Acc Geo

11.1: 5, 6, 9, 10, 15 and 11.2: 3, 11, 15, 16, 18, 20-26

Note Title 2/9/2016

11.1 -- Understanding Area

11.2 -- Areas of Parallelograms & Triangles

11.1 UNDERSTANDING AREA

Objectives

After studying this section, you will be able to

- Understand the concept of area
- Find the areas of rectangles and squares
- Use the basic properties of area

FTC — rate (speed) — area

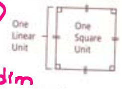
Part One: Introduction

The Concept of Area

When we measure lengths of line segments, we use such standard units as meters, yards, miles, centimeters, and kilometers. These are often called ***linear units*** because they are measures of length. → 1 dim

The standard units of ***area*** are ***square units*** such as square meters, square yards, and square miles. A square meter, for example, is the space enclosed by a square whose sides are each one meter in length.

units² ← 2 dim



Definition The area of a closed region is the number of square units of space within the boundary of the region.

We can estimate the area of a region by determining the approximate number of square units it would take to fill the region.

Estimated Area = 10 sq units

Estimated Area = 18 sq units

Estimated Area = 19 sq units

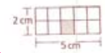
Counting squares, however, is neither the easiest nor the best way to find the area of a region. We will develop formulas for computing the areas of regions bounded by the common geometrical figures. Such regions are usually named by their boundaries, as when we speak of "the area of a rectangle."

The Areas of Rectangles and Squares

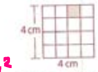
In the figures to the right, there are two ways to find the areas:

1 The numbers of square units can be counted individually.

2 The areas can be computed by multiplying the number of columns (the measure of the base) by the number of rows (the height).



2·5 = Area = 10 sq cm



4^2 = Area = 16 sq cm

The second method suggests the following formula, which may be used to compute areas even when the lengths are fractions or irrational numbers.

Postulate *The area of a rectangle is equal to the product of the base and the height for that base.*

$$A_{rect} = bh$$

where b is the length of the base and h is the height.

In a square, the base and the height are equal, so the following formula is used.

Theorem 99 *The area of a square is equal to the square of a side.*

$$A_{sq} = s^2$$

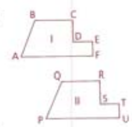
where s is the length of a side.

Basic Properties of Area

We make three basic assumptions about area:

Postulate *Every closed region has an area.*

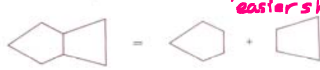
Postulate *If two closed figures are congruent, then their areas are equal.*



If ABCDEF ≅ PQRSTU, then the area of region I = the area of region II.

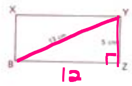
Postulate *If two closed regions intersect only along a common boundary, then the area of their union is equal to the sum of their individual areas.*

You can cut shapes to easier shapes then add



Part Two: Sample Problems

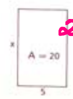
Problem 1 *Find the area of the rectangle.*



5(10+2)

Solution
$A_{rect} = bh$
We need to find base BZ.
$\triangle BZY$ is a right $\triangle$ of the (5, 12, 13) family, so BZ = 12.
$A_{rect} = 12(5) = 60$ sq cm

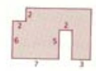
Problem 2 *Given that the area of a rectangle is 20 sq dm and the altitude is 5 dm, find the base.*



20 = 5x

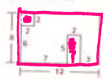
Solution Let x be the number of decimeters in the base.
$A_{rect} = bh$
$20 = x(5)$
$4 = x$
Base = 4 dm

Problem 3 *Find the area of the shaded region.*



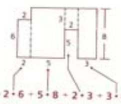
Solution There are two methods of finding the area. One uses subtraction, and the other uses addition.

Method One:



Area of large rectangle = 12 · 8 = 96
Area of square = $2^2 = 4$
Area of small rectangle = 2 · 5 = 10
Shaded area = 96 − 4 − 10 = 82

Method Two ("Divide and Conquer"):

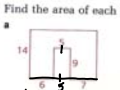


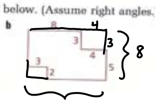
A = 2 · 6 + 5 · 8 + 2 · 3 + 3 · 8
Shaded area = 82

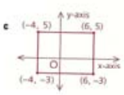
Part Three: Problem Sets

Problem Set A

1 Find the area of each figure below. (Assume right angles.)

a


b


c


14 $\boxed{18}$ − □ 18
14(18) − 5(9)
(10+4)(10+8) − 45
100+80+40+32 −45
252 − 45 = 207 u^2

12
12(8) − 3(2) − (3)(4)
96 − 6 − 12 = 78

11.2 Areas of Parallelograms and Triangles

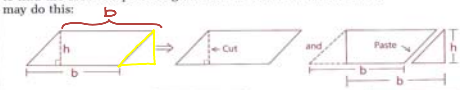
Objectives

After studying this section, you will be able to
- Find the areas of parallelograms
- Find the areas of triangles

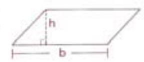
Part One: Introduction

The Area of a Parallelogram

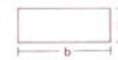
Many areas can be found by a "cut and paste" method. For example, to find the area of a parallelogram with base *b* and altitude *h*, we may do this:



Thus, the area of



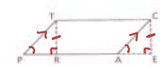
= the area of



Theorem 100 ***The area of a parallelogram is equal to the product of the base and the height.***
$A = bh$
where b is the length of the base and h is the height.

Formal area proofs are often based on the cut-and-paste method. For instance, the key steps in a proof of Theorem 100 could be those below.

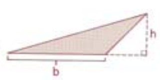
Given: PACT is a ▱.
$\overline{RT}$ is an altitude to $\overline{PA}$.
Prove: A_{PACT} = (PA) (RT)

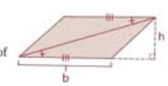


Key Steps:
1 Extend $\overleftrightarrow{PA}$ and draw altitude $\overline{CE}$ to $\overleftrightarrow{PA}$; RECT is a rectangle.
2 $A_{PRT} = A_{AEC}$ because $\triangle PRT \cong \triangle AEC$ by HL.
3 $A_{PACT} = A_{RECT}$, since $A_{CART} + A_{PRT} = A_{CART} + A_{AEC}$.
4 A_{RECT} = (TC)(RT) (Why?)
5 A_{PACT} = (PA)(RT), because PA = TC.

The Area of a Triangle

The area of any triangle can be shown to be one half of the area of a parallelogram with the same base and height.

Area of


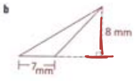
= $\frac{1}{2}$ · area of


Theorem 101 ***The area of a triangle is equal to one-half the product of a base and the height (or altitude) for that base.***
$A_{\triangle} = \frac{1}{2}bh$
where b is the length of the base and h is the altitude.

Part Two: Sample Problems

Problem 1 *Find the area of each triangle.*

a


b


Solution

a $A_{\triangle} = \frac{1}{2}bh$
$= \frac{1}{2}(15)(10)$
$= 75$ sq cm

Note The base of a triangle is not always on the bottom. The 10-cm altitude is the altitude associated with the 15-cm base.

b $A_{\triangle} = \frac{1}{2}bh$
$= \frac{1}{2}(7)(8)$
$= 28$ sq mm

Note The altitude of a triangle is not always inside the triangle.