Name   Ms. Kresovic

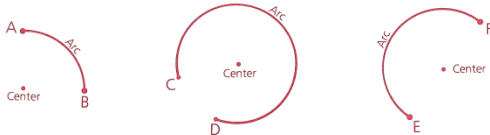Acc Geo -   10-3: Arcs of a Circle   21 January 2016

Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles
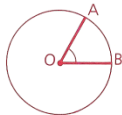
Types of Arcs

| Definition	An **arc** consists of two points on a circle and all points on the circle needed to connect the points by a single path.

Definition	The center of an arc is the center of the circle of which the arc is a part.

Definition	A **central angle** is an angle whose vertex is at the center of a circle.

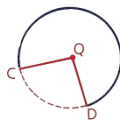Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

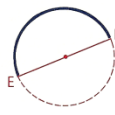| Definition	A **minor arc** is an arc whose points are on or between the sides of a central angle.
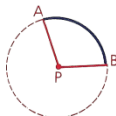
Central angle APB determines minor arc AB.

| Definition	A **major arc** is an arc whose points are on or outside of a central angle.

Central angle CQD determines major arc CD.

Definition	A **semicircle** is an arc whose endpoints are the endpoints of a diameter.

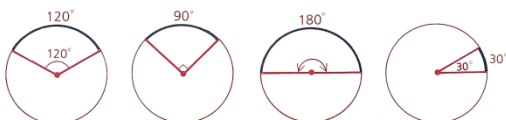Arc EF is a semicircle.

   The symbol ⌢ is used to label arcs. The minor arc joining A and B is called $\overarc{AB}$. The major arc joining A and B is called $\overarc{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)
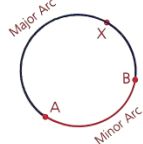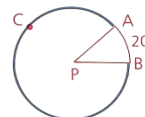
The Measure of an Arc

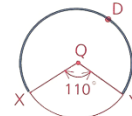| Definition	The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

Definition	The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

Example

a *Given:* m$\overarc{AB}$ = 20
   Find: m$\overarc{ACB}$

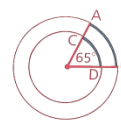**b** *Given:* m∠XQY = 110
   Find: m$\overarc{XDY}$

Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, m$\overarc{AB}$ = 65 and m$\overarc{CD}$ = 65, but $\overarc{AB}$ and $\overarc{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent?

| Definition	Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.
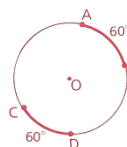
We may conclude that $\overarc{AB} \cong \overarc{CD}$.   If ⊙P ≅ ⊙Q, we may conclude that $\overarc{EF} \cong \overarc{GH}$.
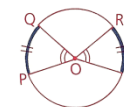
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

   You can readily prove the following theorems.
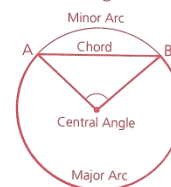
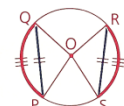| Theorem 79	*If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.*
Theorem 80	*If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.*
Theorem 81	*If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.*
Theorem 82	*If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.*
Theorem 83	*If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.*
Theorem 84	*If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.*

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Problem 1   Given: ⊙B;
                          D is the midpt. of $\overset{\frown}{AC}$.
                  Conclusion: $\overrightarrow{BD}$ bisects ∠ABC.
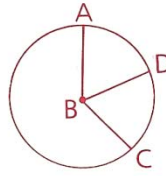
Proof

1 ⊙B; D is the midpt. of $\overset{\frown}{AC}$.	1 Given
2 $\overset{\frown}{AD} \cong \overset{\frown}{DC}$	2
3 ∠ABD ≅ ∠DBC	3
4 $\overrightarrow{BD}$ bisects ∠ABC.	4

Problem 2   If m$\overset{\frown}{AB}$ = 102 in ⊙O, find m∠A and m∠B in △AOB.

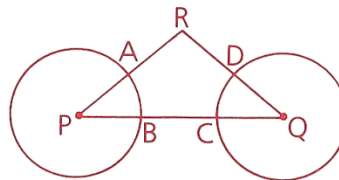**Solution** $\overset{\frown}{AB}$ = 102°
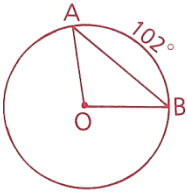
Problem 3   **a** What fractional part of a circle is an arc of 36°? Of 200°?
                  b Find the measure of an arc that is $\frac{7}{12}$ of its circle.

Solution   There are 360° in a whole ⊙.

                  a   **b**

Problem 4   Given: ⊙ P and Q,
                          ∠P ≅ ∠Q, $\overline{AR} \cong \overline{RD}$
                  Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ (Hint: First prove
                          that ⊙P ≅ ⊙Q.)

Proof

1 ⊙ P and Q	1 Given
2 ∠P ≅ ∠Q	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3
4 $\overline{AR} \cong \overline{RD}$	4
5 $\overline{AP} \cong \overline{DQ}$	5
6 ⊙P ≅ ⊙Q	6
7 $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	7

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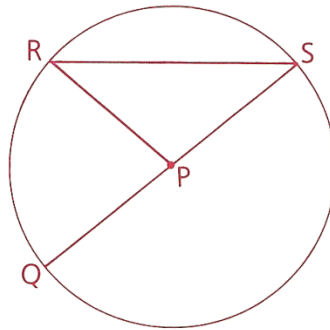Acc Geo -   10-3: Arcs of a Circle   21 January 2016

1 Match each item in the left column with the correct term in the right column.

   a $\overset{\frown}{QRS}$ **1** Radius

   b $\overline{QS}$ **2** Diameter

   c $\overset{\frown}{RQS}$ **3** Chord

   d $\overset{\frown}{RS}$ **4** Minor arc

   e $\overline{RS}$ **5** Major arc

   f $\angle RPQ$ **6** Semicircle

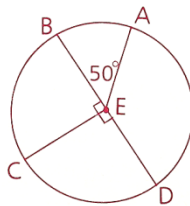   **g** $\overline{PS}$ **7** Central angle

2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

   a Name a major arc of the smaller circle.

   b Name a minor arc of the larger circle.

   c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$?

   d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

   e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?
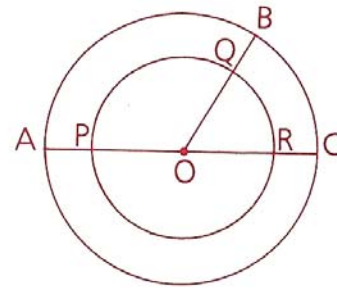
3 In circle E, find each of the following.

   a $m\overset{\frown}{BC}$ **c** $m\overset{\frown}{ACD}$ **e** $m\overset{\frown}{ADC}$

   b $m\overset{\frown}{AD}$ **d** $m\overset{\frown}{BAD}$

4 Given: $\odot Q$, $\angle A = 25°$
Find: $m\overset{\frown}{AB}$
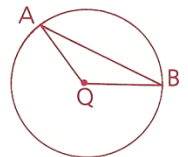
5 Given: $\odot P$,
      $\overset{\frown}{WY} \cong \overset{\frown}{XZ}$
Conclusion: $\overline{WX} \cong \overline{YZ}$
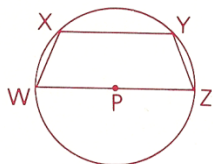
6 Given: $\odot D$, $\angle B \cong \angle C$
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$
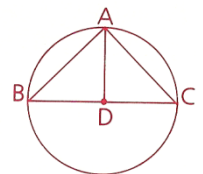
7 Given: $\overline{AB} \cong \overline{CD}$

 Conclusion: $\widehat{AC} \cong \widehat{BD}$
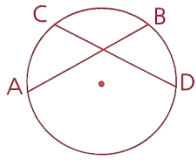
8 Given: $\odot E$,

   $\overline{AB} \cong \overline{CD}$

 Prove: $\overline{BD} \cong \overline{AC}$
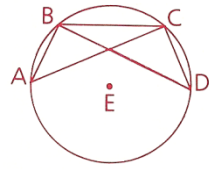
9 What fractional part of a circle is an arc that measures

 a 8   **c** 144

 b 240   **d** 315

10 Find the measure of an arc that is

 a $\frac{3}{5}$ of its circle   **b** $\frac{5}{9}$ of its circle   **c** 70% of its circle
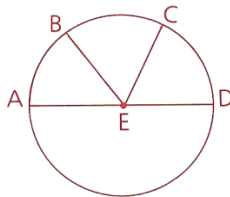
11 Given: $\overline{AD}$ is a diameter of $\odot E$.

   C is the midpoint of $\widehat{BD}$.

   $m\widehat{AB} = 9x + 30$,

   $m\widehat{CD} = 54 - x$

 Find: $m\angle AEC$

12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

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13 Find the length of each arc described. (The length is a fractional part of the circumference.)

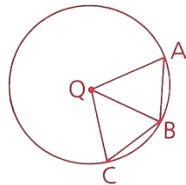**a** An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

b An arc that has a measure of 270 and is part of a circle with radius 12

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\overset{\frown}{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.
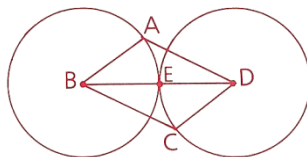
15 Given: $\odot Q$;
    B is the midpt. of $\overset{\frown}{AC}$.
Conclusion: $\angle A \cong \angle C$

16 Given: $\odot B \cong \odot D$,
    $\overset{\frown}{AE} \cong \overset{\frown}{CE}$
Prove: ABCD is a $\square$.

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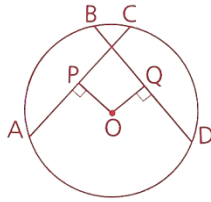
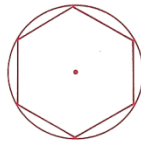Acc Geo -   10-3: Arcs of a Circle   21 January 2016

17 Given: $\odot O$,

$\overline{OP} \perp \overline{AC},\ \overline{OQ} \perp \overline{BD},$

$\overline{OP} \cong \overline{OQ}$

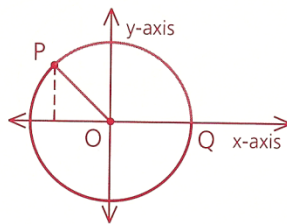Conclusion: $\overarc{AB} \cong \overarc{CD}$

18 A polygon is *inscribed in* a $\odot$ if all its vertices lie on the $\odot$. Find the measure of the arc cut off by a side of each of the following inscribed polygons.

a A regular hexagon

b A regular pentagon

c A regular octagon

19 Point P is located at $(-5, 5)$.

a Find the radius of $\odot O$.

b Find the measure of $\overarc{PQ}$.

20 Given: $\odot P \cong \odot Q,$

$\overline{BC} \cong \overline{CD}$

Conclusion: $\angle A \cong \angle E$