

Objectives

After studying this section, you will be able to

- Apply the addition properties of segments and angles
- Apply the subtraction properties of segments and angles

Last year in algebra, you learned the reflexive property.

Reflexive Property	A quantity is congruent (equal) to itself. $a = a$
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ASN supplement: An angle (or segment) is congruent to itself (Reflexive Property).

Theorem 8 If a segment is added to two congruent segments, the sums are congruent. (Addition Property)

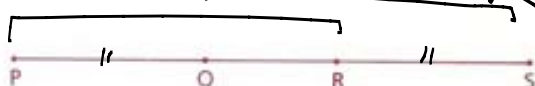
then homework

G: $\overline{PQ} \cong \overline{RS}$
P: $\overline{PQ} \cong \overline{RS}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. REF
3. $\overline{PR} \cong \overline{QS}$	3. Add (1,2)

Given: $\overline{PQ} \cong \overline{RS}$

Conclusion: $\overline{PR} \cong \overline{QS}$



Proof: $\overline{PQ} \cong \overline{RS}$, so by definition of congruent segments, $PQ = RS$. Now, the Addition Property of Equality says that we may add QR to both sides, so $PQ + QR = RS + QR$. Substituting, we get $PR = QS$. Therefore, $\overline{PR} \cong \overline{QS}$ by the definition of congruent segments (reversed).

For System

Proofs of theorems 9-13 can be created by modeling the proof provided for theorem 8. Compare and contrast the theorem you are trying to prove with the proof provided for theorem 8, and make appropriate adjustments.

Theorem 9 If an angle is added to two congruent angles, the sums are congruent. (Addition Property)

Theorem 10 If congruent segments are added to congruent segments, the sums are congruent. (Addition Property)

Theorem 11 If congruent angles are added to congruent angles, the sums are congruent. (Addition Property)

Theorem 12 If a segment (or angle) is subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)

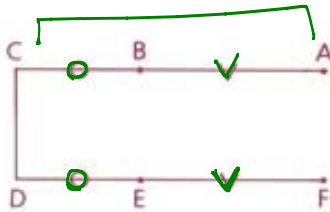
Theorem 13 If congruent segments (or angles) are subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)

Using the Addition and Subtraction Properties in Proofs

- 1 An addition property is used when the segments or angles in the conclusion are greater than those in the given information.
- 2 A subtraction property is used when the segments or angles in the conclusion are smaller than those in the given information.

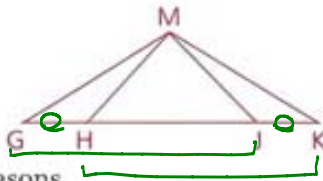
Part Two: Sample Problems

Problem 1 Given: $\overline{AB} \cong \overline{FE}$,
 $\overline{BC} \cong \overline{ED}$
 Prove: $\overline{AC} \cong \overline{FD}$



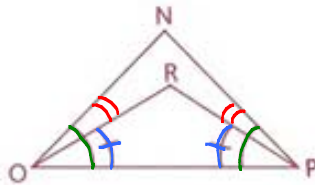
Statements	Reasons
1 $\overline{AB} \cong \overline{FE}$	1 Given
2 $\overline{BC} \cong \overline{ED}$	2 Given
3 $\overline{AC} \cong \overline{FD}$	3 Add

Problem 2 Given: $\overline{GJ} \cong \overline{HK}$
 Conclusion: $\overline{GH} \cong \overline{JK}$



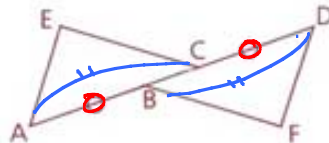
Statements	Reasons
1 $\overline{GJ} \cong \overline{HK}$	1 Given
2 $\overline{HJ} \cong \overline{HJ}$	2 Ref
3 $\overline{GH} \cong \overline{JK}$	3 Subtract

Problem 3 Given: $\angle NOP \cong \angle NPO$,
 $\angle ROP \cong \angle RPO$
 Prove: $\angle NOR \cong \angle NPR$



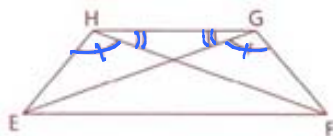
Statements	Reasons
1 $\angle NOP \cong \angle NPO$	1 Given
2 $\angle ROP \cong \angle RPO$	2 Given
3 $\angle NOR \cong \angle NPR$	3 Subtract

Problem 4 Given: $\overline{AB} \cong \overline{CD}$
 Conclusion: ?



Statements	Reasons
1 $\overline{AB} \cong \overline{CD}$	1 Given
2 $\overline{BC} \cong \overline{BC}$	2 Ref
3 $\overline{AC} \cong \overline{BD}$	3 Add

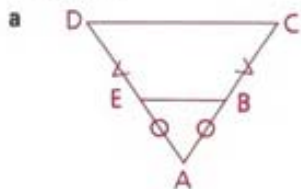
Problem 5 Given: $\angle HEF$ is supp. to $\angle EHG$.
 $\angle GFE$ is supp. to $\angle FGH$.
 $\angle EHF \cong \angle FGE$.
 $\angle GHF \cong \angle HGE$
 Conclusion: $\angle HEF \cong \angle GFE$



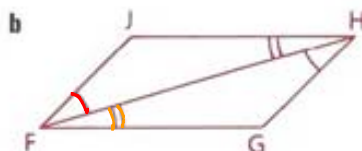
Proof	Statements	Reasons
	1 $\angle HEF$ is supp. to $\angle EHG$.	1 Given
	2 $\angle GFE$ is supp. to $\angle FGH$.	2 Given
	3 $\angle EHF \cong \angle FGE$	3 Given
	4 $\angle GHF \cong \angle HGE$	4 Given
	5 $\angle EHG \cong \angle FGH$	5 Add (3,4)
	6 $\angle HEF \cong \angle GFE$	6 $\angle s$ suppto $\cong \angle s \Rightarrow \cong \angle s$ (1,2,5)

Throughout this problem set, think of addition when you are asked to prove that segments or angles are larger than the given segments or angles. Think of subtraction when you are asked to prove that segments or angles are smaller than the given segments or angles.

1 Name the angles or segments that are congruent by the Addition Property.



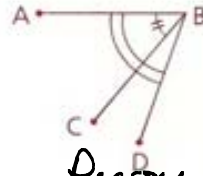
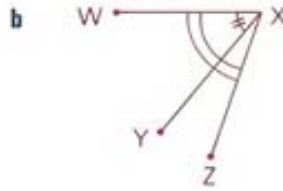
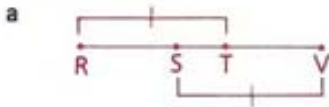
Statements	Reasons
1. $\overline{AE} \cong \overline{DB}$ $\overline{ED} \cong \overline{BC}$	1. Given
2. $\overline{AD} \cong \overline{AC}$	2. Add



Statements	Reasons
1. $\angle JFH \cong \angle GHF$ $\angle HFG \cong \angle FGH$	1. Given
2. $\angle JFG \cong \angle GHJ$	2. Add

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2 Name the angles or segments that are congruent by the Subtraction Property.



Statements

Reasons

1. $\overline{RT} \cong \overline{SV}$
2. $\overline{ST} \cong \overline{ST}$
3. $\overline{RS} \cong \overline{TU}$

1. Given
2. Ref
3. Subtract

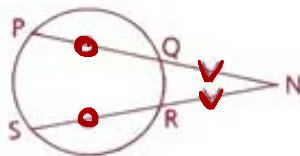
Statements

Reasons

1. $\angle WXYZ \cong \angle ABD$
2. $\angle WXY \cong \angle ABC$
3. $\angle YXZ \cong \angle CBD$

1. Given
2. Given
3. Subtract

3 Given: $\overline{PQ} \cong \overline{SR}$,
 $\overline{QN} \cong \overline{RN}$
 Conclusion: $\overline{PN} \cong \overline{SN}$



Statements

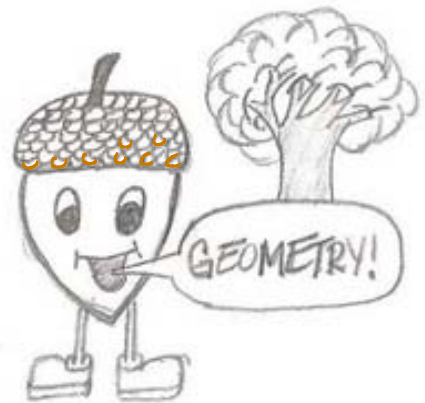
Reasons

1. $\overline{PQ} \cong \overline{SR}$
 $\overline{QN} \cong \overline{RN}$
2. $\overline{PN} \cong \overline{SN}$

1. Given
2. Add

REMINDERS:

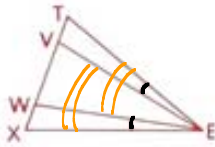
- Tomorrow ~~we will learn about Multiplication and Division Properties (2.6)~~, then you take a ~~quiz~~. When you finish, hand in the quiz and begin the 2.6 homework.
- Tonight: Update the ASN through 2.6. Review 2.4-2.5. We have spent the past two days reviewing this material. All class notes are on the website.
- ~~The test for this chapter is scheduled for next week Thursday.~~



What did the acorn say when it grew up?

Homework

4 Given: $\angle TEV \cong \angle XEW$
Prove: $\angle TEW \cong \angle XEW$

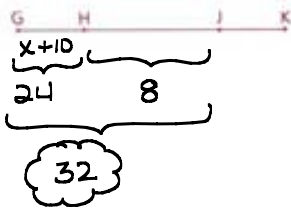


Statements	Reasons
1. $\angle TEV \cong \angle XEW$	1. GIVEN
2. $\angle VEW \cong \angle VEW$	2. REFLEXIVE PROP.
3. $\angle TEW \cong \angle XEW$	3. Add

6 Given: $\overline{GH} \cong \overline{JK}$, $GH = x + 10$,
 $HJ = 8$, $JK = 2x - 4$

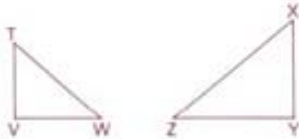
Find: GJ

$$\begin{aligned} GH &= JK \\ x + 10 &= 2x - 4 \\ 14 &= x \end{aligned}$$



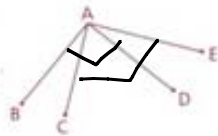
8 Given: $\angle T$ is comp. to $\angle W$,
 $\angle X$ is comp. to $\angle Z$,
 $\angle Z \cong \angle W$

Prove: ?



10 Given: $\angle BAD$ is a right \angle ,
 $\overline{CA} \perp \overline{AE}$

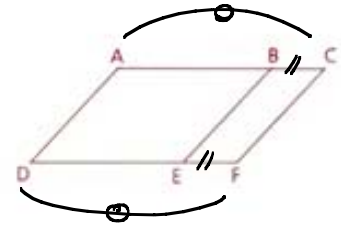
Prove: $\angle BAC \cong \angle EAD$



Statements	Reasons
1. $\angle BAD \text{ rt } \angle$	1. Given
2. $\overline{CA} \perp \overline{AE}$	2. Given
3. $\angle CAE \text{ rt } \angle$	3. $\perp \Rightarrow \text{rt } \angle$ (2)
4. $\angle BAD \cong \angle CAE$	4. $\text{rt } \angle \Rightarrow \cong \angle$ (3)
5. $\angle CAD \cong \angle CAD$	5. ref
6. $\angle BAC \cong \angle DAE$	6. Subtract

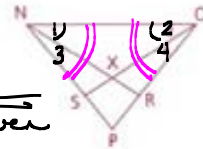
Statements	Reasons
1. $\angle BAD \text{ rt } \angle$	1. Given
2. $\angle BAC \text{ comp } \angle CAD$	2. $\text{rt } \angle \Rightarrow \text{comp } \angle$ (1)
3. $\overline{CA} \perp \overline{AE}$	3. Given
4. $\angle CAE \text{ rt } \angle$	4. $\perp \Rightarrow \text{rt } \angle$ (?)
5. $\angle CAD \text{ comp } \angle DAE$	5. $\text{rt } \angle \Rightarrow \text{comp } \angle$ (4)
6. $\angle BAC \cong \angle DAE$	6. \angle s comp to same \angle are \cong (2, 5)

5 Given: $\overline{AC} \cong \overline{DF}$,
 $\overline{BC} \cong \overline{EF}$
Prove: $\overline{AB} \cong \overline{DE}$



STATEMENTS	REASONS
1. $\overline{AC} \cong \overline{DF}$ $\overline{BC} \cong \overline{EF}$	1. GIVEN
2. $\overline{AB} \cong \overline{DE}$	2. Subtract

7 Given: $\angle PNO \cong \angle PON$,
 $\angle 1 \cong \angle 2$
Conclusion: ?

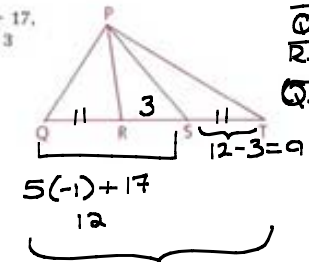


STATEMENTS	REASONS
1. $\angle PND \cong \angle PON$	1. Given
$\angle 1 \cong \angle 2$	
2. $\angle 3 \cong \angle 4$	2. Subtract

9 Given: $\overline{QR} \cong \overline{ST}$, $QS = 5x + 17$,
 $RT = 10 - 2x$, $RS = 3$

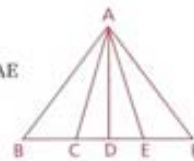
Find: QS and QT
12 & 21

$$\begin{aligned} QS &= RT \\ 5x + 17 &= 10 - 2x \\ 7x &= -7 \\ x &= -1 \end{aligned}$$

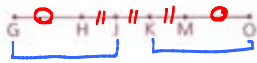


$\overline{QR} \cong \overline{ST}$ (given)
 $\overline{RS} \cong \overline{RS}$ (ref)
 $\overline{QS} \cong \overline{RT}$ (add)

11 Given: $\angle BAD \cong \angle FAD$;
 \overline{AD} bisects $\angle CAE$.
Conclusion: $\angle BAC \cong \angle FAE$



12 Given: J and K are trisection points of \overline{HM} .
* $\overline{GH} \cong \overline{MO}$

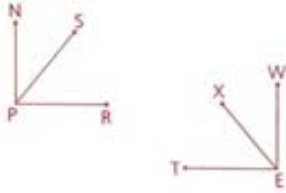


① MARK DIAG

Conclusion: $\overline{GJ} \cong \overline{KO}$

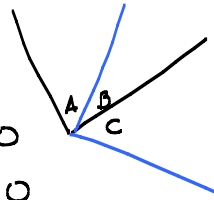
Statements	Reasons
1. J & K trisect \overline{HM}	1. Given
2. $\overline{HJ} \cong \overline{KM}$	2. trisect $\Rightarrow \cong$ segs (1) * Brevity is beautiful
3. $\overline{GH} \cong \overline{MO}$	3. Given
4. $\overline{GJ} \cong \overline{KO}$	4. Add (2,3)

13 Given: $\angle NPR$ is a right \angle .
 $\overline{WE} \perp \overline{ET}$.
 $\angle SPR \cong \angle XET$
Prove: $\angle NPS \cong \angle WEX$



14 Given: $\angle A$ is comp. to $\angle B$.
 $\angle C$ is comp. to $\angle B$.
 $\angle A = (3x + y)^\circ$.
 $\angle B = (x + 4y + 2)^\circ$.
 $\angle C = (3y - 3)^\circ$
Find: $m\angle B$

$\angle s$ comp to same $\angle \Rightarrow \cong \angle s \Rightarrow \angle A \cong \angle C$



$$\begin{aligned} \angle A &= \angle C \\ 3x + y &= 3y - 3 \end{aligned}$$

$$\begin{aligned} \angle A + \angle B &= 90 \\ 4x + 5y + 2 &= 90 \end{aligned}$$

$$3x - 2y = -3$$

$$4x + 5y = 88$$

$$\begin{cases} 3x - 2y = -3 \\ 4x + 5y = 88 \end{cases}$$

Use standard form of lin. eq: $Ax + By = C$
2 methods to solve linear systems

- ① substitute
- ② eliminate

DRAW

$$\begin{cases} (3x - 2y = -3) \cdot 5 \\ (4x + 5y = 88) \cdot 2 \end{cases} \rightarrow \begin{cases} 15x - 10y = -15 \\ 8x + 10y = 176 \end{cases}$$

$$\begin{array}{r} 23x \quad = 161 \\ x \quad = \underline{7} \end{array}$$

Find $\angle B = (x + 4y + 2)^\circ$.

$$3(7) - 2y = -3$$

$$\begin{array}{r} 21 \\ -2y \end{array} = \begin{array}{r} -3 \\ +2y \end{array}$$

$$24 = 2y$$

$$\underline{12} = y$$

$$\angle B = (7 + 48 + 2) = \boxed{57^\circ}$$