

10-2: Congruent Chords

Objective

After studying this section, you will be able to

- Apply the relationship between congruent chords of a circle

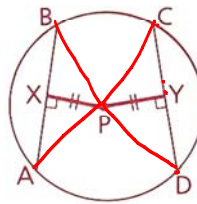
Theorem 77 *If two chords of a circle are equidistant from the center, then they are congruent.*

chds eqdist \Rightarrow \cong chds

Given: $\odot P$, $\overline{PX} \perp \overline{AB}$, $\overline{PY} \perp \overline{CD}$, $\overline{PX} \cong \overline{PY}$

Prove: $\overline{AB} \cong \overline{CD}$

$\triangle APX \cong \triangle CPY$ by HL

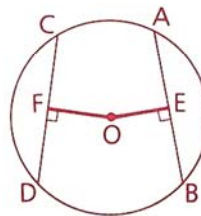


Theorem 78 *If two chords of a circle are congruent, then they are equidistant from the center of the circle.*

\cong chds \Rightarrow chds eqdist.

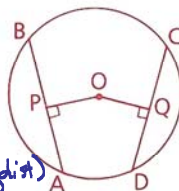
Given: $\odot O$, $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$

Prove: $\overline{OE} \cong \overline{OF}$



Problem 1Given: $\odot O$, $\overline{AB} \cong \overline{CD}$,

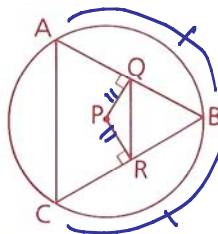
$$OP = 12x - 5, OQ = 4x + 19$$

Find: OP \downarrow
not proof**Solution**

$$\odot O, AB = CD (g) \rightarrow OP = OQ \quad (\cong \text{chds} \rightarrow \text{chds eq dist})$$

$$\begin{array}{r} 12x - 5 = 4x + 19 \\ -4x + 5 \quad -4x + 5 \\ \hline 8x = 24 \\ x = 3 \end{array}$$

$$OP = 12(3) - 5 = \boxed{31}$$

Problem 2Given: $\triangle ABC$ is isosceles, with base \overline{AC} . $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$ Prove: $\triangle PQR$ is isosceles.**Proof**1 $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$ 2 $\triangle ABC$ is isosceles, with base \overline{AC} .3 $\overline{AB} \cong \overline{CB}$ 4 $\overline{PQ} \cong \overline{PR}$ 5 $\triangle PQR$ is isosceles.

1 Given

2 Given

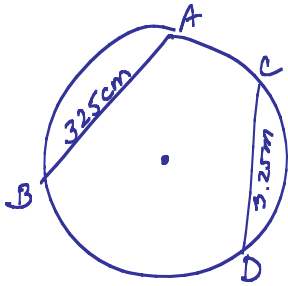
3 $\text{isos} \Rightarrow 2 \text{ sides} \cong$ 4 $\cong \text{chds} \Rightarrow \text{chds eq dist}$ 5 $2 \text{ sides} \cong \text{isos}$ 

NAME
Adv Geo

10-2: 1-12, skip 8 & 10

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W 10 Apr 2013

- 1 In a circle, chord \overline{AB} is 325 cm long and chord \overline{CD} is $3\frac{1}{4}$ m long. Which is closer to the center?

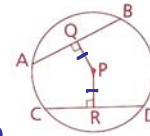


3.25m

$2 \cong \text{chds} \Rightarrow \text{chds}$
equidist
from ctr.

- 2 Given: $\odot P$, $\overline{PQ} \cong \overline{PR}$,
 $AB = 6x + 14$,
 $CD = 4 - 4x$

Find: AB



$$PQ = PR \Rightarrow AB = CD$$

(eq dists $\Rightarrow \cong \text{chds}$)

$$6x + 14 = 4 - 4x$$

$$+4x \quad -14 \quad -14 \quad +14$$

$$10x = -10$$

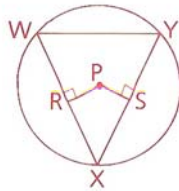
$$x = -1$$

$$AB = 6(-1) + 14 = 8$$

$$\text{or } 4 - 4(-1) = 4 + 4 = 8$$

- 3 Given: $\odot P$, $\overline{PR} \perp \overline{WX}$,
 $\overline{PS} \perp \overline{XY}$, $\overline{PR} \cong \overline{PS}$

Conclusion: $\angle W \cong \angle Y$



1. $\odot P$, $\overline{PR} \perp \overline{WX}$, $\overline{PS} \perp \overline{XY}$

2. $\overline{WX} \cong \overline{XY}$

3. $\triangle WXY$ isos, base WY

4. $\angle W \cong \angle Y$

1. given

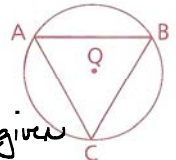
2. eq dist $\Rightarrow \cong \text{chds}$

3. $2 \cong \text{sides} \Rightarrow \text{isos}$

4. $\triangle WXY \Rightarrow \triangle$

- 4 Given: Equilateral $\triangle ABC$ is inscribed in $\odot Q$.

Conclusion: \overline{AB} , \overline{BC} , and \overline{CA} are equidistant from the center.



1. Eq $\triangle ABC$ inscribed in $\odot Q$

2. $\overline{AB} \cong \overline{BC} \cong \overline{CA}$

3. \overline{AB} , \overline{BC} , & \overline{CA} eq dist from ctr

1. given

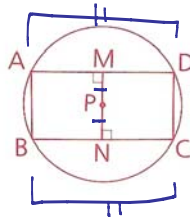
2. Eq $\triangle \Rightarrow \cong \text{sides}$

3. $\cong \text{chds} \Rightarrow$
eq dist chds

- 5 Given: $\odot P$;

P is the midpoint of \overline{MN} .
 $\overline{MN} \perp \overline{AD}$, $\overline{MN} \perp \overline{BC}$

Conclusion: ABCD is a \square .



1. $\odot P$, P mdpt \overline{MN}

2. $\overline{PM} \cong \overline{NP}$

3. $\overline{AD} \cong \overline{BC}$

4. $\overline{MN} \perp \overline{AD}$ & \overline{BC}

5. $\overline{AD} \parallel \overline{BC}$

6. $\square ABCD$

1. given

2. mdpt $\Rightarrow \cong \text{segs}$

3. eq dist $\Rightarrow \cong \text{chds}$

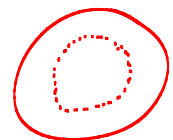
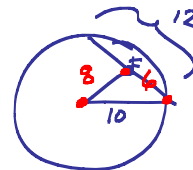
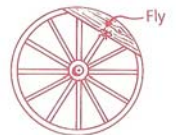
4. given

5. lines \perp to same line $\Rightarrow \parallel$

6. In quad, if 1 pr opp sides
is both \cong & \parallel , then \square

- 6 A fly is sitting at the midpoint of a wooden chord of a circular wheel. The wheel has a radius of 10 cm, and the chord has a length of 12 cm.

- a How far from the hub (center) is the fly? 8 cm
b The wheel is spun. What is the path of the fly?



7 To the nearest hundredth, find

- The area of the circle
- The circumference of the circle

a. $A = r^2 \pi = \left(\frac{19}{2}\right)^2 \pi =$

b. $C = d\pi = 19\pi$



11 In circle O, $PB = 3x - 17$, $CD = 15 - x$, and $OQ = OP = 3$.

a Find AB. = 8

b Find the radius of $\odot O$.

a] eq dist chds $\Rightarrow \cong$ chds

$AB = CD$

$2(PB) = CD$

$2(3x - 17) = 15 - x$

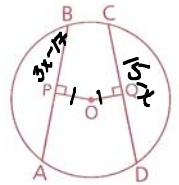
$6x - 34 = 15 - x$

$+x + 34 + 34 + x$

$7x = 49$

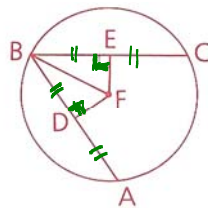
$x = 7$

If $x = 7$, $CD = AB = 15 - 7 = 8$



9 Given: $\odot F$,
 $\overline{FE} \perp \overline{BC}$, $\overline{FD} \perp \overline{AB}$;
 \overline{BF} bisects $\angle ABC$.

Prove: $\overline{BC} \cong \overline{BA}$



1. $OF, \overline{FE} \perp \overline{BC}$ & $\overline{FD} \perp \overline{AB}$
2. $\angle BEF$ & $\angle BDF$ r.t.s
3. $\angle BEF \cong \angle BDF$
4. \overline{BF} bis $\angle ABC$
5. $\angle DBF \cong \angle EBF$
6. $\overline{BF} \cong \overline{BF}$
7. $\triangle DBF \cong \triangle EBF$
8. $\overline{BD} \cong \overline{BE}$
9. $\overline{BC} \cong \overline{BA}$

1. Given
2. $\perp \Rightarrow$ r.t.s
3. r.t.s $\Rightarrow \cong$ \angle s
4. Given
5. bis $\Rightarrow \cong$ \angle s
6. Ref
7. AAS
8. CPCTC
9. Chds = dist $\Rightarrow \cong$ chds

12 A regular hexagon with a perimeter of 24 is inscribed in a circle. How far from the center is each side?

