

NAME  
 Ms. Kresovic  
 Adv Geo -  
 Mon 18 Mar 13

9.8: The Pythagorean Theorem and Space Figures

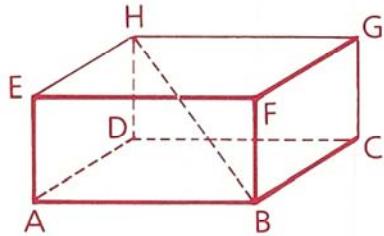
9.8: 413/1-6, 11-15

### Objective

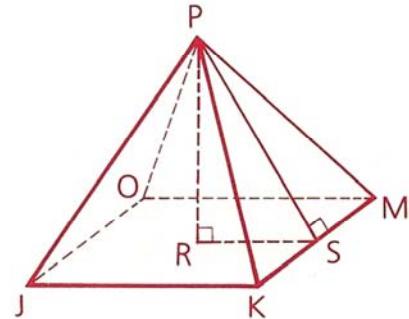
After studying this section, you will be able to

- Apply the Pythagorean Theorem to solid figures

## Part One: Introduction



Rectangular Solid



Regular Square Pyramid

Many of the problems in this section will involve the two figures shown above.

In the rectangular solid:

ABFE is one of the 6 rectangular **faces**

$\overline{AB}$  is one of the 12 **edges**

$\overline{HB}$  is one of the 4 **diagonals** of the solid. (The others are  $\overline{AG}$ ,  $\overline{CE}$ , and  $\overline{DF}$ .)

In the regular square pyramid:

JKMO is a square, and it is called the **base**

P is the **vertex**

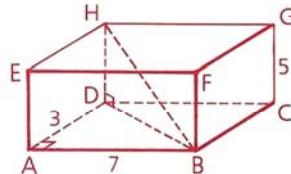
$\overline{PR}$  is the **altitude** of the pyramid and is perpendicular to the base at its center.

$\overline{PS}$  is called a **slant height** and is perpendicular to a side of the base.

**Note** A **cube** is a rectangular solid in which all edges are congruent.

## Part Two: Sample Problems

**Problem 1** The dimensions of a rectangular solid are 3, 5, and 7. Find the diagonal.



**Solution** It does not matter which edges are given the lengths 3, 5, and 7. Let  $AD = 3$ ,  $AB = 7$ , and  $HD = 5$ , and use the Pythagorean Theorem twice.

In  $\triangle ABD$ ,

$$3^2 + 7^2 = (DB)^2$$

$$9 + 49 = (DB)^2$$

$$\sqrt{58} = DB$$

In  $\triangle HDB$ ,

$$5^2 + (\sqrt{58})^2 = (HB)^2$$

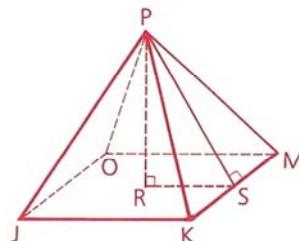
$$25 + 58 = (HB)^2$$

$$\sqrt{83} = HB$$

The measure of the diagonal is  $\sqrt{83}$ .

**Problem 2** Given: The regular square pyramid shown, with altitude  $\overline{PR}$  and slant height  $\overline{PS}$ , perimeter of  $JKMO = 40$ ,  $PK = 13$

Find: **a**  $JK$     **b**  $PS$     **c**  $PR$



### Solution

**a**  $JK = \frac{1}{4}(40) = 10$

**b** The slant height of the pyramid is the  $\perp$  bis. of  $\overline{MK}$ , so  $\triangle PSK$  is a right  $\triangle$ .

$$(SK)^2 + (PS)^2 = (PK)^2$$

$$5^2 + (PS)^2 = 13^2$$

$$PS = 12$$

**c** The altitude of a regular pyramid is perpendicular to the base at its center. Thus,  $RS = \frac{1}{2}(JK) = 5$ , and  $\triangle PRS$  is a right  $\triangle$ .

$$(RS)^2 + (PR)^2 = (PS)^2$$

$$5^2 + (PR)^2 = 12^2$$

$$25 + (PR)^2 = 144$$

$$PR = \sqrt{119}$$

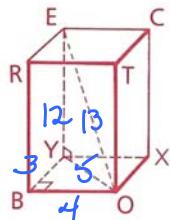
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1 Given: The rectangular solid shown,  
 $BY = 3$ ,  $OB = 4$ ,  $EY = 12$

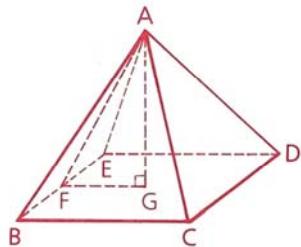
Find: a YO, a diagonal of face BOXY 5  
 b EO, a diagonal of the solid 13



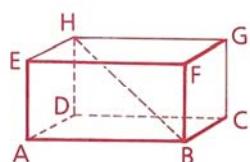
2 Find the diagonal of a rectangular solid whose dimensions are 3, 4, and 5.

3 Given: Regular square pyramid ABCDE,  
 with slant height  $\overline{AF}$ , altitude  $\overline{AG}$ ,  
 and base BCDE;  
 perimeter of BCDE = 40,  
 $\angle AFG = 60^\circ$

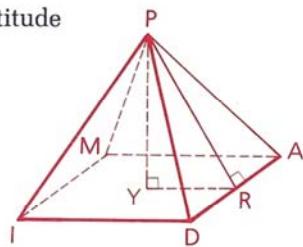
Find: The altitude and the slant height



4 Given: The rectangular solid shown,  
 $GC = 8$ ,  $HG = 12$ ,  $BC = 9$   
 Find: a HB, a diagonal of the solid  
 b AG, another diagonal of the solid



5 Given: The regular square pyramid shown, with altitude  $\overline{PY}$  and slant height  $\overline{PR}$ ,  
 $ID = 14$ ,  $PY = 24$



Find: a AD

b YR

PR

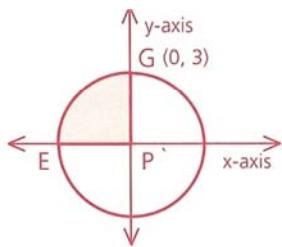
d The perimeter of base AMID

- A diagonal of the base (not shown in the diagram)

6 Find the slant height of a regular square pyramid if the altitude is 12 and one of the sides of the square base is 10.

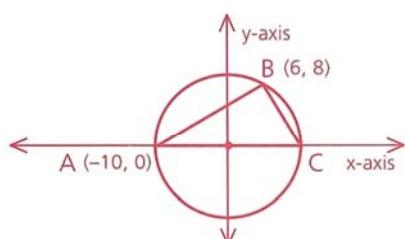
11 Given:  $\odot P$  as shown

Find: a The coordinates of point E  
 b The area of sector EPG to the nearest tenth  
 c The length of  $\widehat{GE}$  to the nearest tenth



12 Given: Diagram as marked

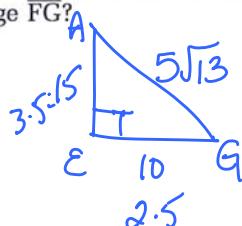
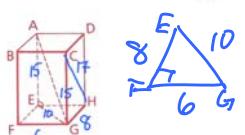
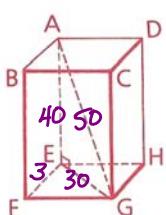
Find: AB (the length of  $\overline{AB}$ )



13 ABCDEFGH is a rectangular solid.

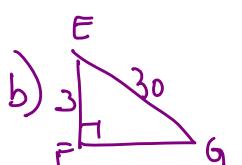
a If face diagonal  $\overline{CH}$  measures 17, edge  $\overline{GH}$  measures 8, and edge  $\overline{FG}$  measures 6, how long is diagonal  $\overline{AG}$ ?  $5\sqrt{13}$

b) If diagonal  $\overline{AG}$  measures 50, edge  $\overline{AE}$  measures 40, and edge  $\overline{EF}$  measures 3, how long is edge  $\overline{FG}$ ?



$$5(2, 3, \sqrt{13})$$

$$5(2^2 + 3^2 = 13)$$



$$3\left(1, \frac{1}{1}, 10\right)$$

$$3(1^2 + \frac{1}{1}^2 = 10^2)$$

$$3(\frac{?}{?} = 100 - 1)$$

$$3(\frac{?}{?} = \sqrt{99})$$

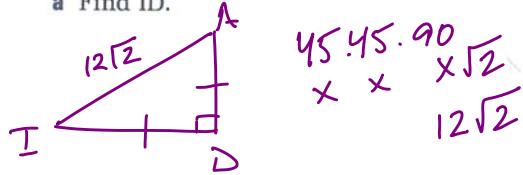
$$3(\sqrt{9 \cdot 11})$$

$$3(3\sqrt{11})$$

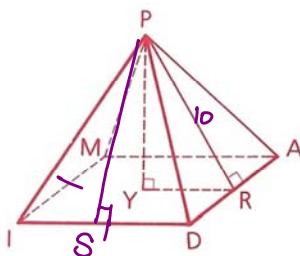
$$9\sqrt{11}$$

14 PADIM is a regular square pyramid. Slant height PR measures 10, and the base diagonals measure  $12\sqrt{2}$ .

a Find ID.

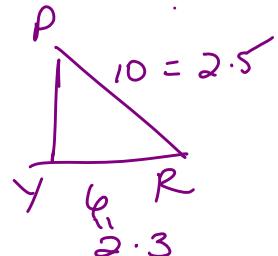


$$ID = 12$$

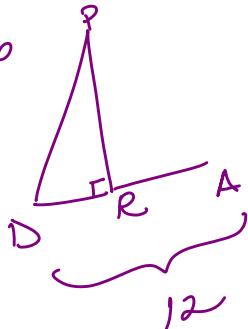


b Find the altitude of the pyramid.  $YR = \frac{1}{2}(12) = 6$

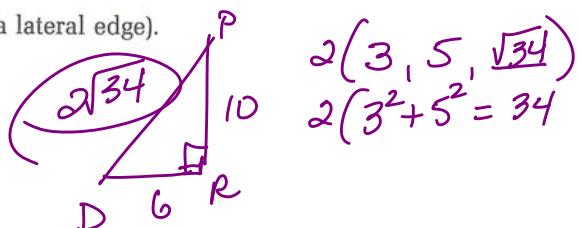
$$PY = 2 \cdot 4 = 8$$



c Find RD. = 6

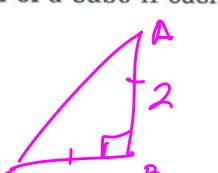
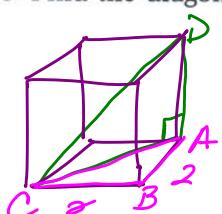


d Find PD (length of a lateral edge).

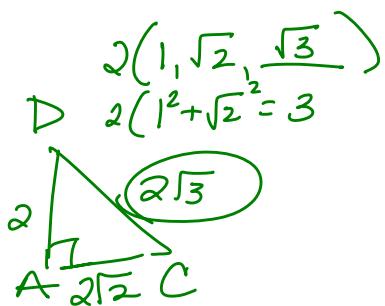


$$2(3^2 + 5^2 = 34)$$

15 Find the diagonal of a cube if each edge is 2.



$$\sqrt{2^2 + 2^2} = 2\sqrt{2}$$



$$2(1^2 + \sqrt{2}^2 = 3)$$

