AMDG

Name Ms. Kresovic

Acc Alg 2 – 2 16: Radian Measure and Circular Functions Date

What is a function?

What's *x?*

What's *y?*

16.1: Radian Measure

What is a radian?

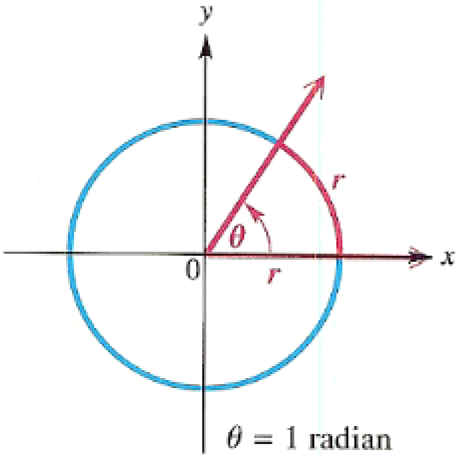


FIGURE 1

Radian

An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of **1 radian.**

$360° = 2\pi$ radians.

Converting Between Degrees and Radians

1. Multiply a degree measure by $\frac{\pi}{180}$ radian and simplify to convert to radians.
2. Multiply a radian measure by $\frac{180°}{\pi}$ and simplify to convert to degrees.

EXAMPLE 1 Converting Degrees to Radians

Convert each degree measure to radians.

(a) 45° **(b)** −270° **(c)** 249.8°

EXAMPLE 2 Converting Radians to Degrees

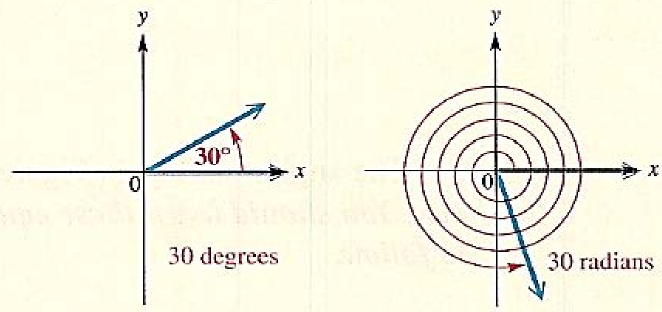
Convert each radian measure to degrees.

(a) $\frac{9\pi}{4}$ **(b)** $-\frac{5\pi}{6}$ **(c)** 4.25

Agreement on Angle Measurement Units

If no unit of angle measure is specified, then the angle is understood to be measured in radians.

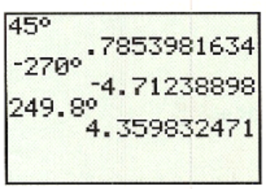
For example, Figure 2(a) shows an angle of 30°, while Figure 2(b) shows an angle of 30 (which means 30 radians).



Note the difference between an angle of 30 *degrees* and an angle of 30 *radians.*

(a) **(b)**

FIGURE 2



A TI-83/84 Plus calculator can convert directly between degrees and radians. This radian mode screen shows the conversions for Example 1. Verify that the first two results are *approximations* for the *exact* values of $\frac{\pi}{4}$ and $-\frac{3\pi}{2}$.

Therefore, you will not be allowed to use the graphing calculator on some assessment problems.

The following table and Figure 3 give some equivalent angle measures in degrees and radians. Keep in mind that **180° = π radians.**

Degrees	Radians		Degrees	Radians	
	Exact	Approximate		Exact	Approximate
0°	0	0	90°	$\frac{\pi}{2}$	1.57
30°	$\frac{\pi}{6}$	0.52	180°	π	3.14
45°	$\frac{\pi}{4}$	0.79	270°	$\frac{3\pi}{2}$	4.71
60°	$\frac{\pi}{3}$	1.05	360°	2π	6.28

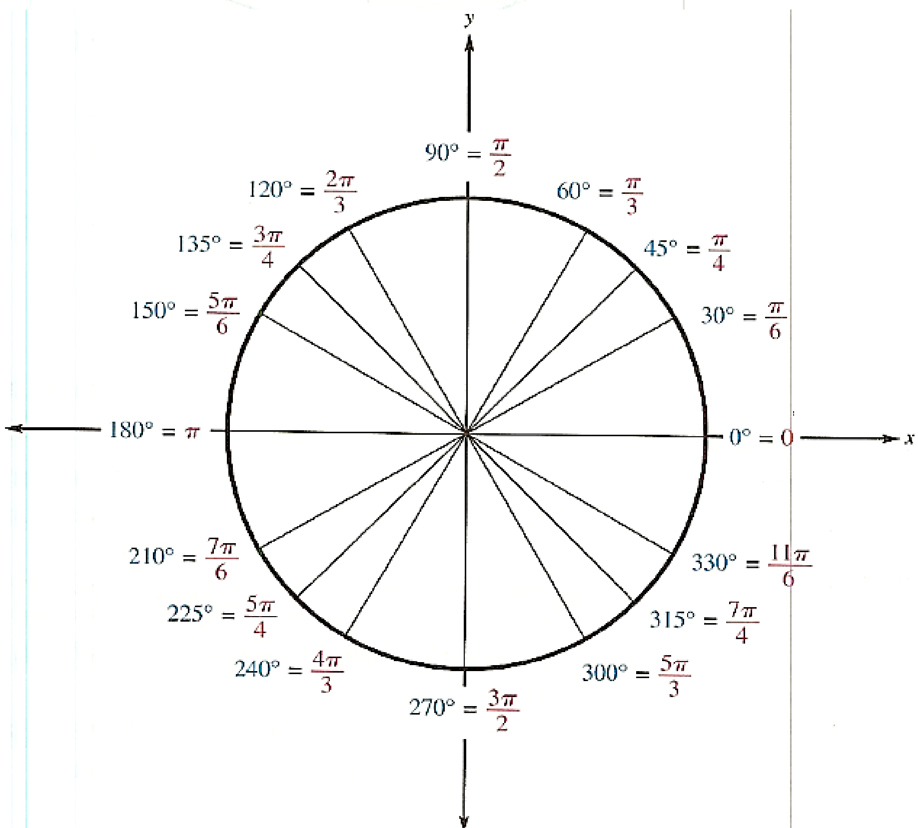


FIGURE 3

The angles marked in Figure 3 are extremely important in the study of trigonometry. ***You should learn these equivalences, as they will appear often in the chapters to follow.***

EXAMPLE 3 Finding Function Values of Angles in Radian Measure

Find each function value.

(a) $\tan \frac{2\pi}{3}$ **(b)** $\sin \frac{3\pi}{2}$ **(c)** $\cos\left(-\frac{4\pi}{3}\right)$

16.2: Applications

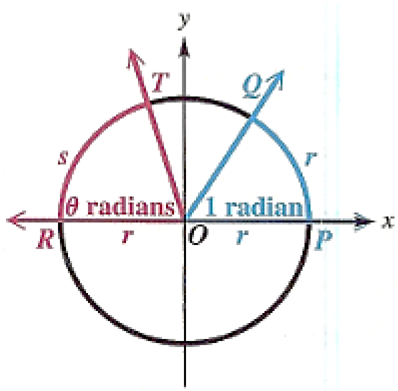


FIGURE 4

$$\frac{s}{r} = \frac{\theta}{1}.$$

Multiplying both sides by r gives the following result.

Arc Length

The length s of the arc intercepted on a circle of radius r by a central angle of measure θ radians is given by the product of the radius and the radian measure of the angle, or

$$s = r\theta, \qquad \theta \text{ in radians.}$$

CAUTION *Avoid the common error of applying this formula with θ in degree mode. When applying the formula $s = r\theta$, the value of θ MUST be expressed in radians.*

EXAMPLE 1 Finding Arc Length Using $s = r\theta$

A circle has radius 18.20 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

(a) $\frac{3\pi}{8}$ radians **(b)** 144°

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EXAMPLE 2 Using Latitudes to Find the Distance between Two Cities

Latitude gives the measure of a central angle with vertex at Earth's center whose initial side goes through the equator and whose terminal side goes through the given location. Reno, Nevada, is approximately due north of Los Angeles. The latitude of Reno is 40° N, while that of Los Angeles is 34° N. (The N in 34° N means *north* of the equator.) The radius of Earth is 6400 km. Find the north—south distance between the two cities.

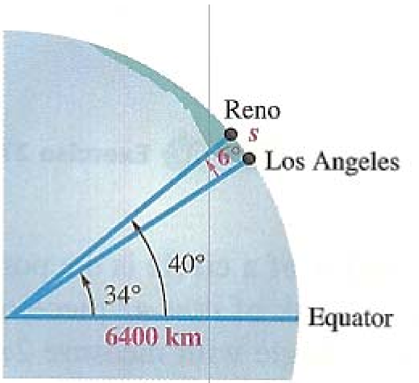


FIGURE 6

EXAMPLE 3 Finding a Length Using $s = r\theta$

A rope is being wound around a drum with radius 0.8725 ft. (See Figure 7.) How much rope will be wound around the drum if the drum is rotated through an angle of 39.72°?

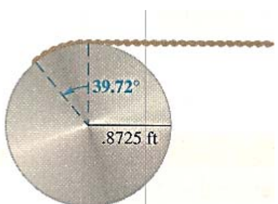


FIGURE 7

EXAMPLE 4 Finding an Angle Measure Using $s = r\theta$

Two gears are adjusted so that the smaller gear drives the larger one, as shown in Figure 8. If the smaller gear rotates through an angle of 225°, through how many degrees will the larger gear rotate?

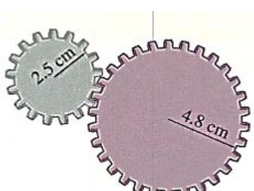
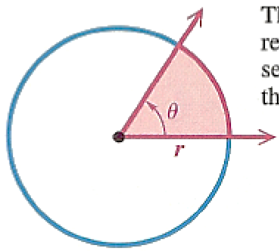


FIGURE 8



The shaded region is a sector of the circle.

FIGURE 9

$$\text{area of the sector} = \frac{\theta}{2\pi}(\pi r^2) = \frac{1}{2}r^2\theta, \qquad \theta \text{ in radians.}$$

This discussion is summarized as follows.

Area of a Sector

The area A of a sector of a circle of radius r and central angle θ is given by

$$A = \frac{1}{2}r^2\theta, \qquad \theta \text{ in radians.}$$

CAUTION As in the formula for arc length, ***the value of θ must be in radians when using this formula for the area of a sector.***

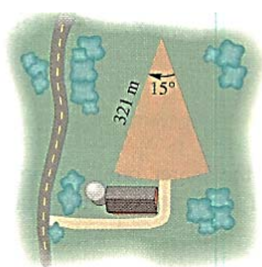


FIGURE 10

EXAMPLE 5 Finding the Area of a Sector-Shaped Field

Find the area of the sector-shaped field shown in Figure 10.

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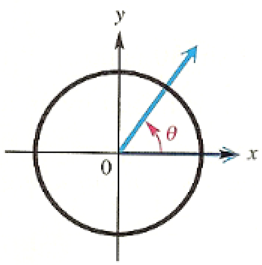
16.1 Exercises

Concept Check *In Exercises 1–6, each angle θ is an integer when measured in radians. Give the radian measure of the angle.*

1.



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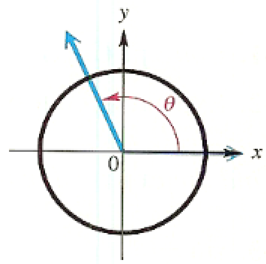

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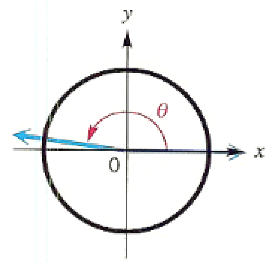
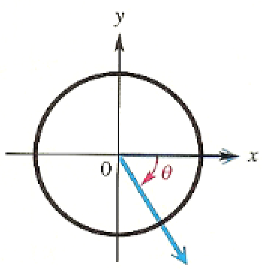
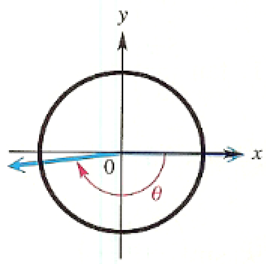


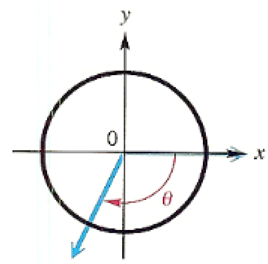
4.



5.



6.



Convert each degree measure to radians. Leave answers as multiples of π. See Examples 1(a) and 1(b).

7. $60°$ **8.** $30°$ **9.** $90°$ **10.** $120°$

11. $150°$ **12.** $270°$ **13.** $-300°$ **14.** $-315°$

15. $450°$ **16.** $480°$ **17.** $1800°$ **18.** $-3600°$

Give a short explanation in Exercises 19–24.

19. In your own words, explain how to convert degree measure to radian measure.

20. In your own words, explain how to convert radian measure to degree measure.

21. In your own words, explain the meaning of radian measure.

22. Explain the difference between degree measure and radian measure.

23. Use an example to show that you can convert from radian measure to degree measure by multiplying by $\frac{180°}{\pi}$.

24. Explain why an angle of radian measure t in standard position intercepts an arc of length t on a circle of radius 1.

Convert each radian measure to degrees. See Examples 2(a) and 2(b).

25. $\frac{\pi}{3}$ **26.** $\frac{8\pi}{3}$ **27.** $\frac{7\pi}{4}$ **28.** $\frac{2\pi}{3}$

29. $\frac{11\pi}{6}$ **30.** $\frac{15\pi}{4}$ **31.** $-\frac{\pi}{6}$ **32.** $-\frac{8\pi}{5}$

33. $\frac{7\pi}{10}$ **34.** $\frac{11\pi}{15}$ **35.** $-\frac{4\pi}{15}$ **36.** $-\frac{7\pi}{20}$

37. $\frac{17\pi}{20}$ **38.** $\frac{11\pi}{30}$ **39.** -5π **40.** 15π

Convert each degree measure to radians. See Example 1(c).

41. 39° **42.** 74° **43.** 42.5° **44.** 264.9°

45. 139° 10′ **46.** 174° 50′ **47.** 64.29° **48.** 85.04°

49. 56° 25′ **50.** 122° 37′ **51.** 47.6925° **52.** 23.0143°

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Convert each radian measure to degrees. Write answers to the nearest minute. See Example 2(c).

53. 2 **54.** 5 **55.** 1.74 **56.** 3.06

57. 0.3417 **58.** 9.84763 **59.** -5.01095 **60.** -3.47189

61. *Concept Check* The value of sin 30 is not $\frac{1}{2}$. Why is this true?

62. Explain in your own words what is meant by an angle of one radian.

Find the exact value of each expression without using a calculator. See Example 3.

63. $\sin \frac{\pi}{3}$ **64.** $\cos \frac{\pi}{6}$ **65.** $\tan \frac{\pi}{4}$ **66.** $\cot \frac{\pi}{3}$

67. $\sec \frac{\pi}{6}$ **68.** $\csc \frac{\pi}{4}$ **69.** $\sin \frac{\pi}{2}$ **70.** $\csc \frac{\pi}{2}$

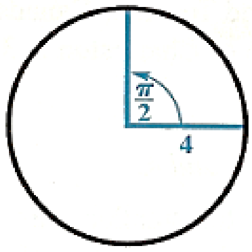
71. $\tan \frac{5\pi}{3}$ **72.** $\cot \frac{2\pi}{3}$ **73.** $\sin \frac{5\pi}{6}$ **74.** $\tan \frac{5\pi}{6}$

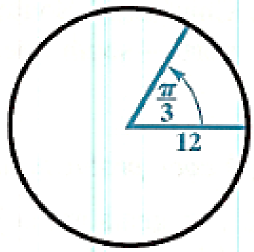
75. $\cos 3\pi$ **76.** $\sec \pi$ **77.** $\sin\left(-\frac{8\pi}{3}\right)$ **78.** $\cot\left(-\frac{2\pi}{3}\right)$

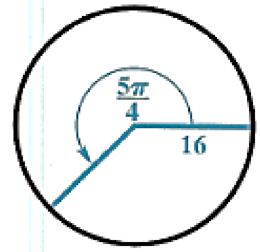
79. $\sin\left(-\frac{7\pi}{6}\right)$ **80.** $\cos\left(-\frac{\pi}{6}\right)$ **81.** $\tan\left(-\frac{14\pi}{3}\right)$ **82.** $\csc\left(-\frac{13\pi}{3}\right)$

16.2 Exercises

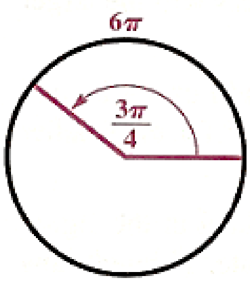
Concept Check *Find the exact length of each arc intercepted by the given central angle.*

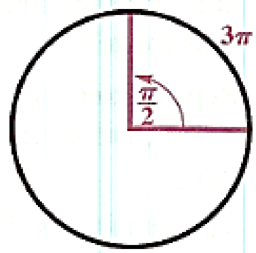
1.


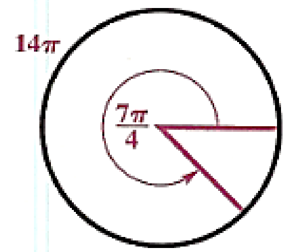
2.


3.


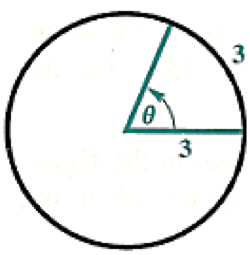
Concept Check *Find the radius of each circle.*

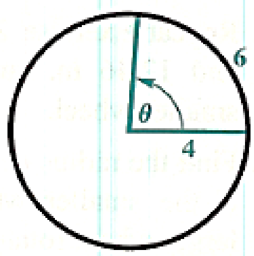
4.


5.


6.


Concept Check *Find the measure of each central angle (in radians).*

7.


8.


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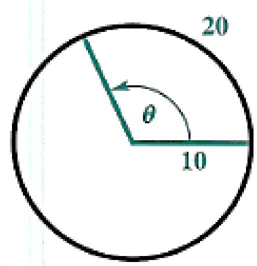

10. Explain in your own words how to find the *degree* measure of a central angle in a circle if both the radius and the length of the intercepted arc are known.

Unless otherwise directed, give calculator approximations in your answers in the rest of this exercise set.

Find the length to three significant digits of each arc intercepted by a central angle θ in a circle of radius r. See Example 1.

11. $r = 12.3$ cm, $\theta = \dfrac{2\pi}{3}$ radians

12. $r = 0.892$ cm, $\theta = \dfrac{11\pi}{10}$ radians

13. $r = 1.38$ ft, $\theta = \dfrac{5\pi}{6}$ radians

14. $r = 3.24$ mi, $\theta = \dfrac{7\pi}{6}$ radians

15. $r = 4.82$ m, $\theta = 60°$

16. $r = 71.9$ cm, $\theta = 135°$

17. $r = 15.1$ in., $\theta = 210°$

18. $r = 12.4$ ft, $\theta = 330°$

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19. *Concept Check* If the radius of a circle is doubled, how is the length of the arc intercepted by a fixed central angle changed?

20. *Concept Check* Radian measure simplifies many formulas, such as the formula for arc length, $s = r\theta$. Give the corresponding formula when θ is measured in degrees instead of radians.

Find the distance in kilometers between each pair of cities, assuming they lie on the same north—south line. See Example 2.

21. Panama City, Panama, 9° N, and Pittsburgh, Pennsylvania, 40° N

22. Farmersville, California, 36° N, and Penticton, British Columbia, 49° N

23. New York City, New York, 41° N, and Lima, Peru, 12° S

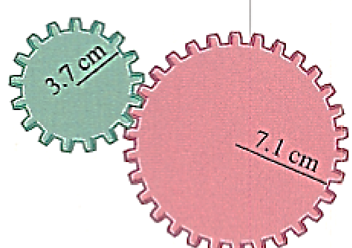
24. Halifax, Nova Scotia, 45° N, and Buenos Aires, Argentina, 34° S

25. Madison, South Dakota, and Dallas, Texas, are 1200 km apart and lie on the same north—south line. The latitude of Dallas is 33° N. What is the latitude of Madison?

26. Charleston, South Carolina, and Toronto, Canada, are 1100 km apart and lie on the same north—south line. The latitude of Charleston is 33° N. What is the latitude of Toronto?

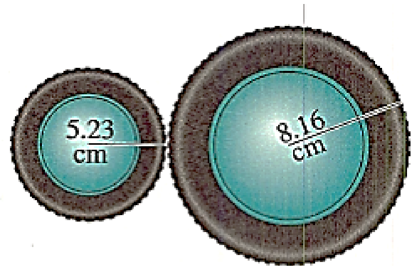
Work each problem. See Examples 3 and 4.

27. Two gears are adjusted so that the smaller gear drives the larger one, as shown in the figure. If the smaller gear rotates through an angle of 300°, through how many degrees will the larger gear rotate?



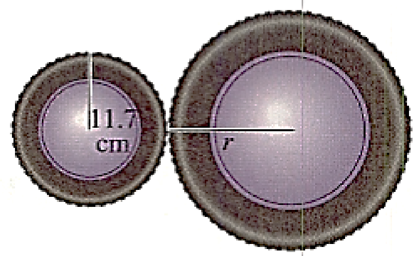
28. Repeat Exercise 27 for gear radii of 4.8 in. and 7.1 in., and for an angle of 315° for the smaller gear.

29. The rotation of the smaller wheel in the figure causes the larger wheel to rotate. Through how many degrees will the larger wheel rotate if the smaller one rotates through 60.0°?



30. Repeat Exercise 29 for wheel radii of 6.84 in. and 12.46 in. and an angle of 150° for the smaller wheel.

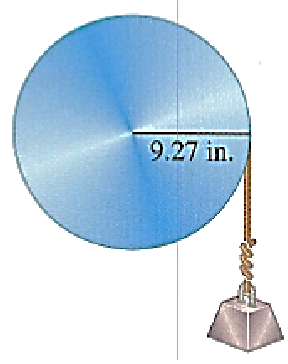
31. Find the radius of the larger wheel in the figure if the smaller wheel rotates 80.0° when the larger wheel rotates 50.0°.



32. Repeat Exercise 31 if the smaller wheel of radius 14.6 in. rotates 120° when the larger wheel rotates 60°.

33. **(a)** How many inches will the weight in the figure rise if the pulley is rotated through an angle of 71° 50′?

(b) Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 6 in.?



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16.1 Classwork

83. ***Concept Check*** The figure shows the same angles measured in both degrees and radians. Complete the missing measures.