AMDG

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Adv Geo – period 5 Date 3/17

10.4 Secants and Tangents

Objectives

After studying this section, you will be able to

- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents

Definition A ***secant*** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.) SEC

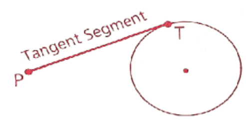
Definition A ***tangent*** is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact***.

Postulate ***A tangent line is perpendicular to the radius drawn to the point of contact.*** RAD $\cap$ TAN $\Rightarrow \perp$

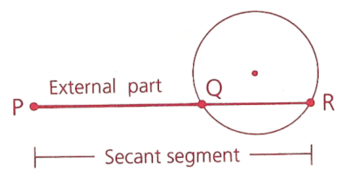
intersects

Postulate ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.*** $\perp \Rightarrow$ RAD $\cap$ TAN

Definition A ***tangent segment*** is the part of a tangent line between the point of contact and a point outside the circle.



Definition A ***secant segment*** is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

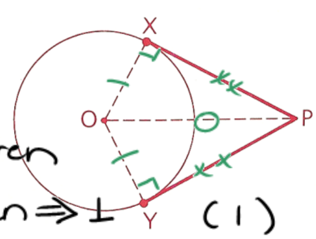


Definition The ***external part*** of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

Theorem 85 ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent.*** (***Two-Tangent Theorem***)

Given: $\odot O$;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

Prove: $\overline{PX} \cong \overline{PY}$



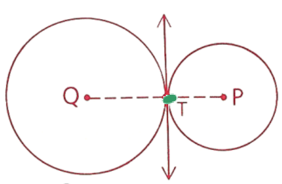
1. $\odot O$, $\overline{PX}$ & $\overline{PY}$ tan $\odot O$ | 1. given
2. $\overline{PX} \perp \overline{OX}$ & $\overline{PY} \perp \overline{OY}$ | 2. tan $\Rightarrow \perp$ (1)
RT∠ 3. $\angle OXP$ & $\angle OYP$ RT∠S | 3. $\perp \Rightarrow$ RT∠S (2)
H 4. $\overline{OP} \cong \overline{OP}$ | 4. REF
L 5. $\overline{OX} \cong \overline{OY}$ | 5. $\odot \Rightarrow \cong$ RADII (1)
6. $\triangle OXP \cong \triangle OYP$ | 6. HL (3,4,5)
7. $\overline{PX} \cong \overline{PY}$ | 7. CPCTC (6)

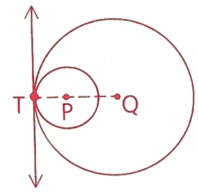
THIS IS THE PROOF OF THM 85. NOW WE CAN USE IT:

2-TAN $\Rightarrow \cong$ SEGS

Tangent Circles

Definition ***Tangent circles*** are circles that intersect each other at exactly one point.

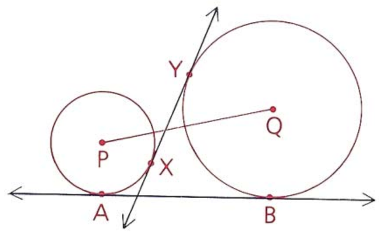




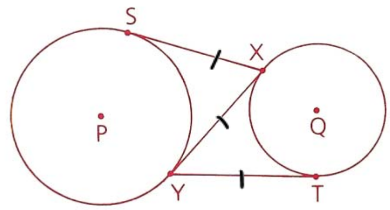
TODAY'S FOCUS

Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers.
$\overleftrightarrow{XY}$ is a ***common internal tangent.***
$\overleftrightarrow{AB}$ is a ***common external tangent.***



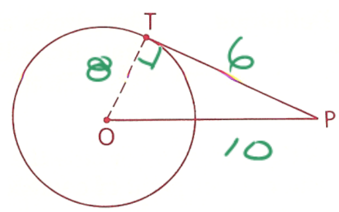
Problem 1 *Given: $\overline{XY}$ is a common internal tangent to ⓢ P and Q at X and Y. $\overline{XS}$ is tangent to ⊙P at S. $\overline{YT}$ is tangent to ⊙Q at T.*
Conclusion: $\overline{XS} \cong \overline{YT}$



Proof

1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q.	1 Given
2 $\overline{XY}$ is tangent to ⓢ P and Q.	2 Given
3 $\overline{XS} \cong \overline{XY}$	3 2 TAN ⇒ ≅ SEGS
4 $\overline{XY} \cong \overline{YT}$	4 2 TAN ⇒ ≅ SEGS
5 $\overline{XS} \cong \overline{YT}$	5 TRANSITIVE (3&4)

Problem 2 *$\overleftrightarrow{TP}$ is tangent to circle O at T. The radius of circle O is 8 mm. Tangent segment $\overline{TP}$ is 6 mm long. Find the length of $\overline{OP}$.*



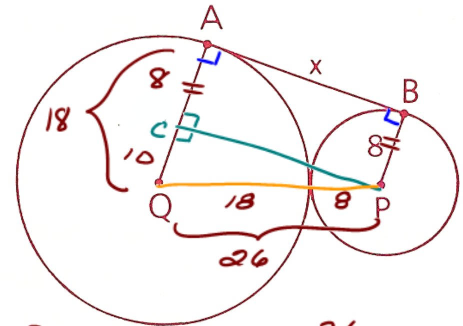
Solution

6, 8, ___

Common-Tangent Procedure

1. Draw the segment joining the centers.
2. Draw the radii to the points of contact.
3. Through the center of the smaller circle, draw a line parallel to the common tangent.
4. Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.
5. Use the Pythagorean Theorem and properties of a rectangle.

Problem 3 *A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.*



Solution

① rad ∩ tan ⇒ ⊥

② Draw Rect, that is DRAW CP ∥ AB

③ Draw Hypotenuse

ΔQCP: x 10 26

2($\underline{12}$ 5 13)

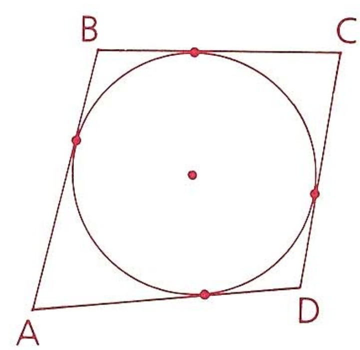
∴ x = 24 cm

SAVE FOR TOMORROW

Problem 4 *A walk-around problem:*

Given: Each side of quadrilateral ABCD is tangent to the circle. AB = 10, BC = 15, AD = 18

Find: CD



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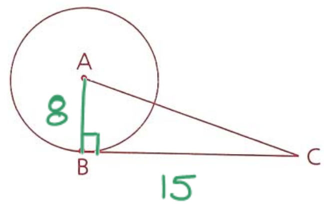
Adv Geo – period _____

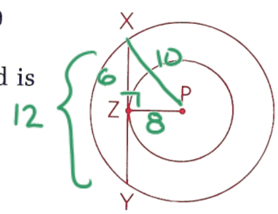
Date ___________________

Homework 10.4 Secants and Tangents

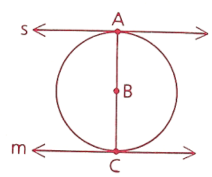
1 The radius of $\odot$A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$. = 17



2 Concentric circles with radii 8 and 10 have center P.
$\overline{XY}$ is a tangent to the inner circle and is a chord of the outer circle.
Find $\overline{XY}$. (Hint: Draw $\overline{PX}$ and $\overline{PY}$.)



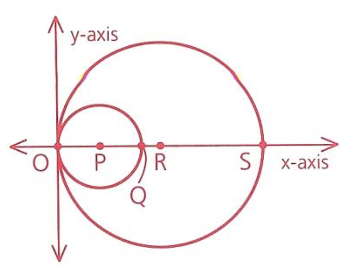
4 Given: $\overline{AC}$ is a diameter of $\odot$B.
Lines s and m are tangents to the $\odot$ at A and C.
Conclusion: s ∥ m



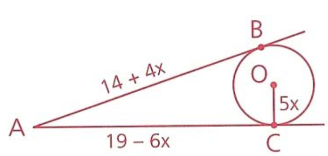
5 $\odot$P and $\odot$R are internally tangent at O.
P is at (8, 0) and R is at (19, 0).

a Find the coordinates of Q and S.

b Find the length of $\overline{QR}$.



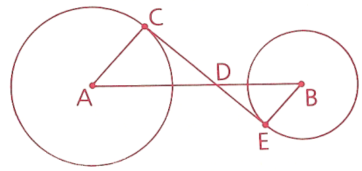
6 $\overline{AB}$ and $\overline{AC}$ are tangents to $\odot$O, and OC = 5x. Find OC.



7 Given: $\overline{CE}$ is a common internal tangent to circles A and B at C and E.

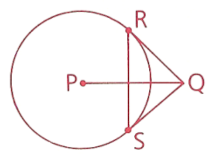
Prove: **a** $\angle A \cong \angle B$

b $\frac{AD}{BD} = \frac{CD}{DE}$



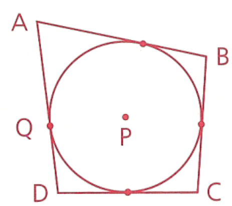
8 Given: $\overline{QR}$ and $\overline{QS}$ are tangent to $\odot P$ at points R and S.

Prove: $\overline{PQ} \perp \overline{RS}$ (Hint: This can be proved in just a few steps.)



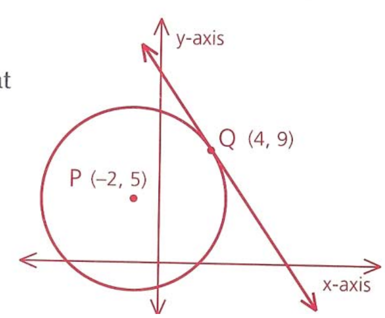
Skip, Save for Friday

10 $\odot P$ is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.



11 **a** Find the radius of $\odot P$.

b Find the slope of the tangent to $\odot P$ at point Q.



12 Two concentric circles have radii 3 and 7. Find, to the nearest hundredth, the length of a chord of the larger circle that is tangent to the smaller circle. (See problem 2 for a diagram.)

Thursday's homework Stops HERE

13 The centers of two circles of radii 10 cm and 5 cm are 13 cm apart.

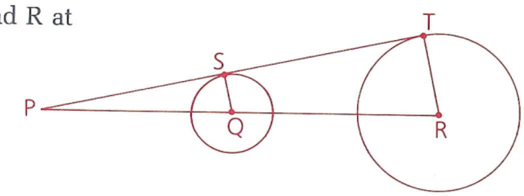
a Find the length of a common external tangent. (Hint: Use the common-tangent procedure.)

b Do the circles intersect?

14 The centers of two circles with radii 3 and 5 are 10 units apart. Find the length of a common internal tangent. (Hint: Use the common-tangent procedure.)

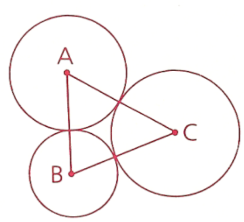
15 Given: $\overline{PT}$ is tangent to ⓢ Q and R at points S and T.

Conclusion: $\frac{PQ}{PR} = \frac{SQ}{TR}$



16 Given: Tangent ⓢ A, B, and C, AB = 8, BC = 13, AC = 11

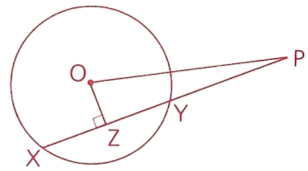
Find: The radii of the three ⓢ (Hint: This is a walk-around problem.)



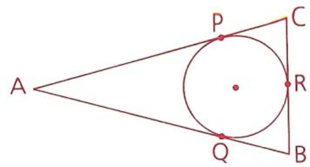
17 The radius of ⊙O is 10.
The secant segment $\overline{PX}$ measures 21 and is 8 units from the center of the ⊙.

a Find the external part (PY) of the secant segment.

b Find OP.



18 Given: △ABC is isosceles, with base $\overline{BC}$.
Conclusion: $\overline{BR} \cong \overline{RC}$



19 If two of the seven circles are chosen at random, what is the probability that the chosen pair are

a Internally tangent?

b Externally tangent?

c Not tangent?