AMDG

Name StuDent | Ms. Kresovic

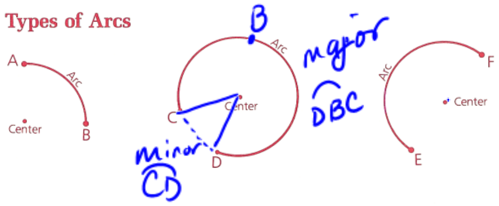
Acc Geo - 2 | 10-3: Arcs of a Circle | 21 January 2016

Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc (vs lengths)
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

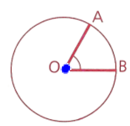
Types of Arcs



Definition An ***arc*** consists of two points on a circle and all points on the circle needed to connect the points by a single path.

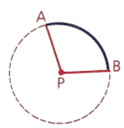
Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A ***central angle*** is an angle whose vertex is at the center of a circle.



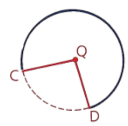
Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

Definition A ***minor arc*** is an arc whose points are on or between the sides of a central angle.



Central angle APB determines minor arc AB.

Definition A ***major arc*** is an arc whose points are on or outside of a central angle.

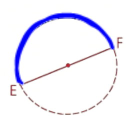


Central angle CQD determines major arc CD.

Definition A ***semicircle*** is an arc whose endpoints are the endpoints of a diameter.

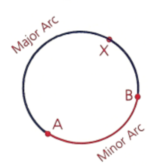
$m\frown = 180°$

length $= \frac{1}{2}C$

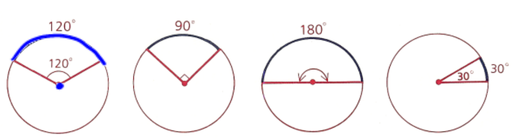


Arc EF is a semicircle.

The symbol ⌒ is used to label arcs. The minor arc joining A and B is called $\overset{\frown}{AB}$. The major arc joining A and B is called $\overset{\frown}{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)



The Measure of an Arc

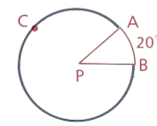


Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

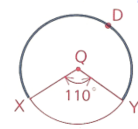
Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

Example

a Given: $m\overset{\frown}{AB} = 20$
Find: $m\overset{\frown}{ACB} = 360 - 20 = 340°$

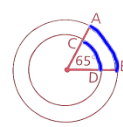


b Given: $m\angle XQY = 110$
Find: $m\overset{\frown}{XDY} = 360 - 110 = 250°$



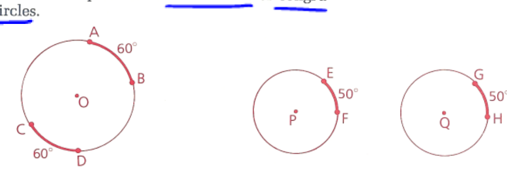
Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\overset{\frown}{AB} = 65$ and $m\overset{\frown}{CD} = 65$, but $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent? If $r_1 = r_2$



$m\overset{\frown}{CD} = m\overset{\frown}{AB} = 65°$

Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.



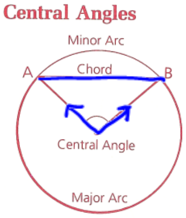
We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

If ⊙P ≅ ⊙Q, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

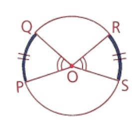
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.

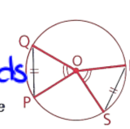


Theorem 79 *If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.* ≅ cent∠s ⇒ ≅ ⌒s



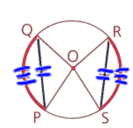
Theorem 80 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* ≅ ⌒s ⇒ ≅ cent∠s

Theorem 81 *If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* ≅ cent∠s ⇒ ≅ chds



Theorem 82 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* ≅ chds ⇒ ≅ cent∠s

Theorem 83 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* ≅ ⌒s ⇒ ≅ chds



Theorem 84 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.* ≅ chds ⇒ ≅ ⌒s

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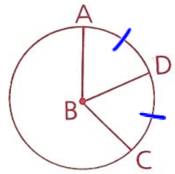
Problem 1　*Given:* $\odot$B;
D is the midpt. of $\widehat{AC}$.
Conclusion: $\overrightarrow{BD}$ *bisects* $\angle ABC$.

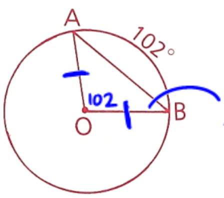


Proof

1 $\odot$B; D is the midpt. of $\widehat{AC}$.	1 Given
2 $\widehat{AD} \cong \widehat{DC}$	2 mdpt $\Rightarrow \cong \frown$s
3 $\angle ABD \cong \angle DBC$	3 $\cong \frown$s $\Rightarrow \cong$ cent $\angle$s
4 $\overrightarrow{BD}$ bisects $\angle ABC$.	4 $\cong \angle$s $\Rightarrow$ bis.

Problem 2　*If* $m\widehat{AB} = 102$ *in* $\odot$O, *find* $m\angle A$ *and* $m\angle B$ *in* $\triangle AOB$.

Solution　$\widehat{AB} = 102°$



$180 - 102 = 78°$

$\frac{78}{2} = 39°$

Problem 3　**a** *What fractional part of a circle is an arc of 36°? Of 200°?*
b *Find the measure of an arc that is* $\frac{7}{12}$ *of its circle.*

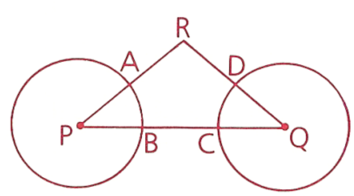
Solution　There are 360° in a whole $\odot$.

a $\frac{36°}{360°} = \frac{1}{10}$

& $\frac{200°}{360°} = \frac{5}{9}$

b

Problem 4　*Given:* Ⓢ P *and* Q,
$\angle P \cong \angle Q$, $\overline{AR} \cong \overline{RD}$
Prove: $\widehat{AB} \cong \widehat{CD}$ (Hint: First prove that $\odot P \cong \odot Q$.)



Proof

1 Ⓢ P and Q	1 Given
2 $\angle P \cong \angle Q$	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3
4 $\overline{AR} \cong \overline{RD}$	4
5 $\overline{AP} \cong \overline{DQ}$	5
6 $\odot P \cong \odot Q$	6
7 $\widehat{AB} \cong \widehat{CD}$	7

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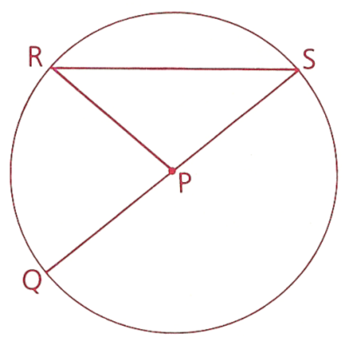
1 Match each item in the left column with the correct term in the right column.

a $\overset{\frown}{QRS}$ 6	**1** Radius	
b $\overline{QS}$ 2	**2** Diameter	
c $\overset{\frown}{RQS}$ 5	**3** Chord	
d $\overset{\frown}{RS}$ 4	**4** Minor arc	
e $\overline{RS}$ 3	**5** Major arc	
f $\angle RPQ$ 7	**6** Semicircle	
g $\overline{PS}$ 1	**7** Central angle	



2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

a Name a major arc of the smaller circle. $\overset{\frown}{PRQ}$ or $\overset{\frown}{QPR}$

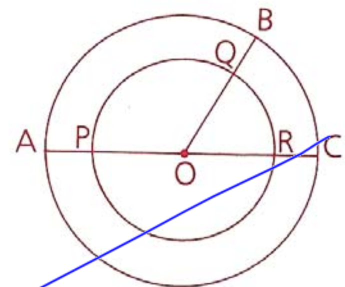
b Name a minor arc of the larger circle. $\overset{\frown}{AB}$ or $\overset{\frown}{CB}$

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$? = 180°

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$? No

same m
but diff lengths

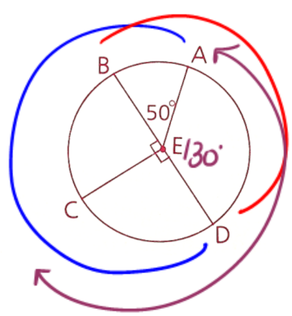


3 In circle E, find each of the following.

a $m\overset{\frown}{BC}$ = 90° **c** $m\overset{\frown}{ACD}$ = 230° **e** $m\overset{\frown}{ADC}$ 220°

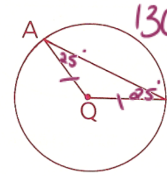
b $m\overset{\frown}{AD}$ = 130° **d** $m\overset{\frown}{BAD}$ = 180°

$m\overset{\frown}{ACD}$ = 180 + 50 = 230°



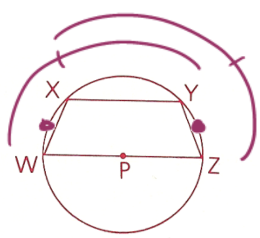
4 Given: $\odot Q$, $\angle A = 25°$
Find: $m\overset{\frown}{AB}$

130°



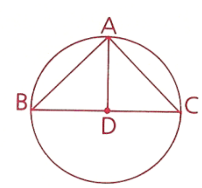
180 − 50 = 130°

5 Given: $\odot P$,
$\overset{\frown}{WY} \cong \overset{\frown}{XZ}$
Conclusion: $\overline{WX} \cong \overline{YZ}$



1. $\overset{\frown}{WY} \cong \overset{\frown}{XZ}$ — 1. Given
2. $\overset{\frown}{XY} \cong \overset{\frown}{XY}$ — 2. Ref → optional
3. $\overset{\frown}{WX} \cong \overset{\frown}{YZ}$ — 3. Subtract
4. $\overline{WX} \cong \overline{YZ}$ — 4. ≅ arcs ⇒ ≅ chds

6 Given: $\odot D$, $\angle B \cong \angle C$
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$



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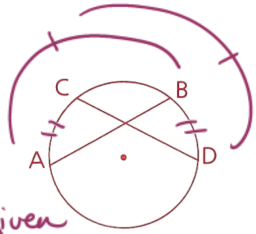
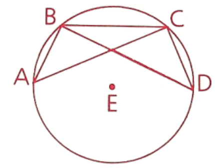
7 Given: $\overline{AB} \cong \overline{CD}$
Conclusion: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$



1. $\overline{AB} \cong \overline{CD}$ 1. given
2. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ 2. $\cong$ chds $\Rightarrow \cong \frown$s
3. $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ 3. subtract

8 Given: $\odot E$,
$\overline{AB} \cong \overline{CD}$
Prove: $\overline{BD} \cong \overline{AC}$



9 What fractional part of a circle is an arc that measures

a 8
b 240 $\frac{240}{360} = \frac{24}{36} = \frac{2}{3}$
c 144
d 315

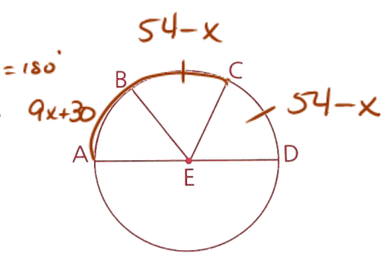
10 Find the measure of an arc that is

a $\frac{3}{5}$ of its circle
$\frac{3}{\cancel{5}} \cdot \cancel{360}^{72} = 216°$
$\frac{3}{5} = \frac{x}{360} \rightarrow \cancel{5}x = \frac{3(360)}{5}$

b $\frac{5}{9}$ of its circle

c 70% of its circle
$\frac{7}{10} = \frac{x}{360}$
$\begin{array}{r} {}^{4}36 \\ 7 \\ \hline 252 \end{array}$
$\boxed{252°}$

11 Given: $\overline{AD}$ is a diameter of $\odot E$. $= 180°$
C is the midpoint of $\overset{\frown}{BD}$.
$m\overset{\frown}{AB} = 9x + 30$,
$m\overset{\frown}{CD} = 54 - x$
Find: $m\angle AEC$



$7x + 138 = 180$
$7x = 42$
$x = 6$

$\overset{\frown}{AB} = 9(6) + 30 = 84$
$\overset{\frown}{BC} = 54 - 6 = 48$
$\boxed{132°}$

12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

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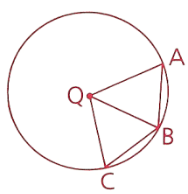
13 Find the length of each arc described. (The length is a fractional part of the circumference.)

a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

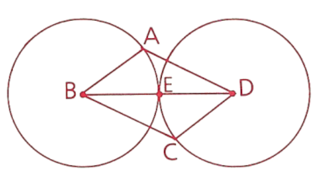
b An arc that has a measure of 270 and is part of a circle with radius 12

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\widehat{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.

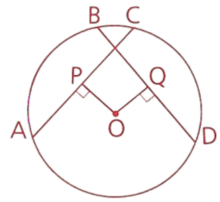
15 Given: $\odot Q$;
B is the midpt. of $\widehat{AC}$.
Conclusion: $\angle A \cong \angle C$



16 Given: $\odot B \cong \odot D$,
$\widehat{AE} \cong \widehat{CE}$
Prove: ABCD is a ▱.

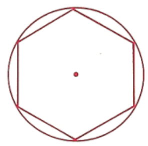


17 Given: $\odot O$,
$\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
$\overline{OP} \cong \overline{OQ}$
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$



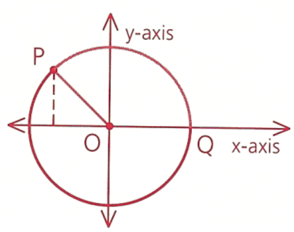
18 A polygon is *inscribed in* a $\odot$ if all its vertices lie on the $\odot$. Find the measure of the arc cut off by a side of each of the following inscribed polygons.

a A regular hexagon
b A regular pentagon
c A regular octagon



19 Point P is located at $(-5, 5)$.

a Find the radius of $\odot O$.
b Find the measure of $\overset{\frown}{PQ}$.



20 Given: $\odot P \cong \odot Q$,
$\overline{BC} \cong \overline{CD}$
Conclusion: $\angle A \cong \angle E$