

## Review

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
IN	$\frac{\text{arc} + \text{arc}}{2}$
ON	$\frac{\text{arc}}{2}$
OUT	$\frac{\text{arc} - \text{arc}}{2}$

## Objectives

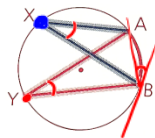
After studying this section, you will be able to

- Recognize congruent inscribed and tangent-chord angles
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

**Theorem 89** If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.

Given: X and Y are inscribed angles intercepting arc AB.

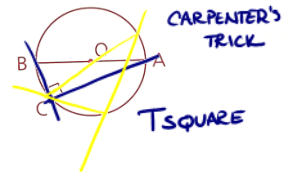
Conclusion:  $\angle X \cong \angle Y$



**Theorem 91** An angle inscribed in a semicircle is a right angle.

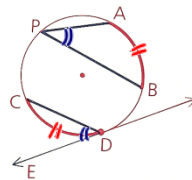
Given:  $\overline{AB}$  is a diameter of  $\odot O$ .

Prove:  $\angle C$  is a right angle.



**Theorem 90** If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.

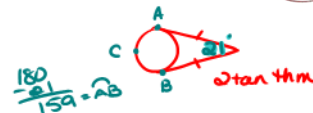
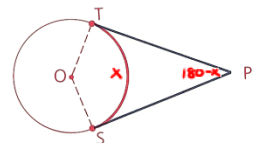
If  $\overleftrightarrow{ED}$  is the tangent at D and  $\widehat{AB} \cong \widehat{CD}$ , we may conclude that  $\angle P \cong \angle CDE$ .



**Theorem 92** The sum of the measures of a tangent-tangent angle and its minor arc is 180.

Given:  $\overline{PT}$  and  $\overline{PS}$  are tangent to circle O.

Prove:  $m\angle P + m\widehat{TS} = 180$

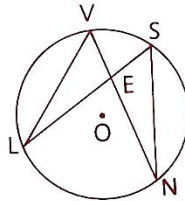


# Classwork

## Problem 1

Given:  $\odot O$

Conclusion:  $\triangle LVE \sim \triangle NSE$ ,  
 $EV \cdot EN = EL \cdot SE$



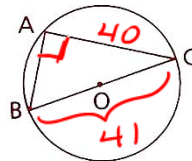
## Proof

- 1  $\odot O$
- 2  $\angle V \cong \angle S$
- 3  $\angle L \cong \angle N$
- 4  $\triangle LVE \sim \triangle NSE$
- 5  $\frac{EV}{SE} = \frac{EL}{EN}$
- 6  $EV \cdot EN = EL \cdot SE$

- 1 GIVEN
- 2 inscribed  $\angle$ s form same arc  $\Rightarrow \cong \angle$ s
- 3 same as 2
- 4  $\triangle A \sim$
- 5  $\sim \triangle \Rightarrow$  corr. sds. prop.
- 6 means extremes product } see ch 7  
cross mult.

## Problem 2

In circle O,  $\overline{BC}$  is a diameter and the radius of the circle is 20.5 mm.  
 Chord  $\overline{AC}$  has a length of 40 mm. Find AB.



## Solution

Since  $\angle A$  is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

$$BC = 20.5(2) = 41$$

$$AB^2 + 40^2 = 41^2$$

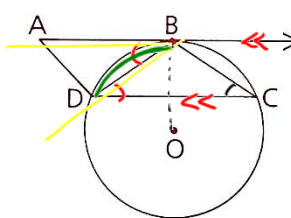
$$AB = \sqrt{41^2 - 40^2}$$

$$(40+1)(40+1) \text{ FOIL}$$

$$1600 + 80 + 1$$

## Problem 3

Given:  $\odot O$  with  $\overleftrightarrow{AB}$  tangent at B,  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$   
 Prove:  $\angle C \cong \angle BDC$



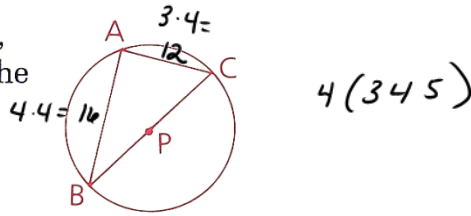
## Proof

- 1  $\overleftrightarrow{AB}$  is tangent to  $\odot O$ .
- 2  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
- 3  $\angle ABD \cong \angle BDC$
- 4  $\angle C \cong \angle ABD$
- 5  $\angle C \cong \angle BDC$

- 1 Given
- 2 Given
- 3  $\parallel \Rightarrow$  alt int  $\angle$ s  $\cong$
- 4 2 inscribed  $\angle$ s form same arc  $\Rightarrow \cong \angle$ s
- 5 transitive

- 3 In  $\odot P$ ,  $\overline{BC}$  is a diameter,  $AC = 12$  mm, and  $BA = 16$  mm. Find the radius of the circle.

$$d = 20, r = 10 \text{ mm}$$

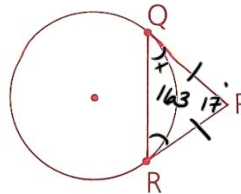


- 4 Given:  $\overline{PQ}$  and  $\overline{PR}$  are tangent segments.  
 $\widehat{QR} = 163^\circ$

Find: a  $\angle P = 180 - 163 = 17^\circ$

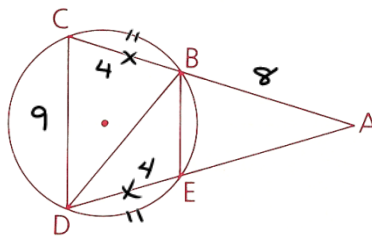
b  $\angle PQR = \frac{163}{2} = 81.5^\circ$

$\hookrightarrow \text{alt } \angle \text{ s } \Delta = 180^\circ \therefore 2x + 17 = 180$   
 $2x = 163$   
 $x = 81.5^\circ$



- 6 Given:  $\widehat{BC} \cong \widehat{ED}$ ,  $AB = 8$ ,  
 $BC = 4$ ,  $CD = 9$

- a Are  $\overline{BE}$  and  $\overline{CD}$  parallel?  
b Find BE.  
c Is  $\triangle ACD$  scalene?



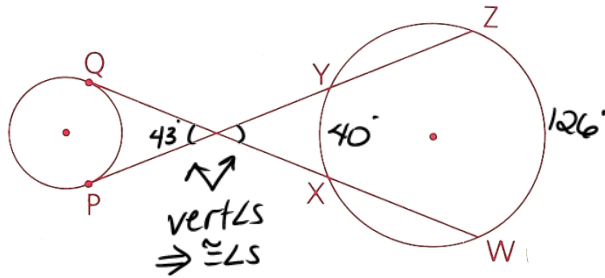
AMDG

- 7 Given:  $\overleftrightarrow{PY}$  and  $\overleftrightarrow{QW}$  are tangents.  
 $\widehat{WZ} = 126^\circ$ ,  $\widehat{XY} = 40^\circ$

Find:  $\widehat{PQ}$

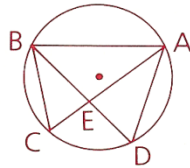
$$\widehat{XY} = \frac{126 - 40}{2} = \frac{86}{2} = 43^\circ$$

$$\widehat{QP} = 180 - 43 = 137^\circ$$



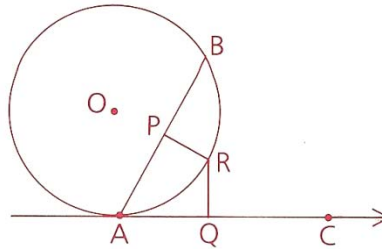
- 18 Given:  $\widehat{BC} \cong \widehat{CD}$

Conclusion:  $\triangle ABC \sim \triangle AED$



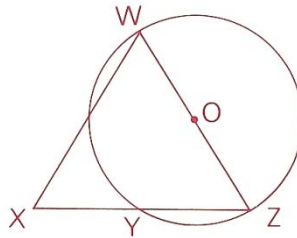
- 19 Given:  $\overleftrightarrow{AC}$  is tangent at A.  $\angle APR$  and  $\angle AQR$  are right  $\angle$ s. R is the midpoint of  $\widehat{AB}$ .

Conclusion:  $\overline{PR} \cong \overline{RQ}$  (Hint: Draw  $\overline{AR}$ .)



- 20 Given:  $\triangle WXZ$  is isosceles, with  $\overline{WX} \cong \overline{WZ}$ .  
 $\overline{WZ}$  is a diameter of  $\odot O$ .

Prove: Y is the midpoint of  $\overline{XZ}$ .  
 (Hint: Draw  $\overline{WY}$ .)

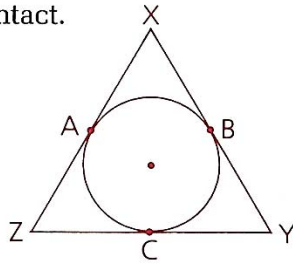


## Homework

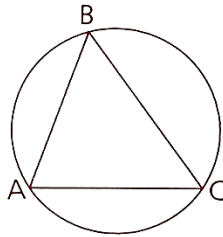
- 5 Given: A, B, and C are points of contact.

$$\widehat{AB} = 145^\circ, \angle Y = 48^\circ$$

Find:  $\angle Z$

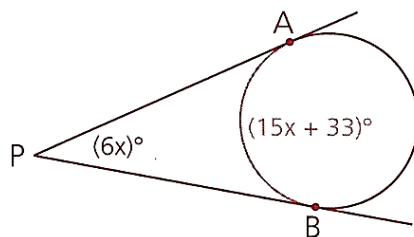


- 8 If  $\triangle ABC$  is inscribed in a circle and  $\widehat{AC} \cong \widehat{AB}$ , tell whether each of the following must be true, could be true, or cannot be true.

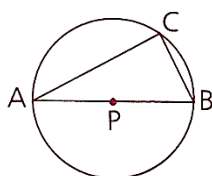


- |  |                                  |
|--|----------------------------------|
| a $\overline{AB} \cong \overline{AC}$  | d $\angle B \cong \angle C$      |
| b $\overline{AC} \cong \overline{BC}$  | e $\angle BAC$ is a right angle. |
| c $\overline{AB}$ and $\overline{AC}$ are equidistant from the center of the circle. | f $\angle ABC$ is a right angle. |

- 9 In the figure shown, find  $m\angle P$ .



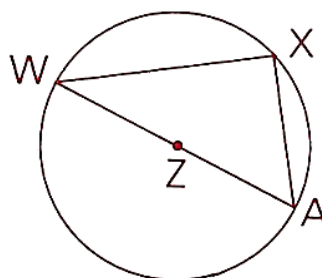
- 10 If  $\overline{AB}$  is a diameter of  $\odot P$ ,  $CB = 1.5$  m, and  $CA = 2$  m, find the radius of  $\odot P$ .



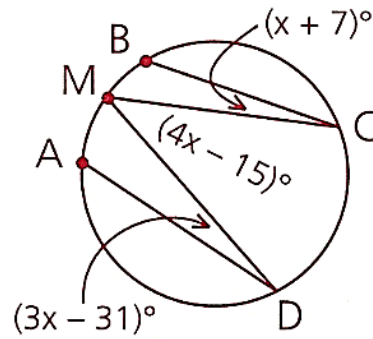
- 11 The radius of  $\odot Z$  is 6 cm and  $\widehat{WX} = 120^\circ$ .

Find: **a** AX

**b** The perimeter of  $\triangle WAX$

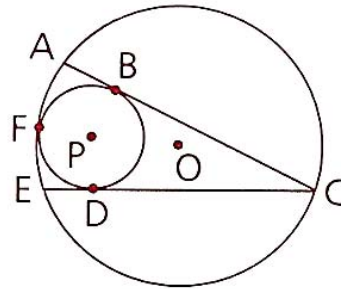


**12** M is the midpoint of  $\widehat{AB}$ . Find  $m\widehat{CD}$ .



**15** Quadrilateral ABCD is inscribed in circle O.  $AB = 12$ ,  $BC = 16$ ,  $CD = 10$ , and  $\angle ABC$  is a right angle. Find the measure of  $\widehat{AD}$  in simplified radical form.

**16** Circles O and P are tangent at F.  $\overline{AC}$  and  $\overline{CE}$  are tangent to  $\odot P$  at B and D. If  $\widehat{DFB} = 223^\circ$ , find  $\widehat{AE}$ .



**17** Given:  $\angle S = 88^\circ$ ,  $\widehat{QT} = 104^\circ$ ,  $\widehat{ST} = 94^\circ$ ,  
tangent  $\overline{PQ}$  AMDG



NAME \_\_\_\_\_

Acc Geo - \_\_\_\_\_

10-6: More Angle-Arc Theorems

Ms. Kresovic

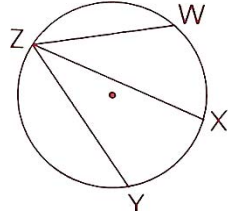
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# Classwork

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Complete the problems on your own. Compare work with a partner. Discuss any differences, and revise. Hand in when completed (before the period ends).

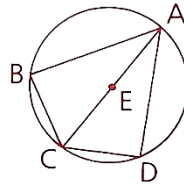
- 1 Given: X is the midpt. of  $\widehat{WY}$ .  
Prove:  $\overrightarrow{ZX}$  bisects  $\angle WZY$ .



(4 points, 1 pt per reason)

S	R
1. X is the midpoint of $\widehat{WY}$	1.
2. $\widehat{WX} \cong \widehat{XY}$	2.
3. $\angle WZX \cong \angle XZY$	3.
4. $\overrightarrow{ZX}$ bisects $\angle WZY$	4.

- 2 Given:  $\odot E$  with diameter  $\overline{AC}$ ,  $\overline{BC} \cong \overline{CD}$   
Conclusion:  $\triangle ABC \cong \triangle ADC$



(5 points, wholistic)

- 13 A rectangle with dimensions 18 by 24 is inscribed in a circle.  
Find the radius of the circle.

(3 pts)