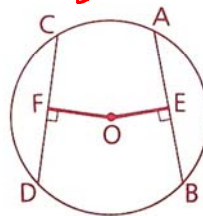
- | | |
|--------------------------------------------------------------------------------|-------------------------------------------------|
| 1. $PX \perp AB, PY \perp CD$ | 1. given |
| 2. $\angle 1, \angle 2, \angle 3, \angle 4$ rt \angle s | 2. $\perp \Rightarrow$ rt \angle |
| 3. $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$ | 3. rt \angle s $\Rightarrow \cong$ \angle s |
| 4. Draw $\overline{PA}, \overline{PB}, \overline{PC}, \overline{PD}$ | 4. Aux |
| 5. $\overline{PA} \cong \overline{PB} \cong \overline{PC} \cong \overline{PD}$ | 5. $\odot \Rightarrow \cong$ rad |
| 6. $\overline{PX} \cong \overline{PY}$ | 6. Given |
| 7. $\triangle PAX \cong \triangle PBX \cong \triangle PCY \cong \triangle PDY$ | 7. HL |
| 8. $\overline{AX} \cong \overline{XB} \cong \overline{CY} \cong \overline{YD}$ | 8. opctc |
| 9. $\overline{AB} \cong \overline{CD}$ | 9. Add |

Theorem 78 *If two chords of a circle are congruent, then they are equidistant from the center of the circle.*

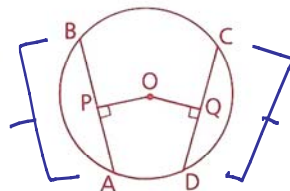
\cong chds \Rightarrow chs eqdist

Given: $\odot O$, $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$

Prove: $\overline{OE} \cong \overline{OF}$



Problem 1 Given: $\odot O$, $\overline{AB} \cong \overline{CD}$,
 $OP = 12x - 5$, $OQ = 4x + 19$
 Find: OP



Solution $AB = CD$ (g), $OP = OQ$ (\cong chds \Rightarrow chds eq dist)

$$12x - 5 = 4x + 19$$

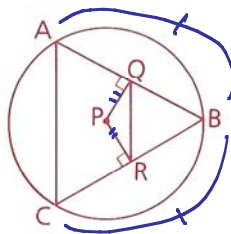
$$\begin{array}{r} -4x \quad +5 \quad -4x \quad +5 \\ \hline \end{array}$$

$$8x = 24$$

$$x = 3$$

$$OP = 12(3) - 5 = \boxed{31}$$

Problem 2 Given: $\triangle ABC$ is isosceles, with base \overline{AC} .
 $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$
 Prove: $\triangle PQR$ is isosceles.



Proof

- 1 $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$
- 2 $\triangle ABC$ is isosceles, with base \overline{AC} .
- 3 $\overline{AB} \cong \overline{CB}$

$$4 \quad \overline{PQ} \cong \overline{PR}$$

5 $\triangle PQR$ is isosceles.

1 Given

2 Given

3 $\text{isos} \Rightarrow 2 \cong \text{sds}$

4 $\cong \text{chds} \Rightarrow \text{chds eq dist}$

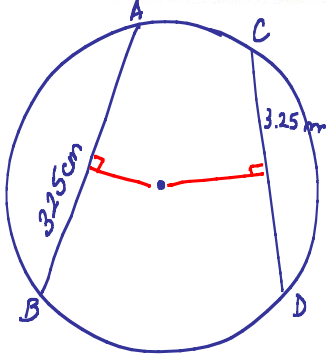
5 $2 \text{ sds} \cong \Rightarrow \text{isos}$

NAME
Adv Geo

10-2: 1-12, skip 8 & 10

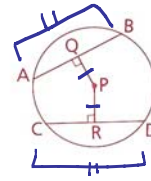
Ms. Kresovic
W 10 Apr 2013

- 1 In a circle, chord \overline{AB} is 3.25 m long and chord \overline{CD} is $3\frac{1}{4}$ m long. Which is closer to the center?

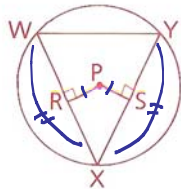


$3.25\text{ m} = 3\frac{1}{4}\text{ m}$
 $\cong \text{chds} \Rightarrow \text{chds eq. dist}$

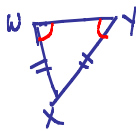
- 2 Given: $\odot P$, $\overline{PQ} \cong \overline{PR}$,
 $AB = 6x + 14$,
 $CD = 4 - 4x$
Find: AB



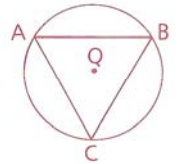
- 3 Given: $\odot P$, $\overline{PR} \perp \overline{WX}$,
 $\overline{PS} \perp \overline{XY}$, $\overline{PR} \cong \overline{PS}$
Conclusion: $\angle W \cong \angle Y$



1. $\odot P$, $\overline{PR} \perp \overline{WX}$,
 $\overline{PS} \perp \overline{XY}$, $\overline{PR} \cong \overline{PS}$
2. $\overline{WX} \cong \overline{YX}$
3. $\angle XWY \cong \angle XYW$
1. given
2. chds eq. dist $\Rightarrow \cong \text{chds}$
3. $\angle X \Rightarrow \angle A$

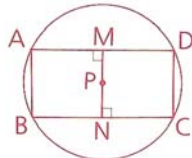


- 4 Given: Equilateral $\triangle ABC$ is inscribed in $\odot Q$.
Conclusion: \overline{AB} , \overline{BC} , and \overline{CA} are equidistant from the center.



In quad IF 1 prop sds both \cong & \parallel , then \square

- 5 Given: $\odot P$;
P is the midpoint of \overline{MN} .
 $\overline{MN} \perp \overline{AD}$, $\overline{MN} \perp \overline{BC}$
Conclusion: ABCD is a \square .



- 6 A fly is sitting at the midpoint of a wooden chord of a circular wheel. The wheel has a radius of 10 cm, and the chord has a length of 12 cm.



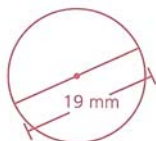
- a How far from the hub (center) is the fly?
b The wheel is spun. What is the path of the fly?

- a. $2(3-4-5)$ 8cm
- b. $\rightarrow \text{circle}$

- 7 ~~To the nearest hundredth~~, find
- The area of the circle
 - The circumference of the circle

$$a) A = \pi r^2 = \left(\frac{19}{2}\right)^2 \pi = \frac{361}{4} \pi$$

$$b) C = d\pi = 19\pi$$



- 11 In circle O, $PB = 3x - 17$, $CD = 15 - x$, and $OQ = OP = 3$.

- Find AB. $= 8$
- Find the radius of $\odot O$.

$$PO = OQ \Rightarrow BA = CD$$

$$2(3x - 17) = 15 - x$$

$$6x - 34 = 15 - x$$

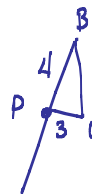
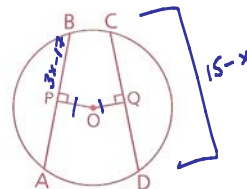
$$+x + 34 \quad +34 + x$$

$$7x = 49$$

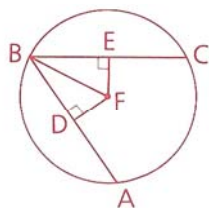
$$x = 7$$

$$a. 15 - 7 = 8$$

$$b. 5$$



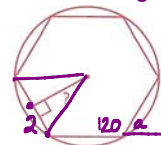
- 9 Given: $\odot F$,
 $\overline{FE} \perp \overline{BC}$, $\overline{FD} \perp \overline{AB}$;
 \overrightarrow{BF} bisects $\angle ABC$.
 Prove: $\overline{BC} \cong \overline{BA}$



- 12 A regular hexagon with a perimeter of 24 is inscribed in a circle. How far from the center is each side?

$$2\sqrt{3}$$

$$P = 24 \rightarrow \frac{24}{6} = 4 = \text{side}$$



$$\frac{360}{6} = 60^\circ$$

