

What do we already know?

$$\frac{1}{\sin \theta} = \csc \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\frac{1}{\tan \theta} = \cot \theta$$

These are called "Reciprocal Identities".

And these are "Quotient Identities"

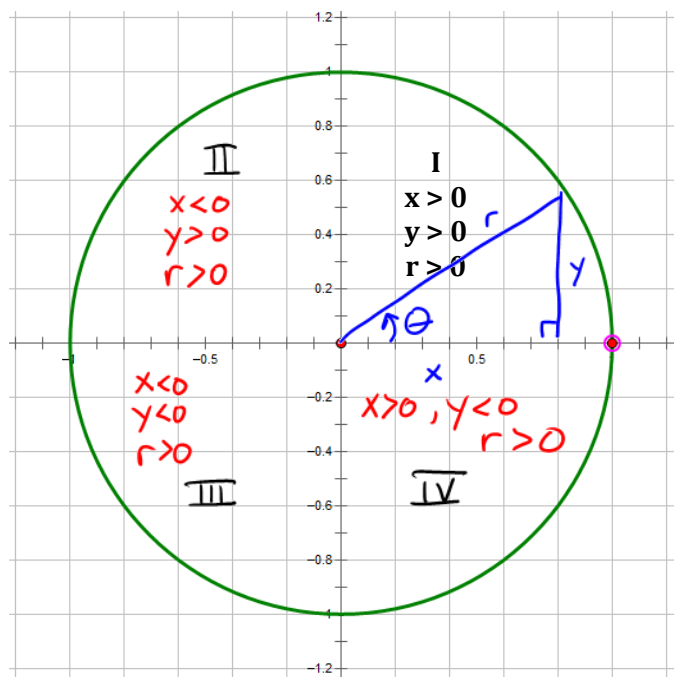
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

CAUTION:

$$\frac{1}{\sin \theta} \neq \sin^{-1} \theta.$$

$\sin^{-1} \theta = \arcsin \theta$ (used when we know the ratio but not the angle measure).



We know definitions:

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y \quad \csc \theta = \frac{r}{y} = \frac{1}{y}$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x \quad \sec \theta = \frac{r}{x} = \frac{1}{x}$$

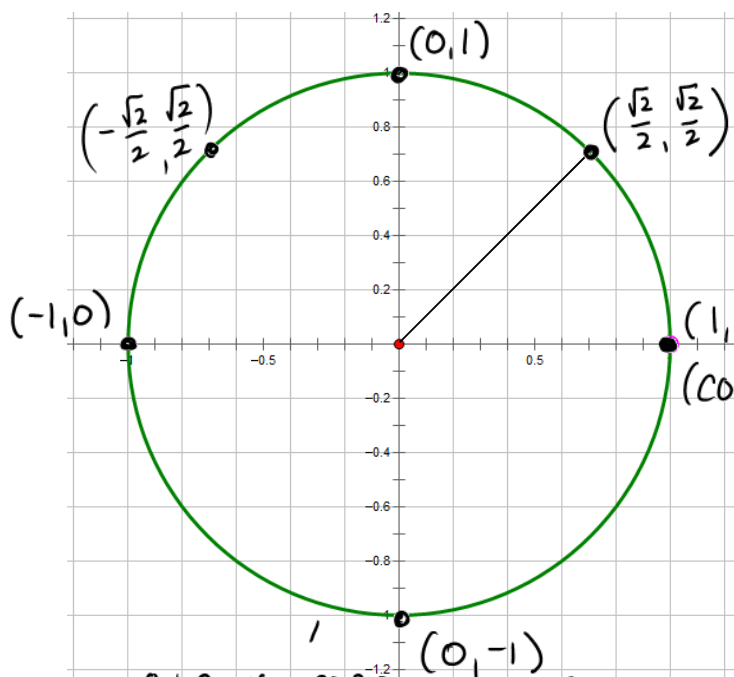
$$\tan \theta = \frac{y}{x} = \frac{y}{x} \quad \cot \theta = \frac{x}{y} = \frac{x}{y}$$

Signs of function values

θ in Quad	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
I	+	+	+	+	+	+
II	+	-	-	+	-	-
III	-	-	+	-	-	+
IV	-	+	-	-	+	-

Can $r \leq 0$? Explain.

No ☹ it's the length of the radius.



What are the possible ranges of each function?
Is there a minimum and maximum value that each function can have?

see pg
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$$\boxed{-1 \leq \cos \theta \leq 1}$$

$$-1 \leq \sin \theta \leq 1$$

$$-\infty \leq \tan \theta \leq \infty$$

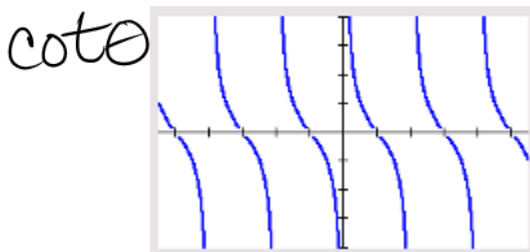
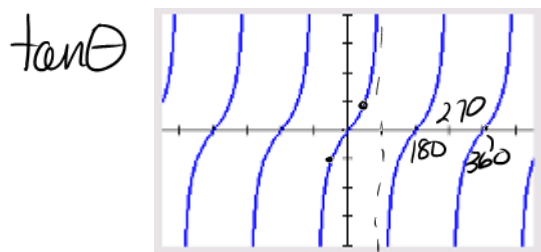
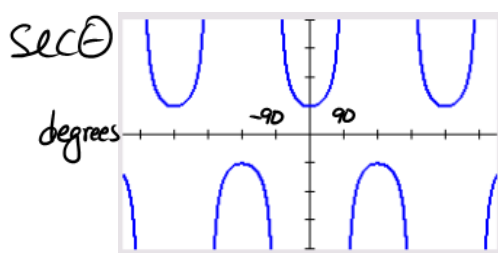
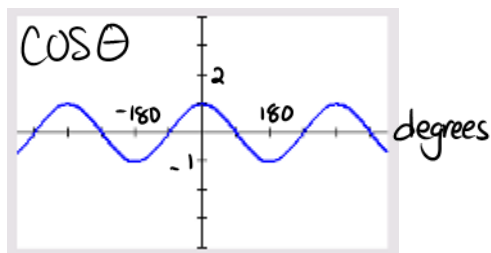
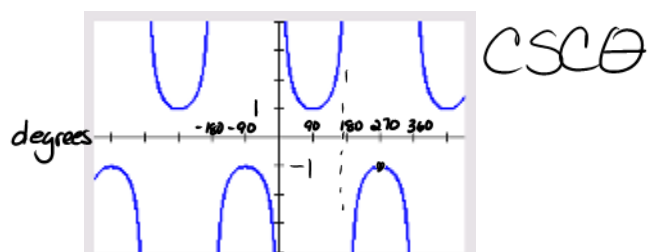
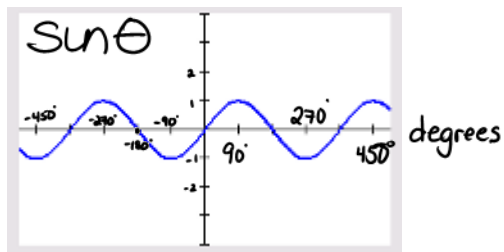
$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \tan \theta = \frac{\pm 1}{\text{close to } 0}$$

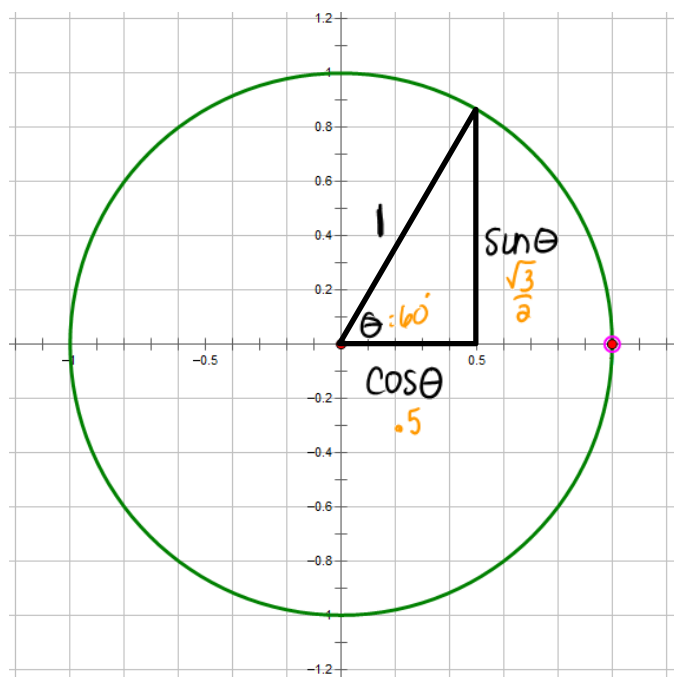
$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$$

	0	90	180	270	360
y	0	1	0	-1	0
x	1	0	-1	0	1

	0	90	180	270	360
cos θ	1	0	-1	0	1

Using the circle graph (above), make inferences about the graphs of the trigonometric functions on the Cartesian plane. Justify your choices. Label the axes. (This is really beyond our curriculum at this point. I just think it's fun. It's also helpful to make connections. You are responsible for the circle graphs, not Cartesian.)





Pythagorean Identities

Let $r = 1$. Then in this right triangle, $x^2 + y^2 = 1$.

$x = \cos \theta$ & $y = \sin \theta$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1 \text{ true}$$

What if?!

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Use the appropriate reciprocal identity to find each function value. Rationalize denominators when applicable.

1. $\sec \theta$, given $\cos \theta = \frac{2}{3}$ $\sec \theta = 3/2$
 5. $\cot \theta$, given $\tan \theta = 5$ $\cot \theta = 1/5$
 7. $\cos \theta$, given $\sec \theta = -\frac{5}{2}$ $\cos \theta = -2/5$
 9. $\sin \theta$, given $\csc \theta = \frac{\sqrt{8}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ $\sin \theta = \frac{\sqrt{2}}{2}$

13. $\sin \theta$, given $\csc \theta = 1.42716321$ $\sin \theta = \frac{1}{1.42716321} = .700690708$

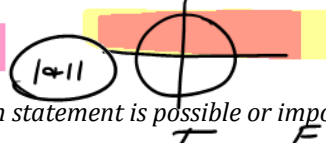
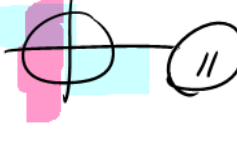
15. What's wrong with the following statement? "Find $\sec \theta$, given $\cos \theta = 3/2$."

$\cos \theta > 1$
outside range

Determine the signs of the trigonometric functions of an angle in standard position with the given measure.

19. 74°  This angle is in Quad I, so all are positive. $\left(\frac{\sin \theta}{\cos \theta} = \tan \theta \right) > 0$ $(\cos \theta, \sin \theta)$
 (x, y)
 23. 178°  This angle is in Quad II. $\sin \theta, \csc \theta$ are positive, $\cos \theta, \sec \theta, \tan \theta, \cot \theta$ are negative.
 26. -15°  This angle is in Quad IV. $\cos \theta, \sec \theta$ are positive, all others are negative.

Identify the quadrant (or possible quadrants) of an angle θ that satisfies the given conditions.

31. $\sin \theta > 0, \csc \theta > 0$  35. $\tan \theta < 0, \cos \theta < 0$ 

Decide whether each statement is possible or impossible for angle θ .

46. $\sin \theta = 3$ **F** 47. $\cos \theta = -0.96$ **T**
 55. $\cot \theta = -4$ **T** 58. $\tan \theta = 2$ AND $\cot \theta = -2$ (for the same angle θ)
 IF $\tan \theta = 2 \Rightarrow \cot \theta = \frac{1}{2} \therefore$ **F**
 60. Explain why no θ satisfies $\cos \theta = \frac{1}{2}$ and $\sec \theta = -2$.
 should be recip. These are opp & recip.

Use identities to solve the following.

61. Find $\cos \theta$, given $\sin \theta = 3/5$ and θ is in quadrant II.



$$\cos \theta = -\frac{4}{5}$$

Homework: 14.4: 3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75