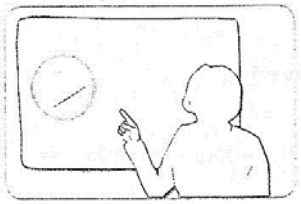
A trinomial that is the square of a binomial, such as the trinomial in Example 5, is called a **perfect square trinomial.** From Chapter 5, there are special product formulas we can use to help us recognize and factor these trinomials. To study these formulas further, see Section 6.3, Objective 3. **Remember:** A perfect square trinomial, such as the one in Example 5, may be factored by special product formulas or by other methods of factoring trinomials, such as by grouping.

Vocabulary, Readiness & Video Check

For each trinomial $ax^2 + bx + c$, choose two numbers whose product is $a \cdot c$ and whose sum is b.

1. $x^2 + 6x + 8$
   a. 4, 2 **b.** 7, 1 **c.** 6, 2 **d.** 6, 8
2. $x^2 + 11x + 24$
   a. 6, 4 **b.** 24, 1 **c.** 8, 3 **d.** 2, 12
3. $2x^2 + 13x + 6$
   a. 2, 6 **b.** 12, 1 **c.** 13, 1 **d.** 3, 4
4. $4x^2 + 8x + 3$
   a. 4, 3 **b.** 4, 4 **c.** 12, 1 **d.** 2, 6

Martin-Gay Interactive Videos



See Video 6.4

See video answer section.

Watch the section lecture video and answer the following question.

OBJECTIVE 1

5. In the lecture following Example 1, why does writing a term as the sum or difference of two terms suggest we'd then try to factor by grouping?

Exercise Set

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Factor each polynomial by grouping. Notice that Step 3 has already been done in these exercises. See Examples 1 through 5.

1. $x^2 + 3x + 2x + 6$
2. $x^2 + 5x + 3x + 15$
3. $y^2 + 8y - 2y - 16$
4. $z^2 + 10z - 7z - 70$
5. $8x^2 - 5x - 24x + 15$
6. $4x^2 - 9x - 32x + 72$
7. $5x^4 - 3x^2 + 25x^2 - 15$
8. $2y^4 - 10y^2 + 7y^2 - 35$

MIXED PRACTICE

Factor each trinomial by grouping. Exercises 9–12 are broken into parts to help you get started. See Examples 1 through 5.

9. $6x^2 + 11x + 3$
   a. Find two numbers whose product is $6 \cdot 3 = 18$ and whose sum is 11.
   b. Write $11x$ using the factors from part (a).
   c. Factor by grouping.
10. $8x^2 + 14x + 3$
    a. Find two numbers whose product is $8 \cdot 3 = 24$ and whose sum is 14.
    b. Write $14x$ using the factors from part (a).
    c. Factor by grouping.
11. $15x^2 - 23x + 4$
    a. Find two numbers whose product is $15 \cdot 4 = 60$ and whose sum is -23.
    b. Write $-23x$ using the factors from part (a).
    c. Factor by grouping.
12. $6x^2 - 13x + 5$
    a. Find two numbers whose product is $6 \cdot 5 = 30$ and whose sum is -13.
    b. Write $-13x$ using the factors from part (a).
    c. Factor by grouping.
13. $21y^2 + 17y + 2$
14. $15x^2 + 11x + 2$

15. $7x^2 - 4x - 11$
16. $8x^2 - x - 9$
17. $10x^2 - 9x + 2$
18. $30x^2 - 23x + 3$
19. $2x^2 - 7x + 5$
20. $2x^2 - 7x + 3$
21. $12x + 4x^2 + 9$
22. $20x + 25x^2 + 4$
23. $4x^2 - 8x - 21$
24. $6x^2 - 11x - 10$
25. $10x^2 - 23x + 12$
26. $21x^2 - 13x + 2$
27. $2x^3 + 13x^2 + 15x$
28. $3x^3 + 8x^2 + 4x$
29. $16y^2 - 34y + 18$
30. $4y^2 - 2y - 12$
31. $-13x + 6 + 6x^2$
32. $-25x + 12 + 12x^2$
33. $54a^2 - 9a - 30$
34. $30a^2 + 38a - 20$
35. $20a^3 + 37a^2 + 8a$
36. $10a^3 + 17a^2 + 3a$
37. $12x^3 - 27x^2 - 27x$
38. $30x^3 - 155x^2 + 25x$
39. $3x^2y + 4xy^2 + y^3$
40. $6r^2t + 7rt^2 + t^3$
41. $20z^2 + 7z + 1$
42. $36z^2 + 6z + 1$
43. $5x^2 + 50xy + 125y^2$
44. $3x^2 + 42xy + 147y^2$
45. $24a^2 - 6ab - 30b^2$
46. $30a^2 + 5ab - 25b^2$
47. $15p^4 + 31p^3q + 2p^2q^2$
48. $20s^4 + 61s^3t + 3s^2t^2$
49. $162a^4 - 72a^2 + 8$
50. $32n^4 - 112n^2 + 98$
51. $35 + 12x + x^2$
52. $33 + 14x + x^2$
53. $6 - 11x + 5x^2$
54. $5 - 12x + 7x^2$

REVIEW AND PREVIEW

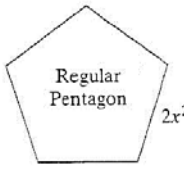
Multiply. See Sections 5.3 and 5.4.

55. $(x - 2)(x + 2)$
56. $(y - 5)(y + 5)$
57. $(y + 4)(y + 4)$
58. $(x + 7)(x + 7)$
59. $(9z + 5)(9z - 5)$
60. $(8y + 9)(8y - 9)$
61. $(x - 3)(x^2 + 3x + 9)$
62. $(2z - 1)(4z^2 + 2z + 1)$

CONCEPT EXTENSIONS

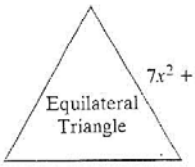
Write the perimeter of each figure as a simplified polynomial. Then factor the polynomial.

63. $10x^2 - 45x - 45; 5(2x - 3)(x - 3)$



$2x^2 + 9x + 9$

64. $21x^2 - 33xy - 12y^2; 3(7x - 4y)(x - y)$



$7x^2 + 11xy + 4y^2$

Factor each polynomial by grouping.

65. $x^{2n} + 2x^n + 3x^n + 6$
(***Hint:*** Don't forget that $x^{2n} = x^n \cdot x^n$.) $(x^n + 2)(x^n + 3)$
66. $x^{2n} + 6x^n + 10x^n + 60$ $(x^n + 6)(x^n + 10)$
67. $3x^{2n} + 16x^n - 35$ $(3x^n - 5)(x^n + 7)$
68. $12x^{2n} - 40x^n + 25$ $(2x^n - 5)(6x^n - 5)$
69. In your own words, explain how to factor a trinomial by grouping. answers may vary

6.5 Factoring Binomials

OBJECTIVES

1. Factor the Difference of Two Squares.
2. Factor the Sum or Difference of Two Cubes.

OBJECTIVE 1

Factoring the Difference of Two Squares

When learning to multiply binomials in Chapter 5, we studied a special product, the product of the sum and difference of two terms, a and b:

$$(a + b)(a - b) = a^2 - b^2$$

For example, the product of $x + 3$ and $x - 3$ is

$$(x + 3)(x - 3) = x^2 - 9$$

The binomial $x^2 - 9$ is called a **difference of squares**. In this section, we reverse the pattern for the product of a sum and difference to factor the binomial difference of squares.

EXAMPLE 4 Factor $x^3 + 27y^3$.

Solution

Step 1. The terms of this binomial contain no common factor (other than 1).

Step 2. This binomial is the sum of two cubes.

$$\begin{aligned} x^3 + 27y^3 &= (x)^3 + (3y)^3 \\ &= (x + 3y)[x^2 - x(3y) + (3y)^2] \\ &= (x + 3y)(x^2 - 3xy + 9y^2) \end{aligned}$$

Step 3. No factor can be factored further.

Step 4. We check by multiplying.

$$\begin{aligned} (x + 3y)(x^2 - 3xy + 9y^2) &= x(x^2 - 3xy + 9y^2) + 3y(x^2 - 3xy + 9y^2) \\ &= x^3 - 3x^2y + 9xy^2 + 3x^2y - 9xy^2 + 27y^3 \\ &= x^3 + 27y^3 \end{aligned}$$

Thus, $x^3 + 27y^3$ factored completely is $(x + 3y)(x^2 - 3xy + 9y^2)$.

PRACTICE 4 Factor $8a^3 + b^3$. $(2a + b)(4a^2 - 2ab + b^2)$

EXAMPLE 5 Factor $30a^2b^3 + 55a^2b^2 - 35a^2b$.

Solution

Step 1. $30a^2b^3 + 55a^2b^2 - 35a^2b = 5a^2b(6b^2 + 11b - 7)$ Factor out the GCF.

Step 2. $= 5a^2b(2b - 1)(3b + 7)$ Factor the resulting trinomial.

Step 3. No factor can be factored further.

Step 4. Check by multiplying.

The trinomial factored completely is $5a^2b(2b - 1)(3b + 7)$.

PRACTICE 5 Factor $60x^3y^2 - 66x^2y^2 - 36xy^2$. $6xy^2(5x - 2)(2x + 3)$

Factor the following completely.

1. $x^2 + 2xy + y^2$
2. $x^2 - 2xy + y^2$
3. $a^2 + 11a - 12$
4. $a^2 - 11a + 10$
5. $a^2 - a - 6$
6. $a^2 - 2a + 1$
7. $x^2 + 2x + 1$
8. $x^2 + x - 2$
9. $x^2 + 4x + 3$
10. $x^2 + x - 6$
11. $x^2 + 7x + 12$
12. $x^2 + x - 12$
13. $x^2 + 3x - 4$
14. $x^2 - 7x + 10$
15. $x^2 + 2x - 15$
16. $x^2 + 11x + 30$
17. $x^2 - x - 30$
18. $x^2 + 11x + 24$
19. $2x^2 - 98$
20. $3x^2 - 75$
21. $x^2 + 3x + xy + 3y$
22. $3y - 21 + xy - 7x$
23. $x^2 + 6x - 16$
24. $x^2 - 3x - 28$
25. $4x^3 + 20x^2 - 56x$
26. $6x^3 - 6x^2 - 120x$
27. $12x^2 + 34x + 24$
28. $8a^2 + 6ab - 5b^2$
29. $4a^2 - b^2$
30. $28 - 13x - 6x^2$
31. $20 - 3x - 2x^2$
32. $x^2 - 2x + 4$
33. $a^2 + a - 3$
34. $6y^2 + y - 15$
35. $4x^2 - x - 5$
36. $x^2y - y^3$
37. $4t^2 + 36$
38. $x^2 + x + xy + y$
39. $ax + 2x + a + 2$
40. $18x^3 - 63x^2 + 9x$
41. $12a^3 - 24a^2 + 4a$
42. $x^2 + 14x - 32$
43. $x^2 - 14x - 48$
44. $16a^2 - 56ab + 49b^2$
45. $25p^2 - 70pq + 49q^2$
46. $7x^2 + 24xy + 9y^2$
47. $125 - 8y^3$
48. $64x^3 + 27$

49. $-x^2 - x + 30$
50. $-x^2 + 6x - 8$
51. $14 + 5x - x^2$
52. $3 - 2x - x^2$
53. $3x^4y + 6x^3y - 72x^2y$
54. $2x^3y + 8x^2y^2 - 10xy^3$
55. $5x^3y^2 - 40x^2y^3 + 35xy^4$
56. $4x^4y - 8x^3y - 60x^2y$
57. $12x^3y + 243xy$
58. $6x^3y^2 + 8xy^2$
59. $4 - x^2$
60. $9 - y^2$
61. $3rs - s + 12r - 4$
62. $x^3 - 2x^2 + 3x - 6$
63. $4x^2 - 8xy - 3x + 6y$
64. $4x^2 - 2xy - 7yz + 14xz$
65. $6x^2 + 18xy + 12y^2$
66. $12x^2 + 46xy - 8y^2$
67. $xy^2 - 4x + 3y^2 - 12$
68. $x^2y^2 - 9x^2 + 3y^2 - 27$
69. $5(x + y) + x(x + y)$
70. $7(x - y) + y(x - y)$
71. $14t^2 - 9t + 1$
72. $3t^2 - 5t + 1$
73. $3x^2 + 2x - 5$
74. $7x^2 + 19x - 6$
75. $x^2 + 9xy - 36y^2$
76. $3x^2 + 10xy - 8y^2$
77. $1 - 8ab - 20a^2b^2$
78. $1 - 7ab - 60a^2b^2$
79. $9 - 10x^2 + x^4$
80. $36 - 13x^2 + x^4$
81. $x^4 - 14x^2 - 32$
82. $x^4 - 22x^2 - 75$
83. $x^2 - 23x + 120$
84. $y^2 + 22y + 96$
85. $6x^3 - 28x^2 + 16x$
86. $6y^3 - 8y^2 - 30y$
87. $27x^3 - 125y^3$
88. $216y^3 - z^3$
89. $x^3y^3 + 8z^3$
90. $27a^3b^3 + 8$
91. $2xy - 72x^3y$
92. $2x^3 - 18x$
93. $x^3 + 6x^2 - 4x - 24$
94. $x^3 - 2x^2 - 36x + 72$
95. $6a^3 + 10a^2$
96. $4n^2 - 6n$
97. $a^2(a + 2) + 2(a + 2)$
98. $a - b + x(a - b)$
99. $x^3 - 28 + 7x^2 - 4x$
100. $a^3 - 45 - 9a + 5a^2$

CONCEPT EXTENSIONS

Factor. 105. answers may vary

101. $(x - y)^2 - z^2$ $(x - y + z)(x - y - z)$
102. $(x + 2y)^2 - 9$ $(x + 2y + 3)(x + 2y - 3)$
103. $81 - (5x + 1)^2$ $(9 + 5x + 1)(9 - 5x - 1)$
104. $b^2 - (4a + c)^2$ $(b + 4a + c)(b - 4a - c)$
105. Explain why it makes good sense to factor out the GCF first, before using other methods of factoring.
106. The sum of two squares usually does not factor. Is the sum of two squares $9x^2 + 81y^2$ factorable? yes: $9(x^2 + 9y^2)$
107. Which of the following are equivalent to $(x + 10)(x - 7)$? a, c
    a. $(x - 7)(x + 10)$ b. $-1(x + 10)(x - 7)$
    c. $-1(x + 10)(7 - x)$ d. $-1(-x - 10)(7 - x)$
108. Which of the following are equivalent to $(x - 2)(x - 5)$? b, c
    a. $-1(x + 2)(x + 5)$ b. $(x - 5)(x - 2)$
    c. $(5 - x)(2 - x)$ d. $-1(x + 2)(x - 5)$

6.6 Solving Quadratic Equations by Factoring

OBJECTIVES

1. Solve Quadratic Equations by Factoring.
2. Solve Equations with Degree Greater than 2 by Factoring.
3. Find the x-Intercepts of the Graph of a Quadratic Equation in Two Variables.

In this section, we introduce a new type of equation—the **quadratic equation.**

Quadratic Equation

A quadratic equation is one that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$.

Some examples of quadratic equations are shown below.

$$x^2 - 9x - 22 = 0 \qquad 4x^2 - 28 = -49 \qquad x(2x - 7) = 4$$

The form $ax^2 + bx + c = 0$ is called the **standard form** of a quadratic equation. The quadratic equation $x^2 - 9x - 22 = 0$ is the only equation above that is in standard form.