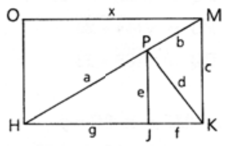
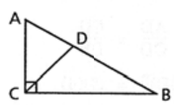
11 $\overline{GJ}$ alt $\overline{FH}$	11 If a ray is drawn from a vertex, $\perp$ to a side, then it is an alt.
12 $\dfrac{JH}{GH} = \dfrac{GH}{HF}$	12 Each leg of a rt $\triangle$ is the mean propor betwn the adj seg of the hyp and the total hyp.

23 Given: HKMO is a rect.
$\overline{PK} \perp \overline{HM}$
$\overline{PJ} \perp \overline{HK}$
Prove: $ab = fx$



1 HKMO rect	1 Given
2 $\overline{PK} \perp \overline{HM}, \overline{PJ} \perp \overline{HK}$	2 Given
3 $d^2 = f(f+g)$	3 Each leg of a rt $\triangle$ is the mean propor btwn the adj seg of the hyp and the total hyp.
4 $f + g = x$	4 Opp sides of a rect are $\cong$.
5 $d^2 = fx$	5 Substitution
6 $d^2 = ab$	6 Alt to hyp of rt $\triangle$ is mean propor betwn segs of hyp.
7 $ab = fx$	7 Transitive prop

24 Arithmetic Mean $= \frac{1}{2}(a+b)$,
Geometric Mean: $\dfrac{AD}{CD} = \dfrac{CD}{DB}$



a 1 $AM = \frac{1}{2}(2+8)$ GM: $\frac{2}{x} = \frac{x}{8}, x^2 = 16$

$AM = 5$ GM is 4.

2 $AM = \frac{1}{2}(3+12)$ GM: $\frac{3}{x} = \frac{x}{12}, x^2 = 36$

$AM = 7\frac{1}{2}$ GM is 6.

3 $AM = \frac{1}{2}(4+25)$ GM: $\frac{4}{x} = \frac{x}{25}, x^2 = 100$

$AM = 14\frac{1}{2}$ GM is 10.

1 HM: $\dfrac{2}{\frac{1}{2}+\frac{1}{8}} = \dfrac{2}{\frac{5}{8}} = \dfrac{16}{5} = 3.2$

2 HM: $\dfrac{2}{\frac{1}{3}+\frac{1}{12}} = \dfrac{2}{\frac{5}{12}} = \dfrac{24}{5} = 4.8$

3 HM: $\dfrac{2}{\frac{1}{4}+\frac{1}{25}} = \dfrac{2}{\frac{29}{100}} = \dfrac{200}{29} = 6\frac{26}{29}$

b Two positive numbers are unequal in the same order as their squares.

$$\left(\frac{a+b}{2}\right)^2 \ ? \ (\sqrt{ab})^2$$
$$\frac{a^2+2ab+b^2}{4} \ ? \ ab$$
$$a^2+2ab+b^2 \ ? \ 4ab$$
$$a^2-2ab+b^2 \ ? \ 0$$
$$(a-b)^2 \ ? \ 0$$

$(a-b)^2$ must be ≥ 0 for real numbers a, b,
so $\left(\frac{a+b}{2}\right)^2 \geq (\sqrt{ab})^2$

Pages 387–391 (Section 9.4)

1 a
$$\begin{aligned} x^2+y^2 &= r^2 \\ 4^2+5^2 &= r^2 \\ 16+25 &= r^2 \\ 41 &= r^2 \\ r &= \sqrt{41} \end{aligned}$$

b
$$\begin{aligned} x^2+y^2 &= r^2 \\ 15^2+y^2 &= 17^2 \\ 225+y^2 &= 289 \\ y^2 &= 64 \\ y &= 8 \end{aligned}$$

c
$$\begin{aligned} x^2+y^2 &= r^2 \\ x^2+9^2 &= 15^2 \\ x^2+81 &= 225 \\ x^2 &= 144 \\ x &= 12 \end{aligned}$$

d
$$\begin{aligned} x^2+y^2 &= r^2 \\ 12^2+y^2 &= 13^2 \\ 144+y^2 &= 169 \\ y^2 &= 144 \\ y &= 5 \end{aligned}$$

e
$$\begin{aligned} x^2+y^2 &= r^2 \\ 5^2+(5\sqrt{3})^2 &= r^2 \\ 25+25\sqrt{9} &= r^2 \\ 25+75 &= r^2 \\ 100 &= r^2 \\ r &= 10 \end{aligned}$$

f
$$\begin{aligned} x^2+y^2 &= r^2 \\ 5^2+y^2 &= (\sqrt{29})^2 \\ 25+y^2 &= 29 \\ y^2 &= 4 \\ y &= 2 \end{aligned}$$

g
$$\begin{aligned} x^2+y^2 &= r^2 \\ (2\sqrt{5})^2+y^2 &= (\sqrt{38})^2 \\ 20+y^2 &= 38 \\ y^2 &= 18 \\ y &= \sqrt{18} = 3\sqrt{2} \end{aligned}$$

2 All sides of a square are $\cong$, so each side $= \frac{12}{4} = 3$

All $\angle$s of a square are rt $\angle$s

If a and b are sides of the square and c is the diagonal,

$$\begin{aligned} c^2 &= a^2+b^2 \\ c^2 &= 3^2+3^2 \\ c^2 &= 9+9 \\ c^2 &= 18 \\ c &= \sqrt{18} = 3\sqrt{2} \end{aligned}$$

3 In a rhombus, diagonals are $\perp$ bis of each other.

$BE = ED = 8, AE = EC = 6$

$\overline{AC} \perp \overline{BD}$

$$\begin{aligned} AB^2 &= 6^2 + 8^2 \\ AB^2 &= 36 + 64 \\ AB^2 &= 100 \\ AB &= \sqrt{100} = 10 \end{aligned}$$

In a rhombus all sides are $\cong$, so perimeter $= 4(10) = 40$.

4

$$\begin{aligned} 15^2 + b^2 &= 17^2 \\ 225 + b^2 &= 289 \\ b^2 &= 64 \\ b &= 8 \\ \text{Perimeter} &= 2(15) + 2(8) \\ P &= 30 + 16 = 46 \end{aligned}$$

5 $\triangle JGF \cong \triangle JGH$ (AAS), therefore $FG = 12$.

Since $\angle JGF$ is a rt $\angle$ (Pythagorean Theorem)

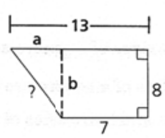
$$\begin{aligned} FG^2 + JG^2 &= JF^2 \\ 12^2 + JG^2 &= 15^2 \\ 144 + JG^2 &= 225 \\ JG^2 &= 81 \\ JG &= \sqrt{81} = 9 \end{aligned}$$

6 In Section 3.7, problem 7 proved that the altitude to the base of an equilateral $\triangle$ is the median to the base. An altitude forms rt $\angle$s, so the equilateral $\triangle$ is divided into 2 rt $\triangle$s and the sides are 2 and x. The hypotenuse is 4.

$$\begin{aligned} 2^2 + x^2 &= 4^2 \quad \text{(Pythagorean Theorem)} \\ 4 + x^2 &= 16 \\ x^2 &= 12 \\ x &= \sqrt{12} = 2\sqrt{3} \end{aligned}$$

7 Draw the dotted segment.

$$\begin{aligned} a &= 13 - 7 = 6, b = 8 \\ x^2 &= a^2 + b^2 \\ x^2 &= 6^2 + 8^2 \\ x^2 &= 36 + 64 \\ x &= \sqrt{100} = 10 \end{aligned}$$



8 Since the wall is perpendicular to the ground, the $\triangle$ formed is a rt $\triangle$. Use the Pythagorean Theorem.

$$\begin{aligned} 48^2 + x^2 &= 50^2 \\ 2304 + x^2 &= 2500 \\ x^2 &= 196 \\ x &= \sqrt{196} = 14 \end{aligned}$$

The foot of the ladder is 14 dm from the wall.

9 **a** $C(2, 3)$

b $AC = 11 - 3 = 8$

$CB = 8 - 2 = 6$

c

$$\begin{aligned} (AB)^2 &= 6^2 + 8^2 \\ (AB)^2 &= 36 + 64 \\ (AB)^2 &= 100 \\ AB &= 10 \end{aligned}$$

d Yes

10

$$\begin{aligned} PQ &= \sqrt{(13-1)^2 + (3-8)^2} \\ PQ &= \sqrt{(12)^2 + (-5)^2} \\ PQ &= \sqrt{144 + 25} \\ PQ &= \sqrt{169} = 13 \end{aligned}$$

11 **a** $AC = x, BC = y$

$$\begin{aligned} AC^2 + BC^2 &= AB^2 \\ x^2 + y^2 &= AB^2 \\ AB &= \sqrt{x^2 + y^2} \end{aligned}$$

b $AC = 2, BC = x$

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ AB^2 &= 2^2 + x^2 \\ AB^2 &= 4 + x^2 \\ AB &= \sqrt{4 + x^2} \end{aligned}$$

c $AC = 3a, BC = 4a$

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ AB^2 &= (3a)^2 + (4a)^2 \\ AB^2 &= 9a^2 + 16a^2 \\ AB^2 &= 25^2 \\ AB &= 5a \end{aligned}$$

d $AB = 13c \quad AC = 5c$

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ (13c)^2 &= (5c)^2 + BC^2 \\ 169c^2 &= 25c^2 + BC^2 \\ BC^2 &= 144c^2 \\ BC &= 12c \end{aligned}$$

12 **a**

$$\begin{aligned} \frac{AD}{CD} &= \frac{CD}{DB} \\ (CD)^2 &= (7)(4) \\ CD^2 &= 28 \\ CD &= \sqrt{28} = 2\sqrt{7} \end{aligned}$$

b

$$\begin{aligned} (CD)^2 + (DB)^2 &= (CB)^2 \\ 64 + 36 &= (CB)^2 \\ 100 &= (CB)^2 \\ CB &= 10 \end{aligned}$$

c

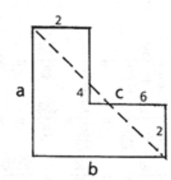
$$\begin{aligned} \frac{DB}{CB} &= \frac{CB}{AB} \\ \frac{2}{8} &= \frac{8}{AB} \\ 2AB &= 64 \\ AB &= 32 \end{aligned}$$

d

$$\begin{aligned} (AC)^2 + (CB)^2 &= (AB)^2 \\ 21^2 + (CB)^2 &= 29^2 \\ 441 + (CB)^2 &= 841 \\ (CB)^2 &= 400 \\ CB &= 20 \end{aligned}$$

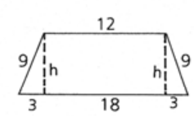
13

$$\begin{aligned} a &= 6, b = 8 \\ c^2 &= 6^2 + 8^2 \\ c^2 &= 36 + 64 \\ c &= \sqrt{100} = 10 \text{ km} \end{aligned}$$



14 Draw in altitudes, making a rectangle and two rt $\triangle$s, $18 - 12 = 6$. The $\triangle$s are $\cong$, so the shorter legs are 3.

$$\begin{aligned} h^2 + 3^2 &= 9^2 \\ h^2 + 9 &= 81 \\ h &= \sqrt{72} = 6\sqrt{2} \end{aligned}$$



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$BE = ED = 8$, $AE = EC = 6$

$\overline{AC} \perp \overline{BD}$

$$AB^2 = 6^2 + 8^2$$
$$AB^2 = 36 + 64$$
$$AB^2 = 100$$
$$AB = \sqrt{100} = 10$$

In a rhombus all sides are $\cong$, so perimeter $= 4(10) = 40$.

4
$$15^2 + b^2 = 17^2$$
$$225 + b^2 = 289$$
$$b^2 = 64$$
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$$P = 30 + 16 = 46$$

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Since $\angle JGF$ is a rt $\angle$ (Pythagorean Theorem)
$$FG^2 + JG^2 = JF^2$$
$$12^2 + JG^2 = 15^2$$
$$144 + JG^2 = 225$$
$$JG^2 = 81$$
$$JG = \sqrt{81} = 9$$

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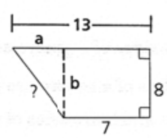
$$2^2 + x^2 = 4^2 \quad \text{(Pythagorean Theorem)}$$
$$4 + x^2 = 16$$
$$x^2 = 12$$
$$x = \sqrt{12} = 2\sqrt{3}$$

7 Draw the dotted segment.
$$a = 13 - 7 = 6, b = 8$$
$$x^2 = a^2 + b^2$$
$$x^2 = 6^2 + 8^2$$
$$x^2 = 36 + 64$$
$$x = \sqrt{100} = 10$$



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$$48^2 + x^2 = 50^2$$
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The foot of the ladder is 14 dm from the wall.

9 **a** $C(2, 3)$

b $AC = 11 - 3 = 8$

$CB = 8 - 2 = 6$

c
$$(AB)^2 = 6^2 + 8^2$$
$$(AB)^2 = 36 + 64$$
$$(AB)^2 = 100$$
$$AB = 10$$

d Yes

10
$$PQ = \sqrt{(13-1)^2 + (3-8)^2}$$
$$PQ = \sqrt{(12)^2 + (-5)^2}$$
$$PQ = \sqrt{144 + 25}$$
$$PQ = \sqrt{169} = 13$$

11 **a** $AC = x, BC = y$
$$AC^2 + BC^2 = AB^2$$
$$x^2 + y^2 = AB^2$$
$$AB = \sqrt{x^2 + y^2}$$

b $AC = 2, BC = x$
$$AB^2 = AC^2 + BC^2$$
$$AB^2 = 2^2 + x^2$$
$$AB^2 = 4 + x^2$$
$$AB = \sqrt{4 + x^2}$$

c $AC = 3a, BC = 4a$
$$AB^2 = AC^2 + BC^2$$
$$AB^2 = (3a)^2 + (4a)^2$$
$$AB^2 = 9a^2 + 16a^2$$
$$AB^2 = 25^2$$
$$AB = 5a$$

d $AB = 13c \quad AC = 5c$
$$AB^2 = AC^2 + BC^2$$
$$(13c)^2 = (5c)^2 + BC^2$$
$$169c^2 = 25c^2 + BC^2$$
$$BC^2 = 144c^2$$
$$BC = 12c$$

12 **a**
$$\frac{AD}{CD} = \frac{CD}{DB}$$
$$(CD)^2 = (7)(4)$$
$$CD^2 = 28$$
$$CD = \sqrt{28} = 2\sqrt{7}$$

b
$$(CD)^2 + (DB)^2 = (CB)^2$$
$$64 + 36 = (CB)^2$$
$$100 = (CB)^2$$
$$CB = 10$$

c
$$\frac{DB}{CB} = \frac{CB}{AB}$$
$$\frac{2}{8} = \frac{8}{AB}$$
$$2AB = 64$$
$$AB = 32$$

d
$$(AC)^2 + (CB)^2 = (AB)^2$$
$$21^2 + (CB)^2 = 29^2$$
$$441 + (CB)^2 = 841$$
$$(CB)^2 = 400$$
$$CB = 20$$

13
$$a = 6, b = 8$$
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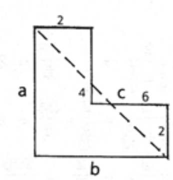

14 Draw in altitudes, making a rectangle and two rt $\triangle$s, $18 - 12 = 6$. The $\triangle$s are $\cong$, so the shorter legs are 3.
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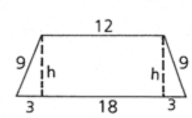


15 Perimeter of $\triangle ACE$ = ?

Using information given and the Pythagorean Theorem,

$FD = AB = 24$, $BC = 7$

Then $AC^2 = 24^2 + 7^2$

$AC = \sqrt{576 + 49} = 25$

A mdpt divides a seg into 2 ≅ segs.

Then $FE = ED = 12$ and $CD = 16 - 7 = 9$

$CE^2 = 12^2 + 9^2$

$CE = \sqrt{144 + 81} = 15$

Opp sides of a rectangle are ≅, so

$AF = BD = 16$ and

$AE^2 = 16^2 + 12^2$

$AE = \sqrt{256 + 144} = 20$

Perimeter of $\triangle ACE = 25 + 15 + 20 = 60$

16 Since $\triangle ACB$ is a rt $\triangle$,

$$\begin{aligned} 6^2 + 8^2 &= AB^2 \\ 36 + 64 &= AB^2 \\ AB &= \sqrt{100} = 10 \end{aligned}$$

$AB = 10$, $DB = y$, so

$$\begin{aligned} AD &= 10 - y, \frac{AD}{AC} = \frac{AC}{AB} \\ \frac{10-y}{6} &= \frac{6}{10} \\ 10(10-y) &= 36 \\ 100 - 10y &= 36 \\ 10y &= 64, y = 6.4 \end{aligned}$$

$\therefore DB = 6.4$

$$\begin{aligned} DB &= 6.4,\ AD = 3.6 \\ \frac{AD}{CD} &= \frac{CD}{DB} \\ \frac{3.6}{CD} &= \frac{CD}{6.4} \\ CD^2 &= (3.6)(6.4) \\ CD &= \sqrt{23.04} \\ CD &= 4.8 \end{aligned}$$

17 Starting with the bottom $\triangle$,

$$\begin{aligned} z^2 &= 1^2 + 1^2 \\ z &= \sqrt{2} \end{aligned}$$

In the second $\triangle$,

$$\begin{aligned} y^2 &= z^2 + 1^2 \\ y^2 &= (\sqrt{2})^2 + 1 = 3 \\ y &= \sqrt{3} \end{aligned}$$

In the third $\triangle$,

$$\begin{aligned} w^2 &= 1^2 + y^2 \\ w^2 &= 1^2 + (\sqrt{3})^2 \\ w^2 &= 1 + 3 = 4 \\ w &= 2 \end{aligned}$$

In the last $\triangle$,

$$\begin{aligned} x^2 &= w^2 + 1^2 \\ x^2 &= 2^2 + 1^2 \\ x^2 &= 5 \\ x &= \sqrt{5} \end{aligned}$$

18 $P = 8\sqrt{5}$

each side $= \frac{1}{4}(8\sqrt{5}) = 2\sqrt{5}$

one diagonal $= 4\sqrt{2}$

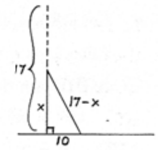
$\frac{1}{2}$ diagonal $= 2\sqrt{2}$

$$\begin{aligned} (2\sqrt{2})^2 + b^2 &= (2\sqrt{5})^2 \\ 8 + b^2 &= 20 \\ b^2 &= 12 \\ b &= \sqrt{12} = 2\sqrt{3} \end{aligned}$$

Other diagonal $= 4\sqrt{3}$

19

$$\begin{aligned} x^2 + 10^2 &= (17 - x)^2 \\ x^2 + 100 &= 289 - 34x + x^2 \\ 34x &= 189 \\ x &= \frac{189}{34} = 5\tfrac{19}{34}\text{ m} \approx 5.56\text{ m} \end{aligned}$$



20 Let x = leg of $\triangle$,

$$\begin{aligned} x^2 + x^2 &= 6^2 \\ 2x^2 &= 6^2 \\ x^2 &= 18 \\ x &= \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} P &= x + x + 6 \\ P &= 3\sqrt{2} + 3\sqrt{2} + 6 \\ P &= (6\sqrt{2} + 6)\text{ cm} \end{aligned}$$

21 $P = 20$, each side = 5

The diagonals bisect each other, so

the $\triangle$ has legs $x, \frac{x}{2}$ and hypotenuse is 5.

$$\begin{aligned} x^2 + \left(\frac{x}{2}\right)^2 &= 5^2 \\ x^2 + \frac{x^2}{4} &= 25 \\ 4x^2 + x^2 &= 100 \\ 5x^2 &= 100 \\ x^2 &= 20 \\ x &= \sqrt{20} = 2\sqrt{5},\ 2x = 4\sqrt{5},\ \text{sum} = 6\sqrt{5} \end{aligned}$$

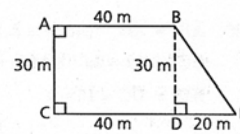
22 Obtuse; Impossible $(3 + 5 \not> 10)$

23 a Draw altitude BD. Then

$A = A_{ABCD} + A_{BDE}$

$A = (30 \cdot 40) + \frac{1}{2}(30 \cdot 20)$

$A = 1{,}200 + 300 = 1{,}500$



b $(BE)^2 = (30)^2 + (20)^2 = 900 + 400$

$BE \approx 36$

24 First find CB.

$$\begin{aligned} CD^2 + CA^2 &= DA^2 \\ 12^2 + CA^2 &= 20^2 \\ 144 + CA^2 &= 400 \\ CA &= \sqrt{256} = 16 \\ CA - BA &= CB \\ CB &= 6 \end{aligned}$$

Now find DB.

$$\begin{aligned} CD^2 + CB^2 &= DB^2 \\ 12^2 + 6^2 &= DB^2 \\ 144 + 36 &= DB^2 \\ \sqrt{180} &= DB \\ 6\sqrt{5} &= DB \end{aligned}$$

$P = 12 + 6\sqrt{5} + 6 = 18 + 6\sqrt{5}$

25 a Let GF = x, HF = y

FE = 21 – x

$\triangle$GHF is a rt $\triangle$, so $\quad$ $\triangle$HFE is a rt $\triangle$, so

$$10^2 = x^2 + y^2 \qquad 17^2 = (21 - x)^2 + y^2$$

$$100 - x^2 = y^2 \qquad 17^2 - (21 - x)^2 = y^2$$

Then:

$$\begin{aligned} 100 - x^2 &= 17^2 - (21 - x)^2 \\ 100 - x^2 &= 289 - 441 + 42x - x^2 \\ 100 &= -152 + 42x \\ 252 &= 42x \\ 6 &= x \\ \therefore GF &= 6 \end{aligned}$$

In $\triangle$GHF, $6^2 + HF^2 = 10^2$

$$\begin{aligned} 36 + HF^2 &= 100 \\ HF &= \sqrt{64} = 8 \end{aligned}$$

b If $\triangle EHF \sim \triangle HGF$, their corr sides must be in proportion

$$\frac{EH}{HF} \stackrel{?}{=} \frac{HG}{GF}$$

$$\frac{17}{8} \stackrel{?}{=} \frac{10}{6}$$

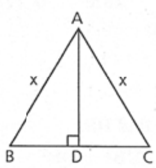
$102 \neq 80$

Their corr sides are not in proportion, so the $\triangle$s are not $\sim$.

26 $\overline{AB} \cong \overline{AC}$ (An isos $\triangle$ has 2 legs $\cong$.)

BC = 32 – 2x $\quad$ $\triangle ABD \cong \triangle ACD$ (ASA)

BD = DC = 16 – x



$$\begin{aligned} AD^2 + BD^2 &= AB^2 \\ 8^2 + (16 - x)^2 &= x^2 \\ 64 + 256 - 32x + x^2 &= x^2 \\ 320 - 32x + x^2 &= x^2 \\ 320 &= 32x \\ 10 &= x \end{aligned}$$

27 In $\triangle$JOM, $JM^2 = JO^2 - MO^2$ $\quad$ PO = x, JK = 2x

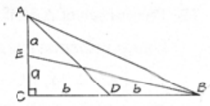
$$\begin{aligned} JM^2 &= 25^2 - 20^2 \qquad KM = 15 - 2x \\ JM^2 &= 225 \qquad PM = 20 + x \\ JM &= 15 \end{aligned}$$

IN $\triangle$PMK, $25^2 = (20 + x)^2 + (15 - 2x)^2$

$$\begin{aligned} 625 &= 400 + 40x + x^2 + 225 - 60x + 4x^2 \\ 0 &= -20x + 5x^2 \\ 0 &= 5x(x - 4) \\ 4 &= x \end{aligned}$$

KM = 15 – 2x, KM = 15 – 2(4) = 7

28 In $\triangle$ACD, $(2a)^2 + (b)^2 = (2\sqrt{13})^2$



$$4a^2 + b^2 = 52$$

In $\triangle$BCE, $(a)^2 + (2b)^2 = (\sqrt{73})^2$

$$a^2 + 4b^2 = 73$$

Solving both equations,

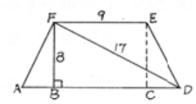
$(4a^2 + b^2 = 52)$ Mult –4 $\quad$ $-16a^2 - 4b^2 = -208$

$a^2 + 4b^2 = 73$ $\quad$ $a^2 + 4b^2 = 73$

$$\begin{aligned} -15a^2 &= -135 \\ a^2 &= 9 \\ a &= 3 \end{aligned}$$

$$\begin{aligned} (2a)^2 + (b)^2 &= (2\sqrt{13})^2 \qquad & AB^2 &= (2a)^2 + (2b)^2 \\ 36 + b^2 &= 52 & AB^2 &= 36 + 64 \\ b^2 &= 16 & AB^2 &= 36 + 64 \\ b &= 4 & AB &= \sqrt{100} = 10 \end{aligned}$$

29 In $\triangle$FBD, $8^2 + (BD)^2 = 17^2$,



$BD^2 = 289 - 64$

BD = 15.

Since BC = FE = 9,

CD = AB = 6

In $\triangle$ECD, $ED^2 = 8^2 + 6^2$

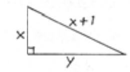
$$\begin{aligned} ED^2 &= 64 + 36 \\ ED &= \sqrt{100} = 10 \text{ and } \overline{ED} \cong \overline{FA}\text{, so} \end{aligned}$$

P = 9 + 2(10) + (15 + 6) = 50

30 a one leg = x

hypotenuse = x + 1

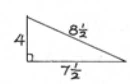
second leg = y



Then $(x + 1)^2 = x^2 + y^2$

$x^2 + 2x + 1 = x^2 + y^2$ and $y^2 = 2x + 1$

b One counterexample would be a rt $\triangle$ with legs 4, $7\frac{1}{2}$; hypotenuse $8\frac{1}{2}$



($7\frac{1}{2}$, $8\frac{1}{2}$ are not integers.)

$4^2 = 7\frac{1}{2} + 8\frac{1}{2}$

31 a slope $\overline{QU} = \frac{15}{8}$

slope $\overline{DA} = \frac{15}{8}$

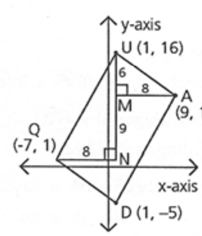
$\therefore \overline{QU} \parallel \overline{DA}$

Slope $\overline{QD} = \frac{-3}{4}$

Slope $\overline{AU} = \frac{-3}{4}$

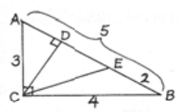
$\overline{QD} \parallel \overline{AU}$

$\therefore$QUAD is a $\square$.



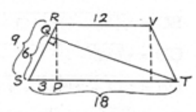
b $(UA)^2 = 6^2 + 8^2$
$UA = 10$
$(QU)^2 = 8^2 + 15^2$
$QU = 17$
Perimeter QUAD = 17 + 17 + 10 + 10 = 54

32 By theorem, AC is the mean prop between AB and AD. $AB = \sqrt{3^2+4^2} = \sqrt{25} = 5$



$AC^2 = (AB)(AD)$
$3^2 = 5(AD)$
$\frac{9}{5} = AD$ and
$DE = 5 - 2 - \frac{9}{5} = \frac{6}{5}$
In $\triangle ACD$, $AC^2 = CD^2 + AD^2$
$3^2 = CD^2 + (\frac{9}{5})^2$
$CD = \sqrt{9 - \frac{81}{25}} = \sqrt{\frac{225}{25} - \frac{81}{25}}$
$CD = \sqrt{\frac{144}{25}} = \frac{12}{5}$
In $\triangle CDE$, $CE^2 = CD^2 + DE^2$
$CE^2 = (\frac{12}{5})^2 + (\frac{6}{5})^2 = \frac{144}{25} + \frac{36}{25}$
$CE = \frac{6\sqrt{5}}{5}$

33 Draw $\overline{RP}$ and $\overline{VX}$

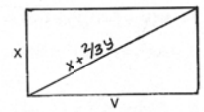


$\overline{SP} \cong \overline{XT}$, so $SP \cong \frac{1}{2}(ST - RV)$.
SP = 3. Draw $\overline{TQ} \perp \overline{SW}$
$\angle Q \cong \angle P$ (rt $\angle$s are $\cong$)
$\angle S \cong \angle S$ (Reflexive prop)
$\triangle SQT \sim \triangle SPR$ (AA~)

$\frac{SR}{ST} = \frac{SP}{SQ}$ $\qquad ST^2 = SQ^2 + QT^2$
$\frac{9}{18} = \frac{3}{SQ}$ $\qquad 18^2 = 6^2 + QT^2$
$9(SQ) = 54$ $\qquad 324 = 36 + QT^2$
$SQ = 6$ $\qquad \sqrt{288} = QT$
$12\sqrt{2} = QT$

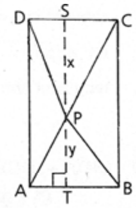
34 In rt $\triangle PRT$, $15^2 + (RT)^2 = 25^2$
$RT = \sqrt{625 - 225}$
$RT = \sqrt{400} = 20$
If ST = x, SR = 20 − x. In rt $\triangle PRS$,
$15^2 + (20 - x)^2 = (x + 12)^2$
$225 + 400 - 40x + x^2 = x^2 + 24x + 144$
$-64x = -481$
$x = \frac{481}{64}$
$SR = 20 - x$, $SR = 20 - \frac{481}{64}$
$SR = \frac{799}{64}$

35 Let x = shorter side
y = longer side



$x^2 + y^2 = (x + \frac{2}{3}y)^2$
$x^2 + y^2 = x^2 + \frac{4}{3}xy + \frac{4}{9}y^2$
$\frac{5}{9}y^2 = \frac{4}{3}xy$
$\frac{5}{9}y = \frac{4}{3}x$ $\qquad \frac{x}{y} = \frac{\frac{5}{9}}{\frac{4}{3}} = \frac{5}{12}$

36 a Show $(BP)^2 + (PD)^2 = (AP)^2 + (CP)^2$



$(BP)^2 = y^2 + (TB)^2$
$(PD)^2 = x^2 + (DS)^2$
$(BP)^2 + (PD)^2 = x^2 + y^2 + (TB)^2 + (DS)^2$
$(AP)^2 = y^2 + (AT)^2$
$(CP)^2 = x^2 + (SC)^2$
$(AP)^2 + (CP)^2 = x^2 + y^2 + (AT)^2 + (SC)^2$
If $\overline{ST}$ is drawn through P, parallel to CB, STBC is a rectangle and $\overline{TB} \cong \overline{SC}$ and $\overline{DS} \cong \overline{AT}$.
b Yes, the same procedure should be followed.

Pages 394–400 (Section 9.5)

1 a $\sqrt{(6-4)^2 + (0-0)^2} = 2$
b $\sqrt{(2-2)^2 + (3-(-1))^2} = 4$
c $\sqrt{(7-4)^2 + (5-1)^2} = \sqrt{9+16} = 5$
d $\sqrt{(4-(-4))^2 + (-8-(-2))^2} = \sqrt{8^2+6^2} = 10$
e $\sqrt{(2-0)^2 + (5-0)^2} = \sqrt{4+25} = \sqrt{29}$
f $\sqrt{(6-2)^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

2 $AB = \sqrt{(5-2)^2 + (10-6)^2} = \sqrt{9+16} = 5$
$BC = \sqrt{(5-0)^2 + (10-13)^2} = \sqrt{25+9} = \sqrt{34}$
$CA = \sqrt{(2-0)^2 + (6-13)^2} = \sqrt{4+49} = \sqrt{53}$
$P = AB + BC + CA = 5 + \sqrt{34} + \sqrt{53} \approx 18.1$

3 a Let A = (8, 4); B = (3, 5); C = (4, 10)
$AB = \sqrt{(8-3)^2 + (4-5)^2} = \sqrt{25+1} = \sqrt{26}$
$BC = \sqrt{(4-3)^2 + (10-5)^2} = \sqrt{1+25} = \sqrt{26}$
$AC = \sqrt{(8-4)^2 + (4-10)^2} = \sqrt{16+36} = \sqrt{52}$
$AB^2 + BC^2 = AC^2$
$(\sqrt{26})^2 + (\sqrt{26})^2 = AC^2$
$26 + 26 = AC^2$
$\sqrt{52} = AC$
b $m(\overline{AB}) = \frac{5-4}{3-8} = -\frac{1}{5}$
$m(\overline{BC}) = \frac{10-5}{4-3} = \frac{5}{1} = 5$
The slopes are negative inverses, so the sides are $\perp$.

4 $DO = \sqrt{(6-0)^2 + (0-0)^2} = 6$
$OG = \sqrt{(3-0)^2 + (3\sqrt{3}-0)^2} = \sqrt{9+27} = 6$
$DG = \sqrt{(3-6)^2 + (3\sqrt{3}-0)^2} = \sqrt{9+27} = 6$
$\therefore \triangle DOG$ is isosceles since all 3 sides are $\cong$.

5 $r = \sqrt{(9-(-3))^2 + (-4-5)^2} = \sqrt{144+81} = \sqrt{225} = 15$

$A = \pi r^2 = \pi(15)^2 = 225\pi$

6 **a** Mdpt $\overline{RV} = (7, -1)$

distance from T to mdpt $\overline{RV}$ =

$\sqrt{(9-7)^2 + (8-(-1))^2} = \sqrt{85}$

b The length of the segment joining the midpoints of two sides of a $\triangle = \frac{1}{2}$ of the length of the third side.

$RV = \sqrt{(13-1)^2 + (2-(-4))^2} = \sqrt{144+36} = \sqrt{180} = 6\sqrt{5}$

length of the segment $= \frac{RV}{2} = 3\sqrt{5}$

7 $(BD)^2 + (DC)^2 = (BC)^2$

$6^2 + 8^2 = (BC)^2$

$36 + 64 = (BC)^2$

$100 = (BC)^2$

$10 = BC$

Let AD = x

So $\frac{8}{10} = \frac{10}{8+x}$

$8(8+x) = 100$

$64 + 8x = 100$

$8x = 36$

$AD = x = 4.5$

8 **a** A (0, 2b) B (2a, 2b) C (2a, 0) O (0, 0)

b $M = \left(\frac{0+0}{2}, \frac{2b+0}{2}\right) = (0, b)$ $P = \left(\frac{2a+2a}{2}, \frac{2b+0}{2}\right) = (2a, b)$

$N = \left(\frac{2a+0}{2}, \frac{2b+2b}{2}\right) = (a, 2b)$ $Q = \left(\frac{2a+0}{2}, \frac{0+0}{2}\right) = (a, 0)$

c slope $\overline{MN} = \frac{2b-b}{a-0} = \frac{b}{a}$ slope $\overline{MQ} = \frac{0-b}{a-0} = -\frac{b}{a}$

slope $\overline{QP} = \frac{b-0}{2a-a} = \frac{b}{a}$ slope $\overline{NP} = \frac{2b-b}{a-2a} = -\frac{b}{a}$

Since $\overline{MN} \parallel \overline{QP}$ and $\overline{MQ} \parallel \overline{NP}$, MNPQ is a ▱.

d $MN = \sqrt{(a-0)^2 + (2b-b)^2} = \sqrt{a^2+b^2}$

$QP = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

$MQ = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

$NP = \sqrt{(2a-a)^2 + (b-2b)^2} = \sqrt{a^2+b^2}$

$MN = QP = MQ = NP$, $\therefore$ MNPQ is a rhombus.

9 **a** $PQ = \sqrt{(-a+c)^2 + (0-b)^2} = \sqrt{c^2 - 2ac + a^2 + b^2}$

$SR = \sqrt{(a-c)^2 + (0-b)^2} = \sqrt{a^2 - 2ac + c^2 + b^2}$

$PQ = SR$, $\therefore$ PQRS is an isos trap.

b $PR = \sqrt{(c+a)^2 + (b-0)^2} = \sqrt{c^2 + 2ac + a^2 + b^2}$

$QS = \sqrt{(a+c)^2 + (0-b)^2} = \sqrt{c^2 + 2ac + a^2 + b^2}$

$PR = QS$, $\therefore \overline{PR} \cong \overline{QS}$.

10 Yes, since $m\angle E = 90$ (by def of ▭, $m\widehat{RTC} = 180$; $\therefore \overline{RC}$ is a diameter.

11 **a** $\triangle$REC is a 5-12-13 $\triangle$, so $\overline{RC} = d = 13$.

$C = \pi d = 13\pi$

b $A = \pi r^2 = \pi(\frac{13}{2})^2 \approx 132.7$

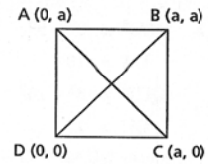
12 $BD = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

$AC = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

slope $\overline{BD} = \frac{a}{-a} = -1$

slope $\overline{AC} = \frac{a}{a} = 1$

$\therefore \overline{AD} \perp \overline{BC}$



13 **a** $m\widehat{TV} = 180 - m\widehat{VW}$

$m\widehat{TV} = 180 - 120 = 60$

Area of sector TRV $= \frac{60}{360}$ (Area of $\odot$ R)

$= \frac{1}{6}(\pi(9)^2) = 13.5\pi$

≈ 42.4

b $\overline{TW} = 18$

$\widehat{VW} = \frac{120}{360}(2\pi(9)) \approx 18.8$

$\widehat{VW} - \overline{TW} \approx 18.8 - 18 \approx 0.8$

14 The distance from the pts to the center would all be equal.

$\sqrt{(7-2)^2 + (11-(-1))^2} = \sqrt{25+144} = \sqrt{169} = 13$

$\sqrt{(7-2)^2 + (-13-(-1))^2} = \sqrt{25+144} = \sqrt{169} = 13$

$\sqrt{(14-2)^2 + (4-(-1))^2} = \sqrt{144+25} = \sqrt{169} = 13$

15 $AB = \sqrt{(7-2)^2 + (3-1)^2} = \sqrt{25+4} = \sqrt{29}$

$BC = \sqrt{(12-7)^2 + (1-3)^2} = \sqrt{25+4} = \sqrt{29}$

$CD = \sqrt{(12-7)^2 + (1-(-4))^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$DA = \sqrt{(7-2)^2 + (-4-1)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$P = 2\sqrt{29} + 10\sqrt{2} \approx 24.9$; Kite (pairs of adj sides =)

16 d (−1, −3) to (3, −2) $= \sqrt{(3-(-1))^2 + (-2-(-3))^2} = \sqrt{17}$

d (2, 1) to (3, −2) $= \sqrt{(3-2)^2 + (-2-1)^2} = \sqrt{10}$

Consecutive sides are not congruent; therefore the parallelogram is not a rhombus.

17 d from (−2, 1) to (5, 5) $= \sqrt{(5-(-2))^2 + (5-1)^2} = \sqrt{65}$

d from (−2, 1) to (−1, −7) $= \sqrt{(-1-(-2))^2 + (-7-1)^2} = \sqrt{65}$

18 One diag has length $= \sqrt{(6-0)^2 + (8-0)^2} = 10$

Since in a ▭ both diagonals are equal, the sum of the two is 20.

19 Let A = (1, 2); B = (4, 6); C = (10, 14)

a $AB = \sqrt{(4-1)^2 + (6-2)^2} = 5$

$BC = \sqrt{(10-4)^2 + (14-6)^2} = 10$

$AC = \sqrt{(10-1)^2 + (14-2)^2} = 15$

AB + BC = AC They are collinear.

b $m(\overleftrightarrow{AB}) = \frac{6-2}{4-1} = \frac{4}{3}$

$m(\overleftrightarrow{BC}) = \frac{14-6}{10-4} = \frac{8}{6} = \frac{4}{3}$

They are collinear.

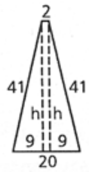
20 $\sqrt{(5-1)^2+(y-4)^2} = \sqrt{(10-5)^2+(-3-y)^2}$

$$\sqrt{16+(4-y)^2} = \sqrt{25+(y+3)^2}$$
$$16+(4-y)^2 = 25+(y+3)^2$$
$$16+16-8y+y^2 = 25+y^2+6y+9$$
$$-2 = 14y$$
$$-\tfrac{1}{7} = y$$

21 $h^2+9^2=41^2$

$h^2+81=1681$

$h=\sqrt{1{,}600}=40$

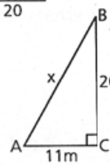


22 $x^2=20^2+11^2$

$x^2=521$

$x\approx 23$

$AB+BC\approx 20+23\approx 43$ m



23 $BC=\sqrt{(a+0)^2+(0-b)^2}=\sqrt{a^2+b^2}$

$BM=CM=\frac{1}{2}BC=\frac{\sqrt{a^2+b^2}}{2}$

M is at $\left(\frac{a}{2},\frac{b}{2}\right)$

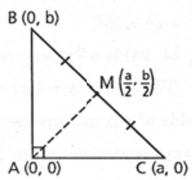
$MA=\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}$

$MA=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$

$2MA=\sqrt{a^2+b^2}$

$MA=\frac{\sqrt{a^2+b^2}}{2}$

$\therefore$ M is equidistant from A, B, and C.



24 By prop of ▱, $B=(a+b, c)$

For diagonals, $BD^2=(a+b)^2+c^2$

$CA^2=(a-b)^2+c^2$

$BD^2+CA^2=a^2+2ab+b^2+c^2+a^2-2ab+b^2+c^2$

$\qquad = 2a^2+2b^2+2c^2$

For sides, $CD^2=a^2$

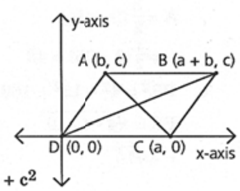
$AB^2=a^2$

$AD^2=b^2+c^2$

$BC^2=b^2+c^2$

So $2a^2+2b^2+2c^2=2a^2+2b^2+c^2$

$\therefore BD^2+CA^2=CD^2+AB^2+AD^2+BC^2$



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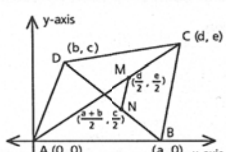


Prove:

$AB^2+BC^2+CD^2+DA^2=AC^2+BD^2+4MN^2$

$AB^2 = a^2$

$BC^2 = (a-d)^2+(e-0)^2$

$\quad = a^2-2ad+d^2+e^2$

$CD^2 = (b-d)^2+(c-e)^2$

$\quad = b^2-2bd+d^2+c^2-2ce+e^2$

$DA^2 = b^2+c^2$

$AC^2 = d^2+e^2$

$BD^2 = (a-b)^2+(0-c)^2$

$\quad = a^2-2ab+b^2+c^2$

$MN^2 = \left(\frac{a+b}{2}-\frac{d}{2}\right)^2+\left(\frac{c}{2}-\frac{e}{2}\right)^2$

$\quad = \frac{a^2+b^2+d^2+2ab-2ad-2bd}{2}+\frac{c^2-2ce+e^2}{4}$

$4MN^2 = a^2+b^2+d^2+2ab-2ad-2bd+c^2-2ce+e^2$

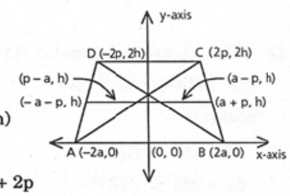
$AB^2+BC^2+CD^2+DA^2 =$

$\quad 2a^2+2b^2+2c^2+2d^2+2e^2-2ad-2bd-2ce$

$AC^2+BD^2+4MN^2 =$

$\quad 2a^2+2b^2+2c^2+2d^2+2e^2-2ad-2bd-2ce$

26



a C (2p, 2h), D (–2p, 2h)

b AB = 4a

c $a+p-(-a-p)=2a+2p$

d $(p-a)-(a-p)=2a-2p$

27 By the dist. formula, the length of each side of the △ is $\sqrt{(6-2)^2+(5-1)^2}=4\sqrt{2}$. If (x, y) is the remaining vertex, $\sqrt{(x-2)^2+(y-1)^2}=4\sqrt{2}$ and $\sqrt{(x-6)^2+(y-5)^2}=4\sqrt{2}$. Therefore,

$$\sqrt{(x-2)^2+(y-1)^2} = \sqrt{(x-6)^2+(y-5)^2}$$
$$x^2-4x+4+y^2-2y+1 = x^2-12x+36+y^2-10y+25$$
$$x+y = 7$$
$$y = 7-x$$

Substituting $7-x$ for y in $\sqrt{(x-2)^2+(y-1)^2}=4\sqrt{2}$ and solving, yields $x=4+2\sqrt{3}$ or $x=4-2\sqrt{3}$. Substituting in $y=7-x$ yields $(4+2\sqrt{3}, 3-2\sqrt{3})$ or $(4-2\sqrt{3}, 3+2\sqrt{3})$.

Pages 401–404 (Section 9.6)

1 a 25 b 36 c 21 d $1\frac{2}{3}$ e 60 **2** a 10 b 78 c 36 d 65 e $6\frac{1}{2}$ **3** a 250 b 48 c 28 d 2.4 e 264 **4** a 51 b 3.4 c 75 d $\frac{4}{5}$ e 80 **5** a 12 b $2\sqrt{7}$ c 10 d 0.5 e 34 f $5\sqrt{7}$ g 72 h 45 i $12\sqrt{7}$

6 (20, 48, ?) belongs to the (5, 12, 13) Family. $4\times 5=20$, $4\times 12=48$, $4\times 13=52$; the diagonal is 52.

7 If the base of the isos △ is 16, the height forms a rt △ with sides (8, 15, ?). This is the (8, 15, 17) family △ so the side of the △ is 17. Perimeter = 17 + 17 + 16 = 50 dm.