Name STU DENT   Ms. Kresovic
Adv Geo - 8   W 3/16/16   10-3: Arcs of a Circle

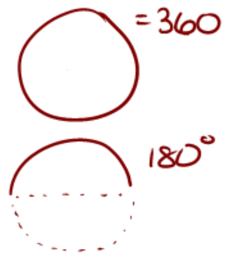
Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

Types of Arcs

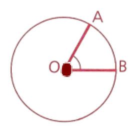
$\overset{\frown}{AB}$ $\overset{\frown}{CZD}$



Definition An ***arc*** consists of two points on a circle and all points on the circle needed to connect the points by a single path.

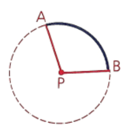
Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A ***central angle*** is an angle whose vertex is at the center of a circle.



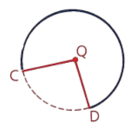
Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

Definition A ***minor arc*** is an arc whose points are on or between the sides of a central angle.



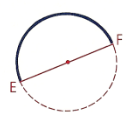
Central angle APB determines minor arc AB.

Definition A ***major arc*** is an arc whose points are on or outside of a central angle.



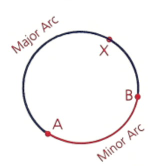
Central angle CQD determines major arc CD.

Definition A ***semicircle*** is an arc whose endpoints are the endpoints of a diameter.

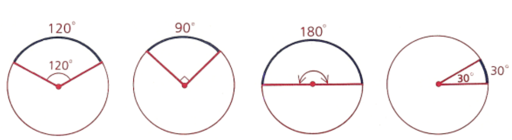


Arc EF is a semicircle.

The symbol ⌒ is used to label arcs. The minor arc joining A and B is called $\overset{\frown}{AB}$. The major arc joining A and B is called $\overset{\frown}{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)



The Measure of an Arc = m central ∠

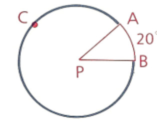


Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

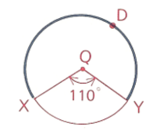
Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

Example

a *Given:* $m\overset{\frown}{AB} = 20$
Find: $m\overset{\frown}{ACB}$



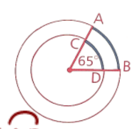
b *Given:* $m\angle XQY = 110$
Find: $m\overset{\frown}{XDY}$



$m\overset{\frown}{CD} = 65°$ & $m\overset{\frown}{AB} = 65°$

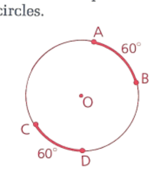
Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\overset{\frown}{AB} = 65$ and $m\overset{\frown}{CD} = 65$, but $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent?

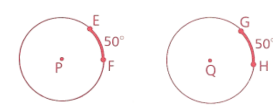


$\overset{\frown}{CD} \not\cong \overset{\frown}{AB}$

Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.



We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

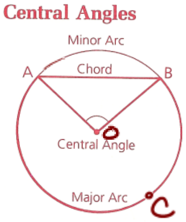


If ⊙P ≅ ⊙Q, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

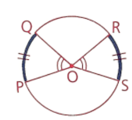
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.

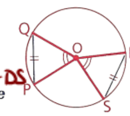


Theorem 79 *If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.* 2 ≅ central ∠s ⇒ 2 ≅ ARCS



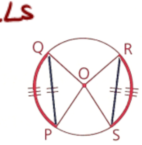
Theorem 80 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* 2 ≅ ARCS ⇒ 2 ≅ CENTRAL ∠s

Theorem 81 *If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* 2 ≅ CENTRAL ∠s ⇒ 2 ≅ CHDS



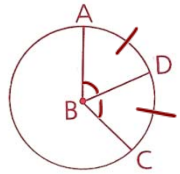
Theorem 82 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* 2 ≅ CHDS ⇒ 2 ≅ CENTRAL ∠S

Theorem 83 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* 2 ≅ ARCS ⇒ 2 ≅ CHDS



Theorem 84 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.* 2 ≅ CHDS ⇒ 2 ≅ ARCS

Problem 1 *Given:* ⊙B;
D is the midpt. of $\widehat{AC}$.
Conclusion: $\overrightarrow{BD}$ *bisects* ∠ABC.

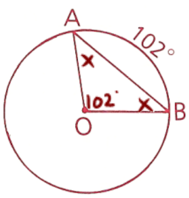


Proof

1 ⊙B; D is the midpt. of $\widehat{AC}$.	1 Given
2 $\widehat{AD} \cong \widehat{DC}$	2 MDPT ⇒ ≅ ARCS (1)
3 ∠ABD ≅ ∠DBC	3 2 ≅ ARCS ⇒ 2 ≅ CENTRAL ∠s (2)
4 $\overrightarrow{BD}$ bisects ∠ABC.	4 2 ≅ ∠s ⇒ BISECTION (3)

Problem 2 *If* m$\widehat{AB}$ = 102 in ⊙O, *find* m∠A *and* m∠B *in* △AOB.

Solution $\widehat{AB}$ = 102°



m∠AOB = 102° (m⌒ = m central ∠)
$\overline{OA} \cong \overline{OB}$ (⊙ ⇒ ≅ RADII)
∠A ≅ ∠B (≅ ⇒ △)
102 + 2x = 180 (Σ∠s△ = 180°)
2x = 78 (Subtract)
x = 39 (Divide)

∴ m∠A = m∠B = 39°

Σ = Sigma means sum

Problem 3 **a** *What fractional part of a circle is an arc of* 36°? *Of* 200°?
b *Find the measure of an arc that is* $\frac{7}{12}$ *of its circle.*

Solution There are 360° in a whole ⊙.

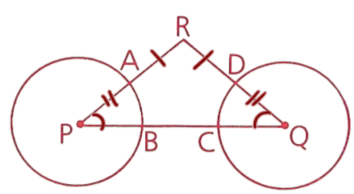
$\frac{\text{PART}}{\text{WHOLE}}$

a $\frac{36}{360} = \boxed{\frac{1}{10}}$; $\frac{200}{360} = \frac{20}{36} = \frac{10}{18} = \boxed{\frac{5}{9}}$

$\frac{7}{12} = \frac{x}{360} = \boxed{210°}$

30

Problem 4 *Given:* Ⓢ P *and* Q,
∠P ≅ ∠Q, $\overline{AR} \cong \overline{RD}$
Prove: $\widehat{AB} \cong \widehat{CD}$ (Hint: First prove that ⊙P ≅ ⊙Q.)



Proof

1 Ⓢ P and Q	1 Given
2 ∠P ≅ ∠Q	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 ≅∠s ⇒ ≅ sides (2)
4 $\overline{AR} \cong \overline{RD}$	4 Given
5 $\overline{AP} \cong \overline{DQ}$	5 Subtract (3, 4)
6 ⊙P ≅ ⊙Q	6 ≅ RADII ⇒ ≅ Ⓢ (5)
7 $\widehat{AB} \cong \widehat{CD}$	7 ≅ CENTRAL ∠s OF ≅ Ⓢ ⇒ ≅ ARCS (2, 6)

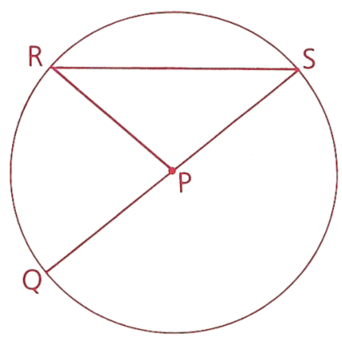
AMDG

Name KEY Ms. Kresovic

Adv Geo - 5, 7, & 8

1 Match each item in the left column with the correct term in the right column.

a $\overset{\frown}{QRS}$ = 6	1 Radius
b $\overline{QS}$ = 2	2 Diameter
c $\overset{\frown}{RQS}$ = 5	3 Chord
d $\overset{\frown}{RS}$ = 4	4 Minor arc
e $\overline{RS}$ = 3	5 Major arc
f $\angle RPQ$ = 7	6 Semicircle
g $\overline{PS}$ = 1	7 Central angle



2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

a Name a major arc of the smaller circle. = $\overset{\frown}{QPR}$

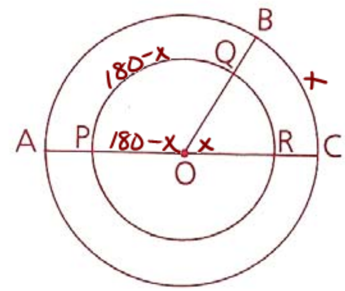
b Name a minor arc of the larger circle. $\overset{\frown}{BC}$

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$? = x + 180 − x = 180

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$? $\overset{\frown}{PQ}$

e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?

They have same measure but ⊙s ≇ ∴ No

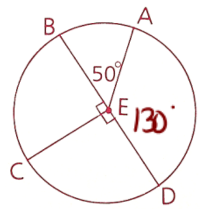


m central ∠ = m ⌒

3 In circle E, find each of the following.

a $m\overset{\frown}{BC}$ = 90° c $m\overset{\frown}{ACD}$ 230° e $m\overset{\frown}{ADC}$ = 220°

b $m\overset{\frown}{AD}$ = 130° d $m\overset{\frown}{BAD}$ = 180°



e) $m\overset{\frown}{ADC} = \overset{\frown}{AD} + \overset{\frown}{DC} = 130° + 90°$

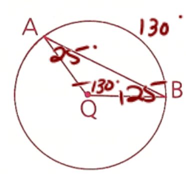
c) $\overset{\frown}{ACD} = \overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CD}$

50 + 90 + 90 =

180
50

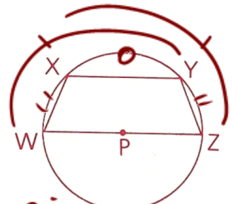
4 Given: ⊙Q, $\angle A = 25°$

Find: $m\overset{\frown}{AB}$



5 Given: ⊙P, $\overset{\frown}{WY} \cong \overset{\frown}{XZ}$

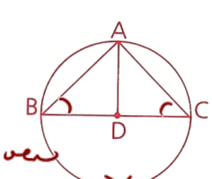
Conclusion: $\overline{WX} \cong \overline{YZ}$



1. ⊙P, $\overset{\frown}{WY} \cong \overset{\frown}{XZ}$	1. Given
2. $\overset{\frown}{XY} \cong \overset{\frown}{XY}$	2. Ref
3. $\overset{\frown}{WX} \cong \overset{\frown}{ZY}$	3. Subtract (1,2)
4. $\overline{WX} \cong \overline{YZ}$	4. 2 ≅ ARCS ⇒ 2 ≅ CHDS

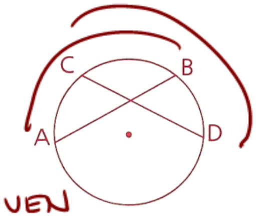
6 Given: ⊙D, $\angle B \cong \angle C$

Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$



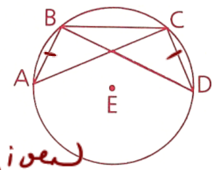
1. ⊙D, ∠B ≅ ∠C	1. Given
2. $\overline{AC} \cong \overline{AB}$	2. △ ⇒ △
3. $\overset{\frown}{AC} \cong \overset{\frown}{AB}$	3. 2 ≅ chds ⇒ 2 ≅ arcs

7 Given: $\overline{AB} \cong \overline{CD}$
Conclusion: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$



1. $\overline{AB} \cong \overline{CD}$ 1. GIVEN
2. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ 2. 2 ≅ CHDS ⇒ 2 ≅ ARCS
3. $\overset{\frown}{CB} \cong \overset{\frown}{CB}$ 3. Ref
4. $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ 4. Subtraction

8 Given: ⊙E,
$\overline{AB} \cong \overline{CD}$
Prove: $\overline{BD} \cong \overline{AC}$



1. ⊙E, $\overline{AB} \cong \overline{CD}$ 1. Given
2. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ 2. 2 ≅ CHDS ⇒ 2 ≅ ARCS
3. $\overset{\frown}{BC} \cong \overset{\frown}{BC}$ 3. REF
4. $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ 4. ADD
5. $\overline{AC} \cong \overline{BD}$ 5. ≅ ARCS ⇒ ≅ CHDS

9 What fractional part of a circle is an arc that measures

a 8 $\frac{8}{360} \Rightarrow \frac{1}{45}$

b 240 $\frac{240}{360} = \frac{2}{3}$

c 144 $\frac{144}{360}$ reduce to $\frac{2}{5}$

d 315 $\frac{315}{360}$ reduce to $\frac{7}{8}$

10 Find the measure of an arc that is Use proportions

a $\frac{3}{5}$ of its circle

$\frac{3}{5} = \frac{x}{360}$

$x = 216°$

b $\frac{5}{9}$ of its circle

$\frac{5}{9} = \frac{x}{360}$

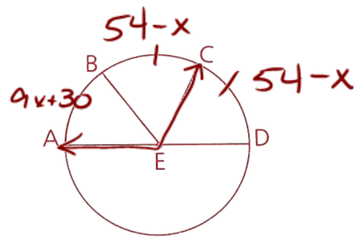
$x = 200°$

c 70% of its circle

$\frac{70}{100} = \frac{x}{360}$

$x = 252°$

11 Given: $\overline{AD}$ is a diameter of ⊙E.
C is the midpoint of $\overset{\frown}{BD}$.
$m\overset{\frown}{AB} = 9x + 30$,
$m\overset{\frown}{CD} = 54 - x$
Find: $m\angle AEC$



$\overset{\frown}{AD} = \overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CD}$

$180 = 7x + 138$

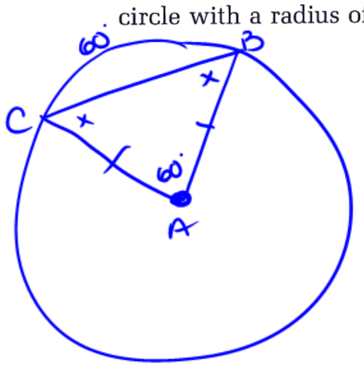
$42 = 7x$

$6 = x$

then $m\overset{\frown}{AEC} = 8x + 84 \Rightarrow 48 + 84 = 132°$

$m \frown = m$ central $\angle$ ∴ $m\angle AEC = 132°$

12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.



$m\overset{\frown}{BC} = 60°$ $m\overset{\frown}{BC} \Rightarrow m$ cent $\angle BAC = 60°$

⊙ ⇒ ≅ radii   so $\overline{AB} \cong \overline{AC}$

isos △ ⇒ △   so $\angle C \cong \angle B$

let them $= x$

$\Sigma \angle s\ \triangle = 180$ so $2x + 60 = 180$

$2x = 120$

$x = 60$

= Angular ⇒ = Lateral ∴ $AB = BC = AC$

$r = AB$ so chd = 12

NAME

→(measure of arc)(circumference)

13 Find the length of each arc described. (The length is a fractional part of the circumference.)

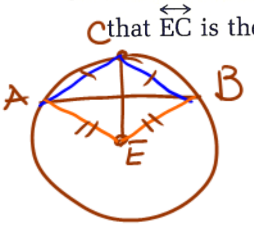
a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

b An arc that has a measure of 270 and is part of a circle with radius 12

$\frac{5}{8}C \Rightarrow \frac{5}{8}\pi d$
$\rightarrow \frac{5}{8} \cdot 24\pi$
$\rightarrow 15\pi$

$\frac{270}{360}\pi 24 \Rightarrow \frac{3}{4} \cdot 24\pi \Rightarrow 18\pi$

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\widehat{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.



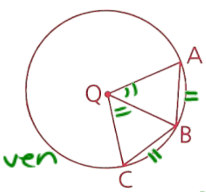
G: ⊙E, C mdpt $\widehat{AB}$
P: $\overleftrightarrow{EC} \perp$ bis $\overline{AB}$

1. ⊙E, C mdpt $\widehat{AB}$	1. GIVEN
2. $\widehat{AC} \cong \widehat{CB}$	2. MDPT ⇒ ≅ ARCS
* 3. $\overline{AC} \cong \overline{CB}$	3. ≅ ARCS ⇒ ≅ CHDS
* 4. $\overline{AE} \cong \overline{EB}$	4. ⊙ ⇒ ≅ RADII
5. $\overleftrightarrow{CE} \perp$ bis $\overline{AB}$	5. = dist ⇒ ⊥ bis

from Ch. 4

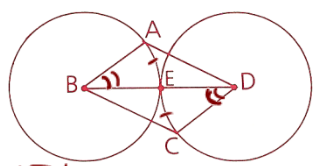
15 Given: ⊙Q;
B is the midpt. of $\widehat{AC}$.
Conclusion: ∠A ≅ ∠C

SSS or SAS



1. ⊙Q, B mdpt $\widehat{AC}$	1. Given
2. $\overline{QA} \cong \overline{QC}$	2. ⊙ ⇒ ≅ radii
3. $\widehat{BA} \cong \widehat{BC}$	3. mdpt ⇒ ≅ arcs
4. ∠AQB ≅ ∠CBQ	4. ≅ arcs ⇒ ≅ central ∠s
5. $\overline{QB} \cong \overline{QB}$	5. ______
6. ______	6. SAS
7. ______	7. CPCTC

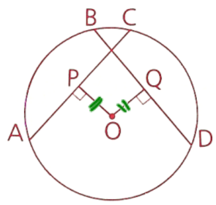
16 Given: ⊙B ≅ ⊙D,
$\widehat{AE} \cong \widehat{CE}$
Prove: ABCD is a ▱.



1. ⊙B ≅ ⊙D, $\widehat{AE} \cong \widehat{CE}$	1. GIVEN
2. ∠ABE ≅ ∠CDE	2. ≅ arcs in ≅ ⓢ ⇒ ≅ central ∠s
3. $\overline{AB} \parallel \overline{DC}$	3. alt int ∠s ⇒ ∥
4. $\overline{AB} \cong \overline{DC}$	4. ≅ ⓢ ⇒ ≅ radii
5. ABCD ▱	5. If one pr opp sds both ∥ & ≅ then ▱

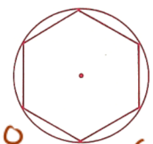
17 Given: ⊙O,
$\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
$\overline{OP} \cong \overline{OQ}$

Conclusion: $\widehat{AB} \cong \widehat{CD}$



Statements	Reasons
1. ⊙O, $\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$, $\overline{OP} \cong \overline{OQ}$	1. Given
2. $\overline{AC} \cong \overline{BD}$	2. = dist chds ⇒ ≅ chds
3. $\widehat{AC} \cong \widehat{BD}$	3. ≅ CHDS ⇒ ≅ ARCS
4. $\widehat{BC} \cong \widehat{BC}$	4. Ref
5. $\widehat{AB} \cong \widehat{CD}$	5. SUBTRACT

18 A polygon is *inscribed in* a ⊙ if all its vertices lie on the ⊙. Find the measure of the arc cut off by a side of each of the following inscribed polygons.



a A regular hexagon , 6 sides ∴ $\frac{360}{6} = 60°$

b A regular pentagon

c A regular octagon

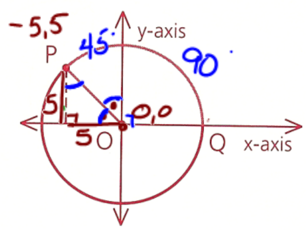
5 sides ∴ $\frac{360}{5} = 72°$

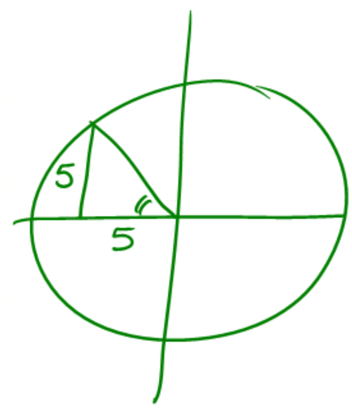
8 sides ∴ $\frac{360}{8} = 45°$

19 Point P is located at (−5, 5).

a Find the radius of ⊙O. $5\sqrt{2}$

b Find the measure of $\widehat{PQ}$.
45+90=135

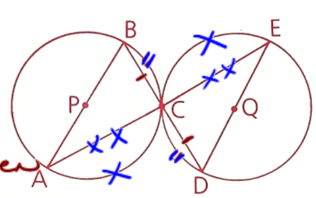




a) 45 45 90
x x $x\sqrt{2}$
5 $\boxed{5\sqrt{2} = r}$

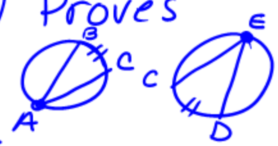
20 Given: ⊙P ≅ ⊙Q,
$\overline{BC} \cong \overline{CD}$

Conclusion: ∠A ≅ ∠E



Statements	Reasons
1. ⊙P ≅ ⊙Q, $\overline{BC} \cong \overline{CD}$	1. given
2. $\widehat{BC} \cong \widehat{CD}$	2. ≅ chds of ≅ ⊙ ⇒ ≅ arcs
3. $\overline{BA} \cong \overline{DE}$	3. ≅ ⊙ ⇒ ≅ DIAMETERS
4. $\widehat{BCA} \cong \widehat{ECD}$	4. same as 2
5. $\widehat{AC} \cong \widehat{EC}$	5. subtract (4, 2)
6. $\overline{AC} \cong \overline{EC}$	6. ≅ arcs ⇒ ≅ chds
7. △ACB ≅ △ECD	7. SSS (1, 3, 6)
8. ∠A ≅ ∠E	8. CPCTC

Proves



≅ ⊙ with ≅ arcs ⇔ ≅ inscribed ∠s