

Objective

After studying this section, you will be able to

- Apply theorems about the interior angles, the exterior angles, and the midlines of triangles.

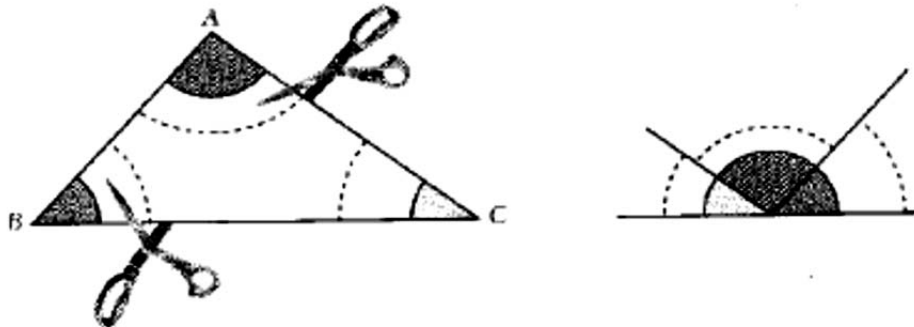
Theorem 50 *The sum of the measures of the three angles of a triangle is 180.*

External reasoning:

An authority (e.g., the book, teacher) told me, so it's true.

① En découpant

- Trace sur papier blanc un triangle ABC comme celui dessiné ci-dessous.
- Découpe chacun de ses angles, puis « regroupe »-les comme l'indique le dessin ci-dessous.



Empirical: single example

- Quelle semble être la valeur de la somme des angles du triangle ?

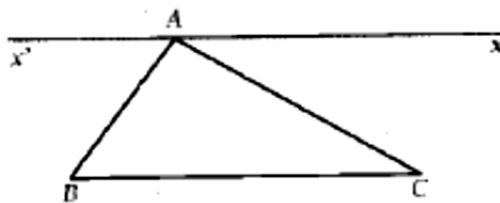
② En mesurant avec ton rapporteur

- Trace trois triangles.
- Mesure les angles de chacun de ces triangles à l'aide d'un rapporteur, puis calcule la somme des angles de chaque triangle.
- Quelle semble être la valeur de cette somme ?

Empirical:
3 examples

③ Une démonstration à présent

La droite $(x'x)$ est parallèle à la droite (BC) et passe par A.

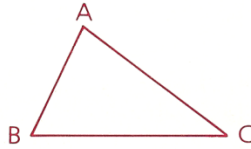


- Compare les angles \widehat{ABC} et $\widehat{BAx'}$, puis \widehat{ACB} et \widehat{CAx} .
- Explique alors pourquoi la somme des trois angles du triangle ABC est égale à 180° .

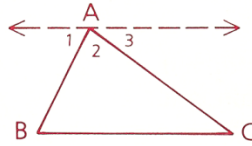
Analytic: Axiomatic

Given: $\triangle ABC$

Prove: $m\angle A + m\angle B + m\angle C = 180$

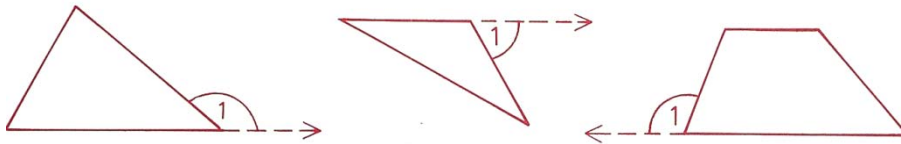


Proof: According to the Parallel Postulate, there exists exactly one line through point A parallel to \overleftrightarrow{BC} , so the figure at the right can be drawn.



Because of the straight angle, we know that $\angle 1 + \angle 2 + \angle 3 = 180^\circ$. Since $\angle 1 \cong \angle B$ and $\angle 3 \cong \angle C$ (by \parallel lines \Rightarrow alt. int. \angle s \cong), we may substitute to obtain $\angle B + \angle 2 + \angle C = 180^\circ$. Hence, $m\angle A + m\angle B + m\angle C = 180$.

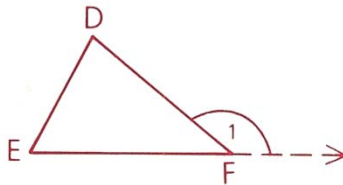
Definition An **exterior angle** of a polygon is an angle that is adjacent to and supplementary to an interior angle of the polygon.



Theorem 51 *The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.*

Given: $\triangle DEF$, with exterior angle 1 at F

Prove: $m\angle 1 = m\angle D + m\angle E$



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7.1: Triangle Application Theorems

Date

Theorem 52 *A segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is one-half the length of the third side. (Midline Theorem)*

The Geometer's Sketchpad - [Triangle Midline]

File Edit Display Construct Transform Measure Number Graph Window Help

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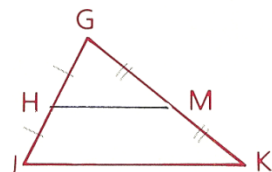
Triangle Midline Lab

Big triangle: $\triangle DEF$	Upper triangle: $\triangle DAB$	Lower left triangle: $\triangle ACE$	Center triangle: $\triangle ABC$	Lower right triangle: $\triangle BCF$
$m\overline{ED} = 7.45$ cm	$m\overline{DB} = 2.37$ cm	$m\overline{EC} = 2.49$ cm	$m\overline{AB} = 2.49$ cm	$m\overline{CF} = 2.49$ cm
$m\overline{DF} = 4.73$ cm	$m\overline{AB} = 2.49$ cm	$m\overline{CA} = 2.37$ cm	$m\overline{BC} = 3.73$ cm	$m\overline{BF} = 2.37$ cm
$m\overline{FE} = 4.97$ cm	$m\overline{AD} = 3.73$ cm	$m\overline{AE} = 3.73$ cm	$m\overline{CA} = 2.37$ cm	$m\overline{BC} = 3.73$ cm
$m\angle AEF = 38.66^\circ$	$m\angle ADB = 41.04^\circ$	$m\angle AEC = 38.66^\circ$	$m\angle CAB = 100.30^\circ$	$m\angle BFC = 100.30^\circ$
$m\angle ADB = 41.04^\circ$	$m\angle DAB = 38.66^\circ$	$m\angle EAC = 41.04^\circ$	$m\angle ABC = 38.66^\circ$	$m\angle CBF = 41.04^\circ$
$m\angle BFC = 100.30^\circ$	$m\angle DBA = 100.30^\circ$	$m\angle ACE = 100.30^\circ$	$m\angle ACB = 41.04^\circ$	$m\angle BCF = 38.66^\circ$
$m\angle AEF + m\angle ADB + m\angle BFC = 180.00^\circ$		$m\angle AEC + m\angle EAC + m\angle ACE = 180.00^\circ$		$m\angle BFC + m\angle CBF + m\angle BCF = 180.00^\circ$
$m\angle ADB + m\angle DAB + m\angle DBA = 180.00^\circ$		$m\angle CAB + m\angle ABC + m\angle ACB = 180.00^\circ$		

Sum of the interior angles of the triangle:

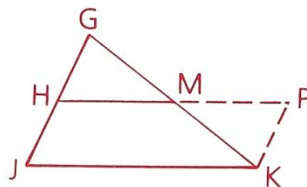
Given: H is a midpoint.
M is a midpoint.

Prove: a $\overline{HM} \parallel \overline{JK}$
b $HM = \frac{1}{2}(JK)$



Proof: Extend \overleftrightarrow{HM} through M to a point P so that $\overline{MP} \cong \overline{HM}$. P is now established, so P and K determine \overleftrightarrow{PK} .

We know that $\overline{GM} \cong \overline{KM}$ (by the definition of midpoint) and that $\angle GMH \cong \angle KMP$ (vertical \angle s are \cong). Thus, $\triangle GMH \cong \triangle KMP$ by SAS.



Since $\angle G = \angle PKM$ by CPCTC, $\overleftrightarrow{PK} \parallel \overleftrightarrow{HJ}$ by alt. int. \angle s \Rightarrow \parallel lines. Also, $\overline{GH} \cong \overline{PK}$ by CPCTC, and $\overline{GH} \cong \overline{HJ}$ (by the definition of midpoint). By transitivity, then, $\overline{PK} \cong \overline{HJ}$.

Two sides, \overline{PK} and \overline{HJ} , are parallel and congruent, so PKJH is a parallelogram. Therefore, $\overleftrightarrow{HP} \parallel \overleftrightarrow{JK}$.

Opposite sides of a parallelogram are congruent, so $HP = JK$.

Also, since we made $MP = HM$, $HM = \frac{1}{2}(HP)$ and, by substitution, $HM = \frac{1}{2}(JK)$.

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7.1: Triangle Application Theorems

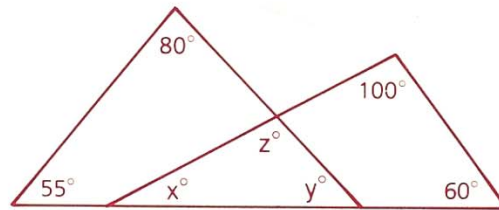
Date

Class Examples

Problem 1

Given: Diagram as marked

Find: x , y , and z



Solution

Since the sum of the measures of the angles of a triangle is 180,

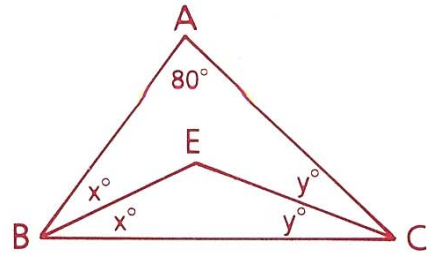
$$\begin{array}{ccc}
 \text{Right triangle} & \text{Left triangle} & \text{Center triangle} \\
 x + 100 + 60 = 180 & 55 + 80 + y = 180 & x + y + z = 180
 \end{array}$$

Problem 2

The measures of the three angles of a triangle are in the ratio 3:4:5.
Find the measure of the largest angle.

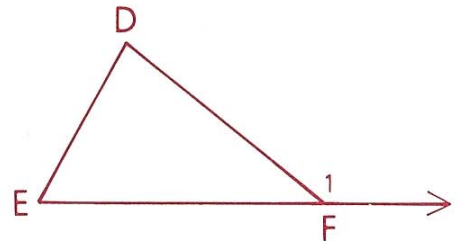
Problem 3

If one of the angles of a triangle is 80° , find the measure of the angle formed by the bisectors of the other two angles.



Problem 4

$\angle 1 = 150^\circ$, and the measure of $\angle D$ is twice that of $\angle E$. Find the measure of each angle of the triangle.



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7.1: Triangle Application Theorems

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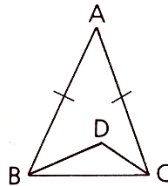
Homework

9 Tell whether each statement is true Always, Sometimes, or Never (A, S, or N).

- a The acute angles of a right triangle are complementary.
- b The supplement of one of the angles of a triangle is equal in measure to the sum of the other two angles of the triangle.
- c A triangle contains two obtuse angles.
- d If one of the angles of an isosceles triangle is 60° , the triangle is equilateral.
- e If the sides of one triangle are doubled to form another triangle, each angle of the second triangle is twice as large as the corresponding angle of the first triangle.

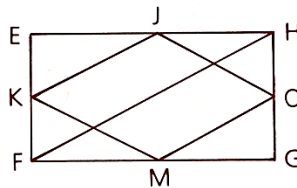
9a	9b	9c	9d	9e
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16 Given: $\angle A = 30^\circ$, $\overline{AB} \cong \overline{AC}$;
 \overrightarrow{CD} bisects $\angle ACB$.
 \overrightarrow{BD} is one of the trisectors of $\angle ABC$.
 Find: $m\angle D$



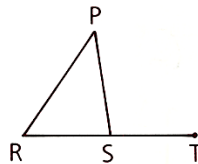
17 Given: EFGH is a rectangle.
 $FH = 20$;
 J, K, M, and O are midpoints.

- a Find the perimeter of JKMO.
- b What is the most descriptive name for JKMO?



18 Given: $\angle PST = (x + 3y)^\circ$,
 $\angle P = 45^\circ$, $\angle R = (2y)^\circ$,
 $\angle PSR = (5x)^\circ$

Find: $m\angle PST$



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7: Polygons

Ms. Kresovic

Course - period

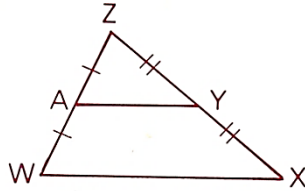
7.1: Triangle Application Theorems

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Classwork

All of the following exercises must be completed and handed in before the class ends.

- 7** In the diagram as marked,
if $WX = 18$, find AY .



- 12** In $\triangle DEF$, the sum of the measures of $\angle D$ and $\angle E$ is 110. The sum of the measures of $\angle E$ and $\angle F$ is 150. Find the sum of the measures of $\angle D$ and $\angle F$.