

**Objectives**

- After studying this section, you will be able to
- Apply the addition properties of segments and angles
  - Apply the subtraction properties of segments and angles

Last year in algebra, you learned the reflexive property.

Reflexive Property	A quantity is congruent (equal) to itself. $a = a$
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ASN supplement: An angle (or segment) is congruent to itself (Reflexive Property). REF

**Theorem 8** *If a segment is added to two congruent segments, the sums are congruent. (Addition Property)*

Given:  $\overline{PQ} \cong \overline{RS}$   
 Conclusion:  $\overline{PR} \cong \overline{QS}$



ASV } Proof:  $\overline{PQ} \cong \overline{RS}$ , so by definition of congruent segments,  $PQ = RS$ .  
 Now, the Addition Property of Equality says that we may add  $QR$  to both sides, so  $PQ + QR = RS + QR$ . Substituting, we get  $PR = QS$ . Therefore,  $\overline{PR} \cong \overline{QS}$  by the definition of congruent segments (reversed).

Proofs of theorems 9-13 can be created by modeling the proof provided for theorem 8. Compare and contrast the theorem you are trying to prove with the proof provided for theorem 8, and make appropriate adjustments.

**Theorem 9** *If an angle is added to two congruent angles, the sums are congruent. (Addition Property)*

**Theorem 10** *If congruent segments are added to congruent segments, the sums are congruent. (Addition Property)*

**Theorem 11** *If congruent angles are added to congruent angles, the sums are congruent. (Addition Property)*

**Theorem 12** *If a segment (or angle) is subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)*

**Theorem 13** *If congruent segments (or angles) are subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)*

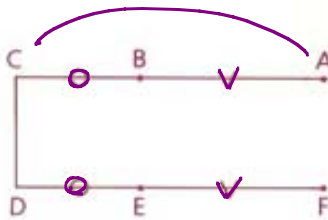
Add or SUBTRACT

**Using the Addition and Subtraction Properties in Proofs**

- 1 An addition property is used when the segments or angles in the conclusion are greater than those in the given information.
- 2 A subtraction property is used when the segments or angles in the conclusion are smaller than those in the given information.

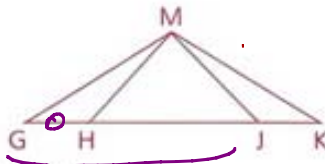
**Part Two: Sample Problems**

**Problem 1** Given:  $\overline{AB} \cong \overline{FE}$ ,  
 $\overline{BC} \cong \overline{ED}$   
 Prove:  $\overline{AC} \cong \overline{FD}$



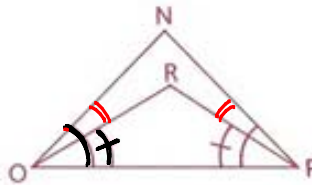
Statements	Reasons
1 $\overline{AB} \cong \overline{FE}$	1 Given
2 $\overline{BC} \cong \overline{ED}$	2 Given
3 $\overline{AC} \cong \overline{FD}$	3 Add

**Problem 2** Given:  $\overline{GJ} \cong \overline{HK}$   
 Conclusion:  $\overline{GH} \cong \overline{JK}$



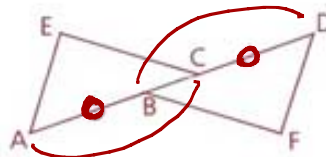
Statements	Reasons
1 $\overline{GJ} \cong \overline{HK}$	1 Given
2 $\overline{HJ} \cong \overline{HJ}$	2 REF.
3. $\overline{GH} \cong \overline{JK}$	3. Subtract

**Problem 3** Given:  $\angle NOP \cong \angle NPO$ ,  
 $\angle ROP \cong \angle RPO$   
 Prove:  $\angle NOR \cong \angle NPR$



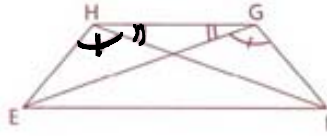
Statements	Reasons
1 $\angle NOP \cong \angle NPO$	1 Given
2 $\angle ROP \cong \angle RPO$	2 Given
3 $\angle NOR \cong \angle NPR$	3 Subtract

**Problem 4** Given:  $\overline{AB} \cong \overline{CD}$   
 Conclusion:  $\underline{\hspace{1cm}}$



Statements	Reasons
1 $\overline{AB} \cong \overline{CD}$	1 Given
2 $\overline{BC} \cong \overline{BC}$	2 REF
3. $\overline{AC} \cong \overline{ED}$	3. Add

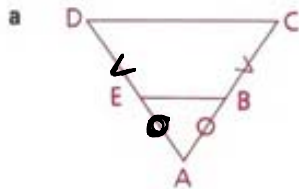
**Problem 5** Given:  $\angle HEF$  is supp. to  $\angle EHG$ .  
 $\angle GFE$  is supp. to  $\angle FGH$ .  
 $\angle EHF \cong \angle FGE$ .  
 $\angle GHF \cong \angle HGE$   
 Conclusion:  $\angle HEF \cong \angle GFE$



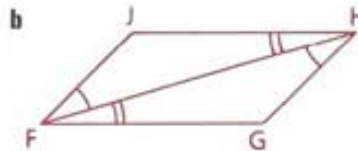
Proof	Statements	Reasons
4	$\angle HEF$ is supp. to $\angle EHG$ .	4 Given
5	$\angle GFE$ is supp. to $\angle FGH$ .	5 Given
1	$\angle EHF \cong \angle FGE$	1 Given
2	$\angle GHF \cong \angle HGE$	2 Given
3	$\angle EHG \cong \angle FGH$	3 Add (1,2)
6	$\angle HEF \cong \angle GFE$	6 $\angle s$ supp $\cong \angle s$ (3,4,5) $\Rightarrow \cong \angle s$

Throughout this problem set, think of addition when you are asked to prove that segments or angles are larger than the given segments or angles. Think of subtraction when you are asked to prove that segments or angles are smaller than the given segments or angles.

1 Name the angles or segments that are congruent by the Addition Property.

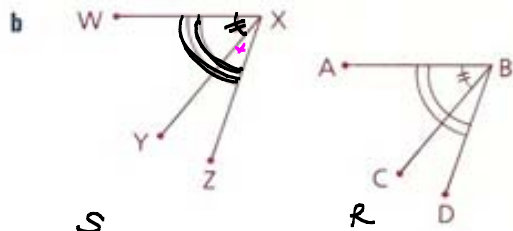
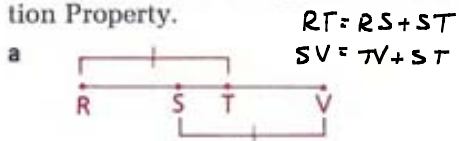


S	R
1. $\overline{AE} \cong \overline{EB}$	1. Given
2. $\overline{EB} \cong \overline{BC}$	2. Given
3. $\overline{DA} \cong \overline{CA}$	3. Add



S	R
1. $\angle JFH \cong \angle HFE$	1. Given
2. $\angle HFG \cong \angle FGH$	2. Given
3. $\angle JFG \cong \angle GHJ$	3. Add

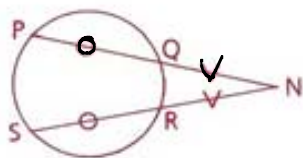
2 Name the angles or segments that are congruent by the Subtraction Property.



S	R
1. $\overline{RT} \cong \overline{SV}$	1. GIVEN
2. $\overline{ST} \cong \overline{ST}$	2. REFLEXIVE PROP
3. $\overline{RS} \cong \overline{TV}$	3. SUBTRACT

S	R
1. $\angle WXZ \cong \angle ABD$	1. Given
2. $\angle WXY \cong \angle CBA$	2. Given
3. $\angle YXZ \cong \angle CBD$	3. Subtract

3 Given:  $\overline{PQ} \cong \overline{SR}$ ,  
 $\overline{QN} \cong \overline{RN}$   
 Conclusion:  $\overline{PN} \cong \overline{SN}$



S	R
1. $\overline{PQ} \cong \overline{SR}$	1. Given
2. $\overline{PN} \cong \overline{SN}$	2. Add

REMINDERS:

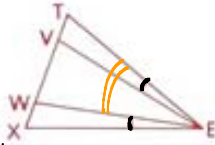
- Tomorrow ~~we will learn about Multiplication and Division Properties (2.6), then you take a Short Quiz on 2.4-2.5.~~ When you finish, hand in the quiz and begin the 2.6 homework.
- ~~Tonight: Update the ASN through 2.6. Review 2.4-2.5. We have spent the past two days reviewing this material. All class notes are on the website.~~
- ~~The test for this chapter is scheduled for next week Thursday. ©~~



What did the acorn say when it grew up?

**Homework**

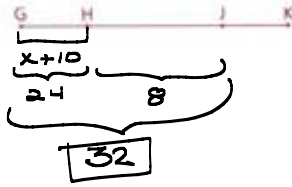
4 Given:  $\angle TEV \cong \angle XEW$   
Prove:  $\angle TEW \cong \angle XEW$



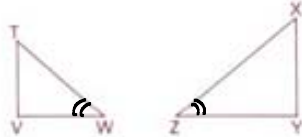
<p>S</p> <p>1. <math>\angle TEV \cong \angle XEW</math></p> <p>2. <math>\angle VEW \cong \angle WEV</math></p> <p>3. <math>\angle TEW \cong \angle XEW</math></p>	<p>R</p> <p>1. Given</p> <p>2. Ref</p> <p>3. Add</p>
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6 Given:  $\overline{GH} \cong \overline{JK}$ ,  $GH = x + 10$ ,  
 $HJ = 8$ ,  $JK = 2x - 4$

Find: GJ  
 $\overline{GH} = \overline{JK}$   
 $x + 10 = 2x - 4$   
 $14 = x$



8 Given:  $\angle T$  is comp. to  $\angle W$ ,  
 $\angle X$  is comp. to  $\angle Z$ ,  
 $\angle Z \cong \angle W$   
Prove: ?

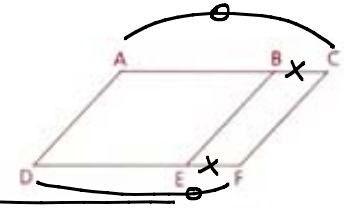


<p>S</p> <p>1. <math>\angle T</math> comp <math>\angle W</math> <math>\angle X</math> comp <math>\angle Z</math> <math>\angle Z \cong \angle W</math></p> <p>2. <math>\angle T \cong \angle X</math></p>	<p>R</p> <p>1. Given</p> <p>2. <math>\angle s</math> comp <math>\cong \angle s \Rightarrow \cong \angle s</math> (1)</p>
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10 Given:  $\angle BAD$  is a right  $\angle$ ,  
 $\overline{CA} \perp \overline{AE}$   
Prove:  $\angle BAC \cong \angle EAD$

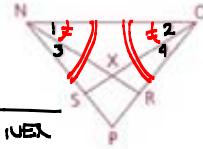


5 Given:  $\overline{AC} \cong \overline{DF}$ ,  
 $\overline{BC} \cong \overline{EF}$   
Prove:  $\overline{AB} \cong \overline{DE}$



<p>S</p> <p>1. <math>\overline{AC} \cong \overline{DF}</math> <math>\overline{BC} \cong \overline{EF}</math></p> <p>2. <math>\overline{AB} \cong \overline{DE}</math></p>	<p>R</p> <p>1. GIVEN</p> <p>2. Subtraction</p>
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7 Given:  $\angle PNO \cong \angle PON$ ,  
 $\angle 1 \cong \angle 2$   
Conclusion: ?

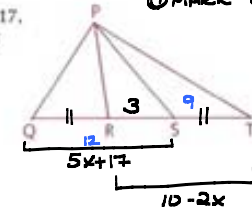


<p>S</p> <p>1. <math>\angle PNO \cong \angle PON</math> <math>\angle 1 \cong \angle 2</math></p> <p>2. <math>\angle 3 \cong \angle 4</math></p>	<p>R</p> <p>1. GIVEN</p> <p>2. SUBTRACT</p>
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9 Given:  $\overline{QR} \cong \overline{ST}$ ,  $QS = 5x + 17$ ,  
 $RT = 10 - 2x$ ,  $RS = 3$   
Find: QS and QT

3/3/1

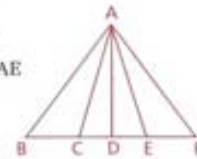
MARK DIAG



$\overline{QR} \cong \overline{ST}$  (g)  
 $QR = ST$  ( $\cong$  seg  $\Rightarrow$  meas)  
 $RS = RS$  (reflexive prop)  
 $QS = RT$  (add)  
 $5x + 17 = 10 - 2x$  (substitute)  
 $7x = -7$   
 $x = -1$

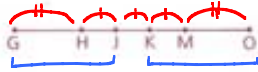
$\rightarrow QS = 5(-1) + 17 = 12$   
 $\rightarrow R = 12 - 3 = 9 = ST$   
therefore  $\therefore QT = 12 + 9 = 21$

11 Given:  $\angle BAD \cong \angle FAD$ ;  
 $\overline{AD}$  bisects  $\angle CAE$ .  
Conclusion:  $\angle BAC \cong \angle FAE$



9/24 Q+A:

12 Given: J and K are trisection points of  $\overline{HM}$ .  
 $\overline{GH} \cong \overline{MO}$



Conclusion:  $\overline{GJ} \cong \overline{KO}$

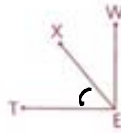
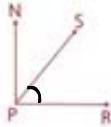
Statements

1. J & K trisect  $\overline{HM}$
2.  $\overline{HJ} \cong \overline{KM}$
3.  $\overline{GH} \cong \overline{MO}$
4.  $\overline{GJ} \cong \overline{KO}$

Reasons

1. Given
2. trisect  $\Rightarrow$   $\cong$  segs (1)
3. Given
4. Add (2,3)

13 Given:  $\angle NPR$  is a right  $\angle$ .  
 $\overline{WE} \perp \overline{ET}$ .  
 $\angle SPR \cong \angle XET$   
 Prove:  $\angle NPS \cong \angle WEX$



Statements

1.  $\angle NPR$  rt  $\angle$
2.  $\overline{WE} \perp \overline{ET}$
3.  $\angle WET$  rt  $\angle$
- 4.

Reasons

1. Given
2. Given
3.  $\perp \Rightarrow$  rt  $\angle$

14 Given:  $\angle A$  is comp. to  $\angle B$ . }  $\angle A \cong \angle C$  ( $\angle$ s comp same  $\angle \Rightarrow \cong$ )  
 $\angle C$  is comp. to  $\angle B$ . }  $3x+4 = 3y-3$   
 $\angle A = (3x+y)^\circ$   
 $\angle B = (x+4y+2)^\circ$   
 $\angle C = (3y-3)^\circ$

Find:  $m\angle B$

$$3x - 2y = -3$$

$$\begin{aligned} \angle A \text{ comp } \angle B \\ \angle A + \angle B &= 90 \\ 3x+y + x+4y+2 &= 90 \\ 4x+5y &= 88 \end{aligned}$$

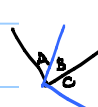
$$\begin{cases} 3x - 2y = -3 \\ 4x + 5y = 88 \end{cases}$$

$$\begin{aligned} \begin{cases} 12x - 8y = -12 \\ -12x - 15y = -264 \end{cases} \\ \hline -23y = -276 \\ y = 12 \end{aligned}$$

$$\begin{aligned} \text{If } y &= 12 \\ 4x + 60 &= 88 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} \angle B &= x + 4y + 2 \\ &= 7 + 4(12) + 2 \\ &= \boxed{57^\circ} \end{aligned}$$

DRAW



$$\begin{aligned} -3(80+8) \\ 240, 24 \end{aligned}$$