



14.3

Trigonometric Functions

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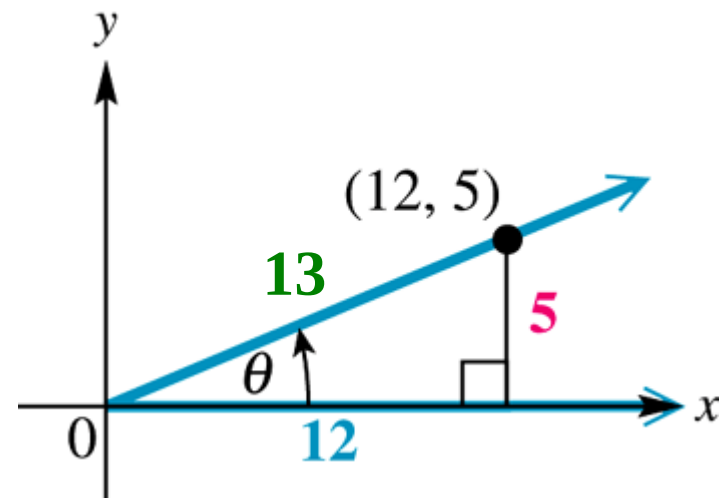
EXAMPLE 1 Finding Function Values of an Angle

The terminal side of an angle θ in standard position passes through the point $(12, 5)$. Find the values of the six trigonometric functions of angle θ .

Answer

$x = 12$ and $y = 5$.

$$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + 12^2} = 13$$



EXAMPLE 1 Finding Function Values of an Angle (continued)

$$\sin \theta = \frac{y}{r} = \frac{5}{13}$$

$$\csc \theta = \frac{r}{y} = \frac{13}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{12}{13}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{12}$$

$$\tan \theta = \frac{y}{x} = \frac{5}{12}$$

$$\cot \theta = \frac{x}{y} = \frac{12}{5}$$

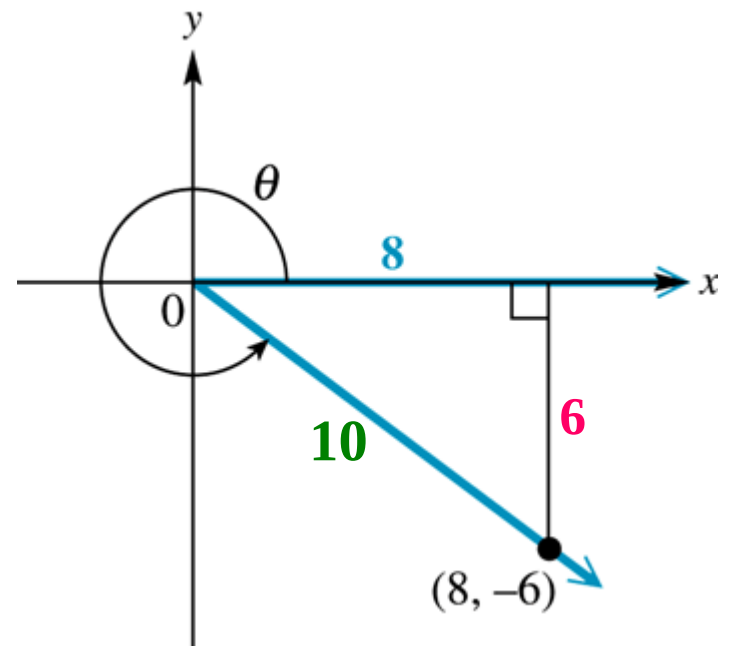
EXAMPLE 2 Finding Function Values of an Angle

The terminal side of an angle θ in standard position passes through the point $(8, -6)$. Find the values of the six trigonometric functions of angle θ .

Answer

$x = 8$ and $y = -6$.

$$r = \sqrt{x^2 + y^2} = \sqrt{8^2 + (-6)^2} = 10$$



EXAMPLE 2

Finding Function Values of an Angle (continued)

$$\sin \theta = \frac{y}{r} = \frac{-6}{10} = -\frac{3}{5}$$

$$\csc \theta = \frac{r}{y} = \frac{10}{-6} = -\frac{5}{3}$$

$$\cos \theta = \frac{x}{r} = \frac{8}{10} = \frac{4}{5}$$

$$\sec \theta = \frac{r}{x} = \frac{10}{8} = \frac{5}{4}$$

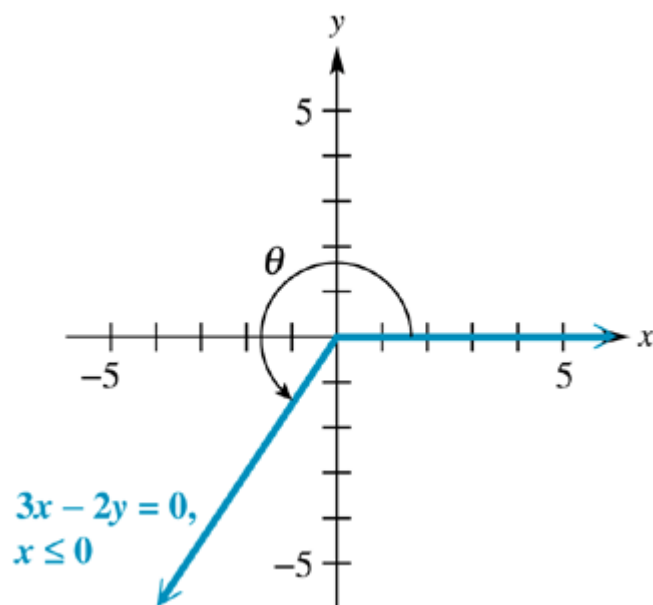
$$\tan \theta = \frac{y}{x} = \frac{-6}{8} = -\frac{3}{4}$$

$$\cot \theta = \frac{x}{y} = \frac{8}{-6} = -\frac{4}{3}$$

EXAMPLE 3 Finding Function Values of an Angle

Find the values of the six trigonometric functions of angle θ in standard position, if the terminal side of θ is defined by $3x - 2y = 0$, $x \leq 0$.

Answer



Since $x \leq 0$, the graph of the line $3x - 2y = 0$ is shown to the left of the y -axis.

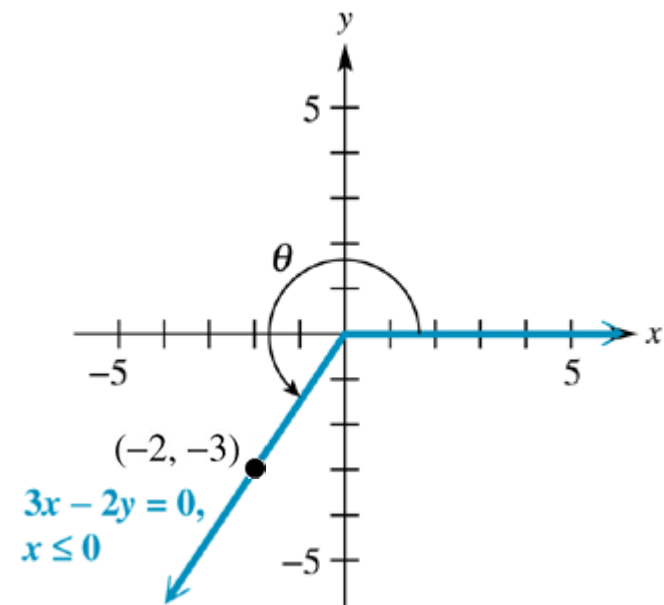
EXAMPLE 3 Finding Function Values of an Angle (continued)

Find a point on the line:

Let $x = -2$. Then $3(-2) - 2y = 0 \Rightarrow y = -3$

A point on the line is $(-2, -3)$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-2)^2 + (-3)^2} = \sqrt{13} \end{aligned}$$



EXAMPLE 3**Finding Function Values of an Angle**
(continued)

$$\sin \theta = \frac{y}{r} = \frac{-3}{\sqrt{13}} = -\frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{-2}{\sqrt{13}} = -\frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{-3}{-2} = \frac{3}{2}$$

$$\cot \theta = \frac{x}{y} = \frac{-2}{-3} = \frac{2}{3}$$

$$\sec \theta = \frac{r}{x} = \frac{\sqrt{13}}{-2} = -\frac{\sqrt{13}}{2}$$

$$\csc \theta = \frac{r}{y} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$

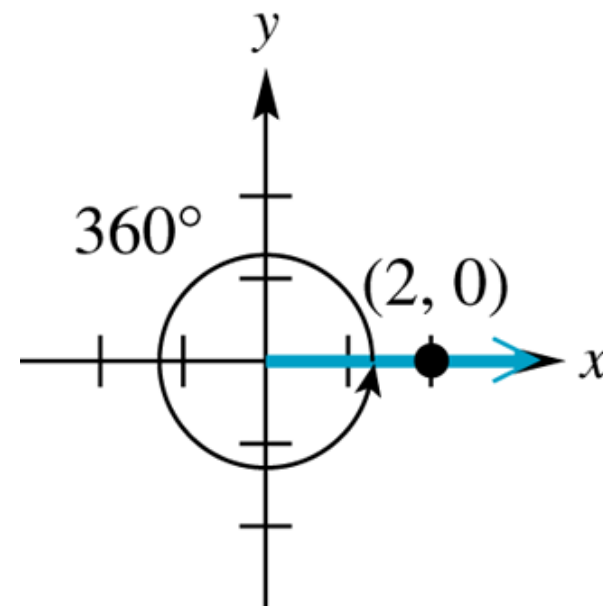
EXAMPLE 4 Finding Function Values of Quadrantal Angles

Find the values of the six trigonometric functions for each angle.

- (a) an angle of 360°
- (b) an angle θ in standard position with terminal side through $(0, -5)$.

Answer

- (a) The terminal side passes through $(2, 0)$. So $x = 2$ and $y = 0$ and $r = 2$.



EXAMPLE 4 Finding Function Values of Quadrantal Angles (continued)

$$\sin \theta = \frac{y}{r} = \frac{0}{2} = 0 \quad \csc \theta = \frac{r}{y} = \frac{2}{0} \Rightarrow \csc \theta \text{ is undefined}$$

$$\cos \theta = \frac{x}{r} = \frac{2}{2} = 1 \quad \sec \theta = \frac{r}{x} = \frac{2}{2} = 1$$

$$\tan \theta = \frac{y}{x} = \frac{0}{2} = 0 \quad \cot \theta = \frac{x}{y} = \frac{5}{0} \Rightarrow \cot \theta \text{ is undefined}$$

EXAMPLE 4 Finding Function Values of Quadrantal Angles (continued)

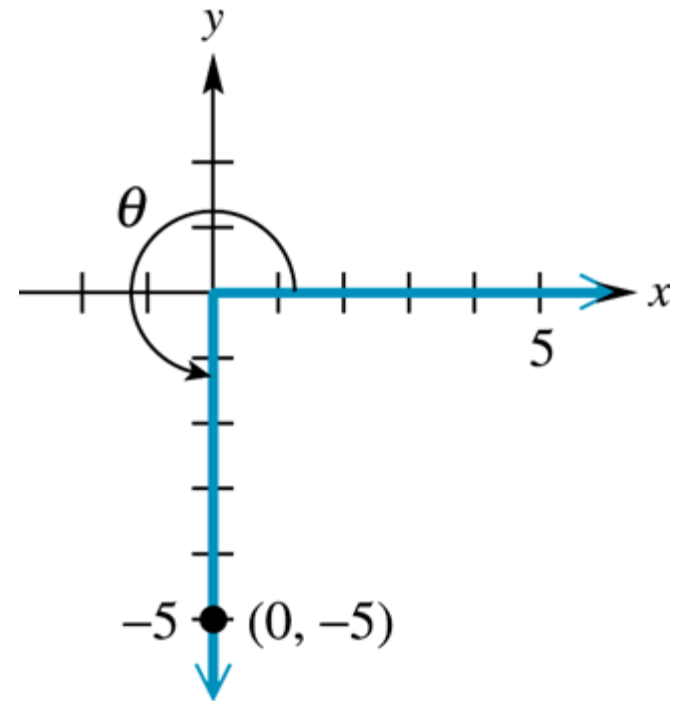
(b) $x = 0$ and $y = -5$ and $r = 5$.

$$\sin \theta = \frac{y}{r} = \frac{-5}{5} = -1$$

$$\csc \theta = \frac{r}{y} = \frac{5}{-5} = -1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{5} = 0$$

$$\sec \theta = \frac{r}{x} = \frac{5}{0} \Rightarrow \sec \theta \text{ is undefined}$$



EXAMPLE 4 Finding Function Values of Quadrantal Angles (continued)

$$\tan \theta = \frac{y}{x} = \frac{-5}{0} \Rightarrow$$

$\tan \theta$ is undefined

$$\cot \theta = \frac{x}{y} = \frac{0}{-5} = 0$$

