Notes:

Objectives

After studying this section, you will be able to

- Apply the Exterior Angle Inequality Theorem
- Use various methods to prove lines parallel

st∠ → supp, supp → 180° } ∠3+∠4=180°

Part One: Introduction 0 < Angle < 180

sum ∠s in Δ → 180° } ∠1+∠2+∠3=180°

Substitute: ∠3+∠4 = ∠1+∠2+∠3

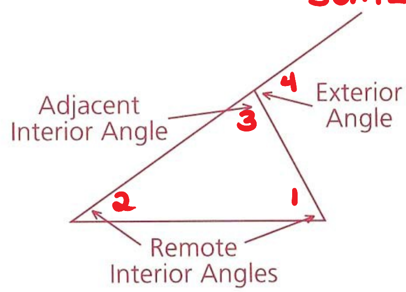
−∠3 −∠3

∠4 = ∠1+∠2

Ext ∠ = sum of rem. int. ∠s

The Exterior Angle Inequality Theorem

An exterior angle of a triangle is formed whenever a side of the triangle is extended to form an angle supplementary to the adjacent interior angle.

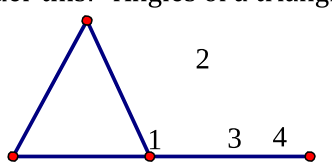


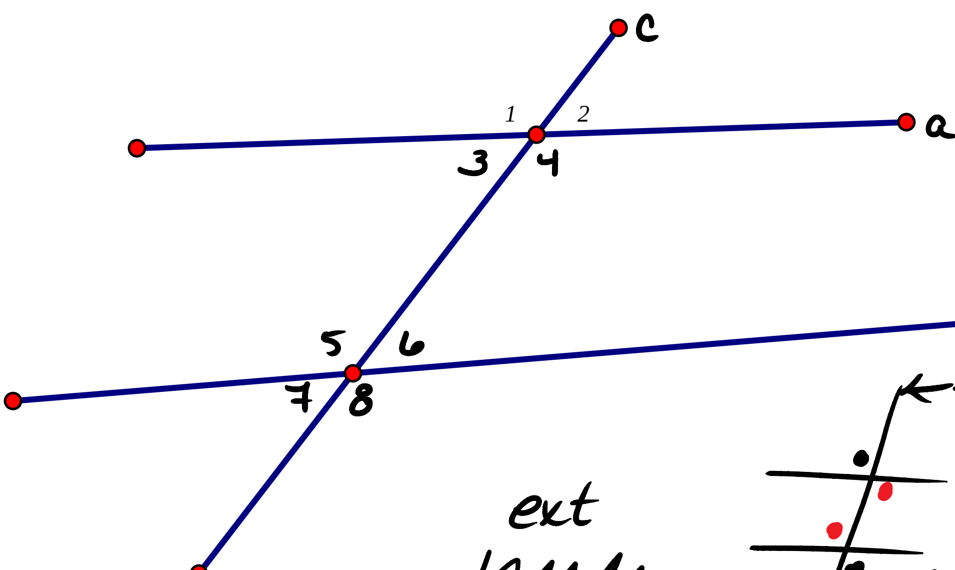
Theorem 30 ***The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.***

180 > Ext∠ > one rem int∠

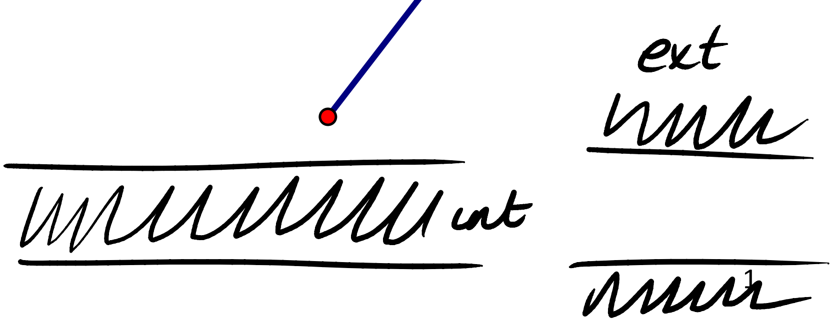
one rem. int∠ < Ext∠ < 180°

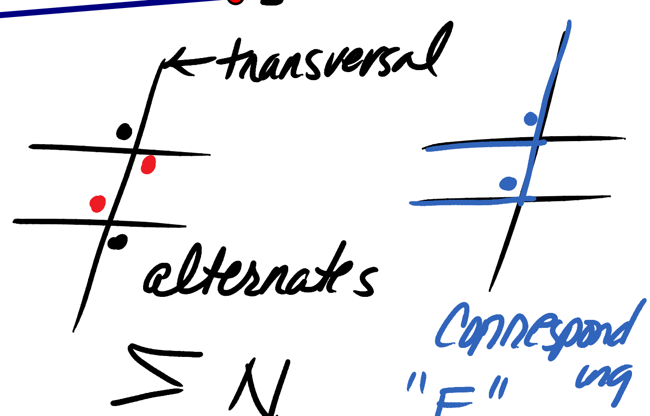
Consider this: Angles of a triangle sum to 180°. Supplementary angles sum to 180°.

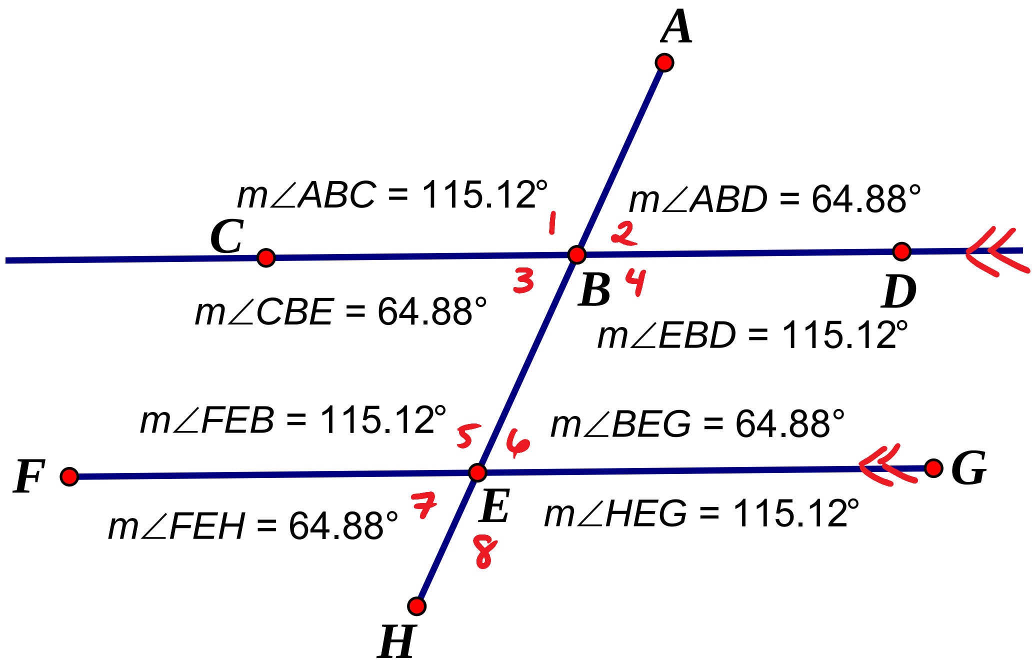




a ∥ b







ALT INT $\angle$S $\cong \Rightarrow \parallel$	$\angle 3$ & $\angle 6$, $\angle 4$ & $\angle 5$
ALT EXT $\angle$S $\cong \Rightarrow \parallel$	$\angle 1$ & $\angle 8$, $\angle 2$ & $\angle 7$
INT $\angle$S SST SUPP $\Rightarrow \parallel$	$\angle 3$ & $\angle 5$, $\angle 4$ & $\angle 6$
CORR $\angle$S $\cong \Rightarrow \parallel$	$\angle 3$ + $\angle 7$, $\angle 4$ + $\angle 8$, $\angle 1$ + $\angle 5$, $\angle 2$ + $\angle 6$

In the GSP demo, we observed:

Theorem 31 *If two lines are cut by a transversal such that two alternate interior angles are congruent, the lines are parallel. (Short form: Alt. int. ∠s ≅ ⇒ ∥ lines)*

Theorem 32 *If two lines are cut by a transversal such that two alternate exterior angles are congruent, the lines are parallel. (Alt. ext. ∠s ≅ ⇒ ∥ lines.)*

Theorem 33 *If two lines are cut by a transversal such that two corresponding angles are congruent, the lines are parallel. (Corr. ∠s ≅ ⇒ ∥ lines)*

Theorem 34 *If two lines are cut by a transversal such that two interior angles on the same side of the transversal are supplementary, the lines are parallel.*

Theorem 35 *If two lines are cut by a transversal such that two exterior angles on the same side of the transversal are supplementary, the lines are parallel.*

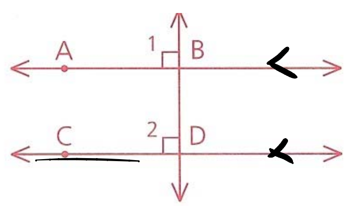
Theorem 36 *If two coplanar lines are perpendicular to a third line, they are parallel.*

Class Examples

Problem 1 Prove Theorem 36.

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{BD}$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



Proof

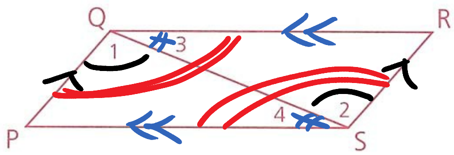
1 $\overleftrightarrow{BD} \perp \overleftrightarrow{AB}$	1 Given
2 ∠1 is a right ∠.	2 ⊥ → rt∠
3 $\overleftrightarrow{BD} \perp \overleftrightarrow{CD}$	3 Given
4 ∠2 is a right ∠.	4 ⊥ → rt∠
5 ∠1 ≅ ∠2	5 rt∠ → ≅∠s
6 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	6 corr ∠s ≅ → ∥

\

Problem 2 *A parallelogram is a four-sided figure with both pairs of opposite sides parallel.*

Given: $\angle 1 \cong \angle 2$,
$\angle PQR \cong \angle RSP$

Prove: PQRS is a parallelogram.



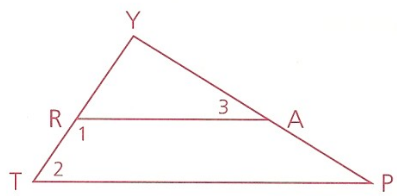
Proof

1 $\angle 1 \cong \angle 2$	1 Given
2 $\overline{PQ} \parallel \overline{RS}$	2 alt int ∠S ⇒ ∥
3 $\angle PQR \cong \angle RSP$	3 GIVEN
4 $\angle 3 \cong \angle 4$	4 SUBTRACT
5 $\overline{QR} \parallel \overline{PS}$	5 ALT INT∠S → ∥
6 PQRS is a parallelogram.	6 BTH PR OPP SDS ∥ → ▱

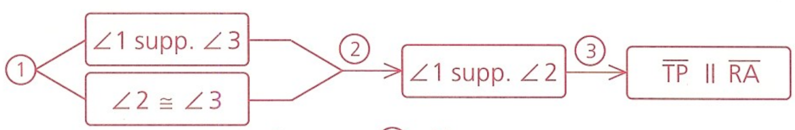
Problem 3 *A trapezoid is a four-sided figure with exactly one pair of parallel sides.*

Given: $\angle 1$ supp. $\angle 3$,
$\angle 2 \cong \angle 3$

Prove: TRAP is a trapezoid.



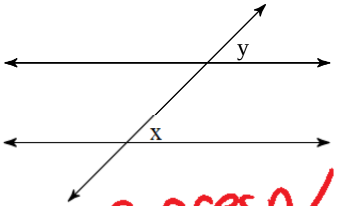
Proof We can use a flow diagram.



Reasons
1. Given
2. Substitution
3. Int. ∠s on same side supp. ⇒ ∥ lines

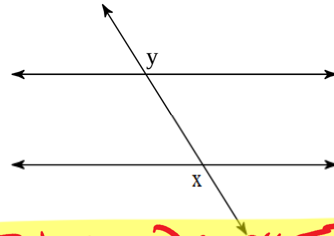
Identify each pair of angles as corresponding, alternate interior, alternate exterior, or consecutive interior.

1)



corresp $\angle$s

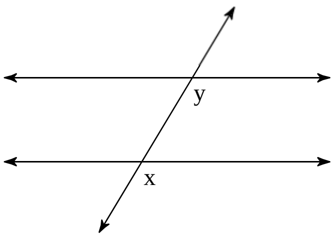
2)



alt ext

FINISH THE PACKET

3)



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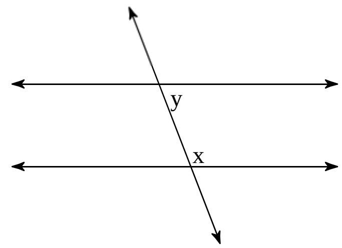

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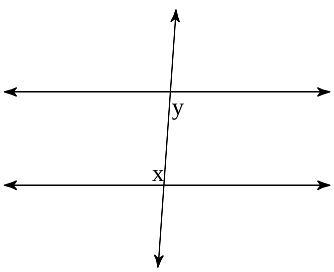

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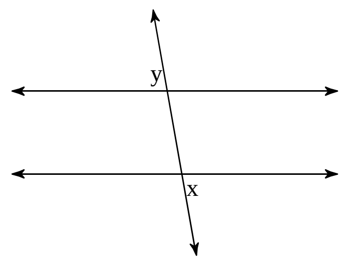

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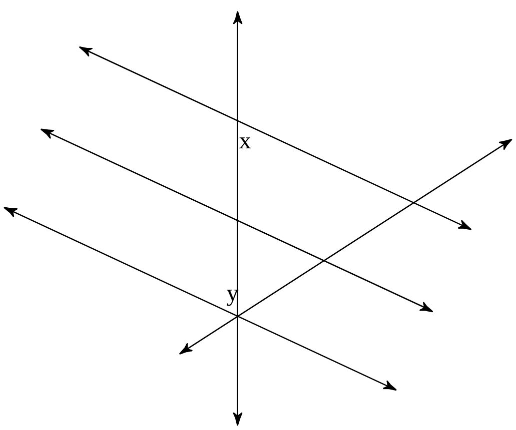

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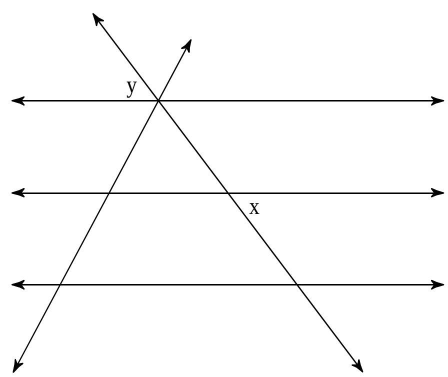

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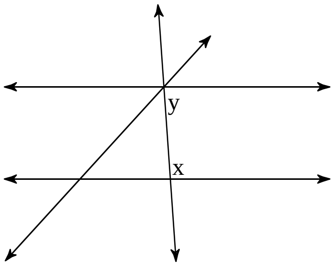

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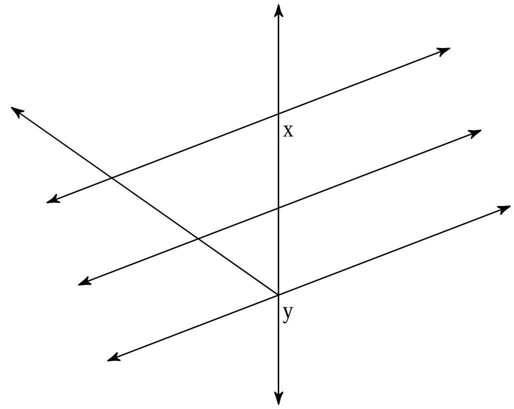
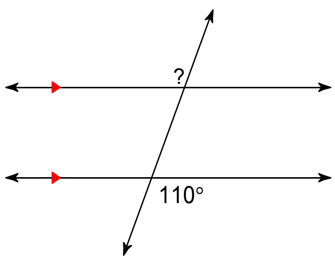


Find the measure of each angle indicated.

11)



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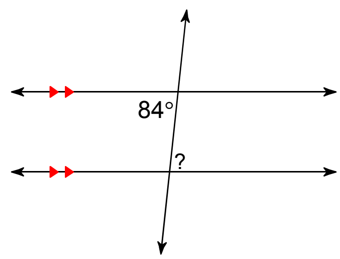

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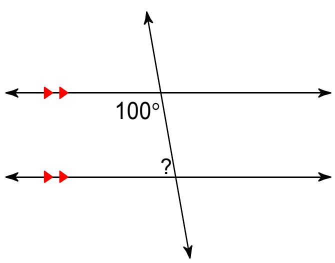

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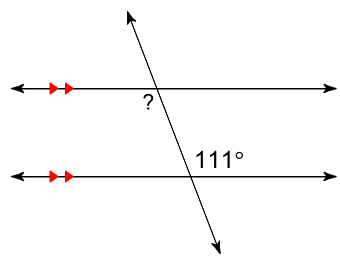

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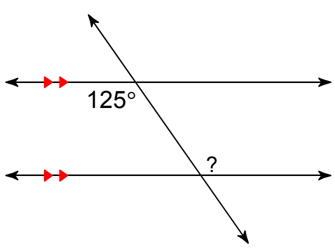

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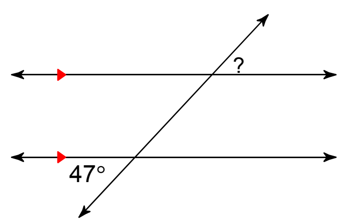

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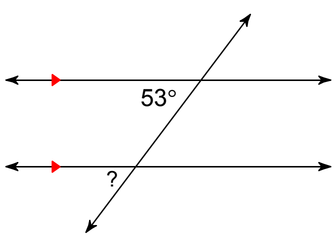

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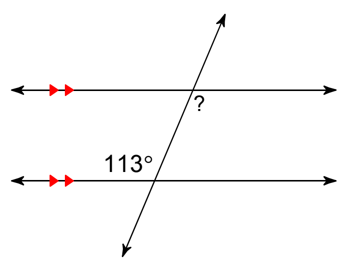
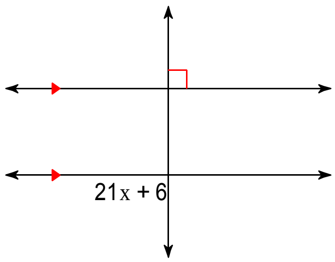


Solve for x.

AMDG

19)



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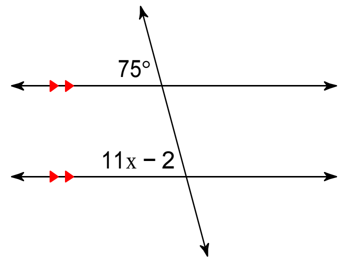

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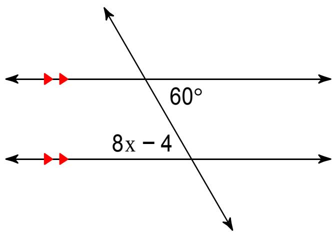

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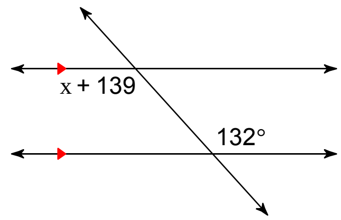

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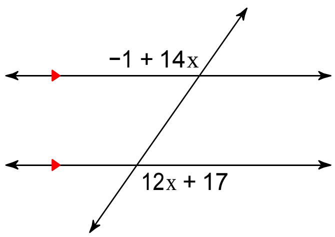

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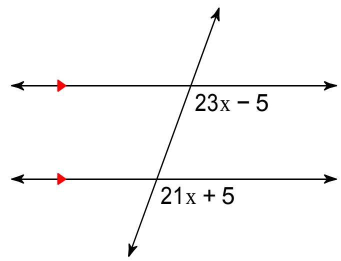
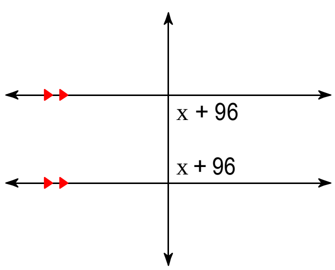


Find the measure of the angle indicated in bold.

25)



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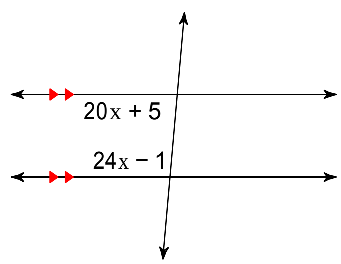

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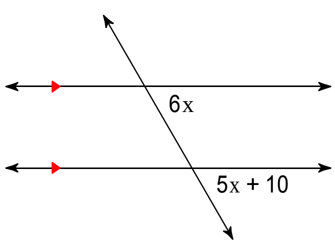

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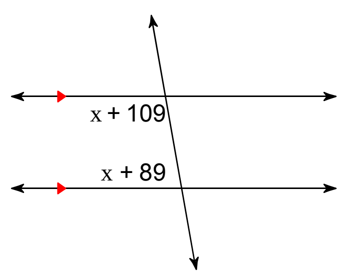
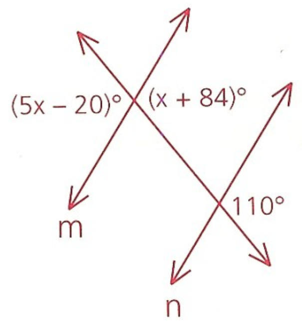
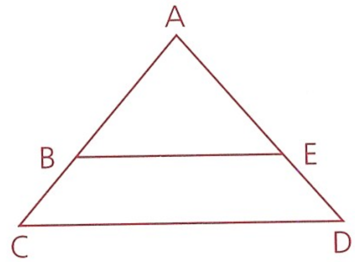


Homework:

16 Solve for x and justify that m ∥ n.



19 Given: ∠D ≅ ∠ABE,
$\overline{BE} \nparallel \overline{CD}$
Prove: $\overline{AC} \ncong \overline{AD}$



Either ________________ or ________________.

Assume ________________________________.

If sides then angles, so ________________________.

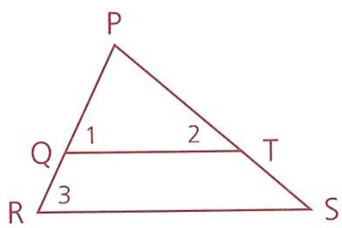
We are given that ∠D ≅ ∠ABE.

By the transitive property________________________.

If corr. ∠s ≅ then ||, so ______________. But this is impossible as it contradicts the given information ____________.

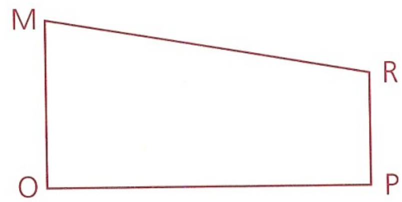
Consequently the assumption is false and ______________ is the only possibility. *QED*

20 Given: ∠1 comp. ∠2,
∠3 comp. ∠2
Prove: $\overline{QT} \parallel \overline{RS}$



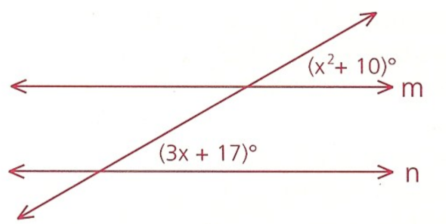
Statements	Reasons
1. ∠1 comp ∠2 & ∠3 comp ∠2	1. Given
2. ∠1 ≅ ∠3	2.
3.	3.

21 Given: $\angle MOP$ is a right angle.
$\overline{RP} \perp \overline{OP}$
Prove: $\overline{MO} \parallel \overline{RP}$



Statements	Reasons
1. $\angle MOP$ rt $\angle$	1. Given
2.	2. Given
3.	3.
4. $\angle MOP$ supp $\angle OPR$	4.
5.	5.

26 Find the value(s) of x (to the nearest tenth) that will allow you to prove that m ∥ n. (Hint: You may wish to review the quadratic formula.)



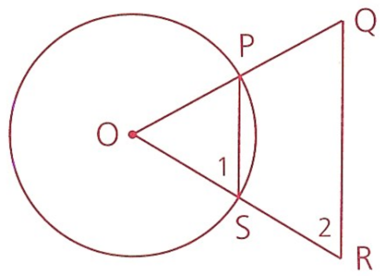
Quadratic Formula: For any quadratic in standard form, that is $ax^2 + bx + c = 0$, the solutions are

x =

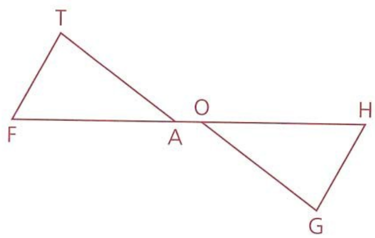
12 Given: $\odot O$,

$\angle 1 \cong \angle 2$

Prove: $\overline{PS} \parallel \overline{QR}$



Statements	Reasons
$\angle 1 \cong \angle 2$	
$\overline{PS} \parallel \overline{QR}$	



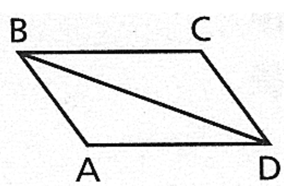
13 Given: $\angle FAT \cong \angle HOG$

Prove: $\overline{AT} \parallel \overline{GO}$

14 Given: $\overline{AB} \cong \overline{CD}$

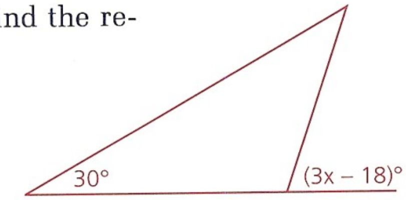
$\overline{BC} \cong \overline{AD}$

Prove: $\overline{AB} \parallel \overline{CD}$

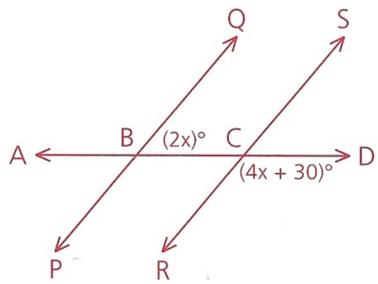


1 $\overline{AB} \cong \overline{CD}$	1
2 $\overline{BC} \cong \overline{AD}$	2
3 $\overline{BD} \cong \overline{BD}$	3
4 $\triangle BAD \cong \triangle DCB$	4
5 $\angle ABD \cong \angle BDC$	5
6 $\overline{AB} \parallel \overline{CD}$	6

17 Write a valid inequality and find the restrictions on x.



23 If $\overleftrightarrow{PQ} \nparallel \overleftrightarrow{RS}$, can x be 25? Explain.



In a problem like this, include reasons (even though it's not a proof).
Remember that geometry is about "explaining why something is true".

Either PQ || RS or PQ is not || RS. Assume PQ || RS. || ⇒ int. ∠s same side supp, so ∠QBC supp ∠SCA.
We know that vertical angles are congruent so ∠SCA ≅ ∠DCR.
Hence, 2x + 4x+30 = 180. (You need to finish this problem.)