14 $V_{sphere} = \frac{4}{3}\pi r^3$

$V_{cyl} = Bh$

$V_{cyl} = \pi r^2(2r)$

$V_{cyl} = 2\pi r^3$

$\frac{V_{sphere}}{V_{cyl}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$

15 $A\odot = \pi r^2$

$= \pi \cdot 6^2$

$= 36\pi$

$A_{rect\ ABGH} = 1 \cdot 12 = 12$

$\therefore (\frac{12}{36\pi} \cdot 100)\% = \frac{100}{3\pi}\% \approx 11\%$

16 $V_{hemi} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi r^3$

$V_{cone} = \frac{1}{3}\pi r^2(r) = \frac{1}{3}\pi r^3$

$\frac{V_{hemi}}{V_{cone}} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^3} = \frac{2}{1}$

17 The height of both prisms is the same, so by Cavalieri's principle, if $A_{shell} = A_{cyl}$, then $V_{shell} = V_{cyl}$.

$A_{shell} = A_{cyl} - A_{inner\ cyl}$

$A_{shell} = \pi(2r)^2 \cdot h - \pi r^2 \cdot h$

$A_{shell} = 3\pi r^2 h$

$A_{cyl} = \pi(r\sqrt{3})^2 h$

$A_{cyl} = 3\pi r^2 h$

18 a $A_{annulus} = \pi R^2 - \pi d^2$

The radius of the $\odot$ is $\sqrt{R^2 - d^2}$ by Pythagorean Theorem.

$A_{circle} = \pi(\sqrt{R^2 - d^2})^2$

$A_{circle} = \pi R^2 - \pi d^2$

b By Cavalieri's principle the volume of the hemisphere is equal to the volume of the cylinder minus the volume of the cone.

$V_{hemi} = V_{cyl} - V_{cone}$

$V_{hemi} = \pi R^2(R) - \frac{1}{3}\pi R^2(R)$

$V_{hemi} = \frac{2}{3}\pi R^3$

Therefore, the volume of a sphere $= \frac{4}{3}\pi R^3$.

Pages 594–597 Chapter 12 Review Problems

1 a Slant height is 4.

$LA = \frac{1}{2}(6)(4)(4) = 48$

$TA = LA + A_{base}$

$= 48 + 36$

$= 84$

b $LA = 2\pi(4)(7) = 56\pi$

$TA = 56\pi + 2\pi(16)$

$= 88\pi$

2 a $V_{cube} = s^3$

$V_{cube} = 8^3$

$V_{cube} = 512$

b $V_{rect} = Bh$

$V_{rect} = 24(4\frac{1}{2})$

$V_{rect} = 108$

c $V_{cyl} = Bh$

$V_{cyl} = 49\pi(2)$

$V_{cyl} = 98\pi$

d $V_{pyr} = \frac{1}{3}Bh$

$V_{pyr} = \frac{1}{3}(12)(5)$

$V_{pyr} = 20$

e $V_{prism} = Bh$

$V_{prism} = 12(5)$

$V_{prism} = 60$

f $V_{sphere} = \frac{4}{3}\pi r^3$

$V_{sphere} = \frac{4}{3}\pi(8)$

$V_{sphere} = \frac{32}{3}\pi$

3 a $V_{cyl} = Bh$

$= \pi r^2 h$

$= \pi(6)^2 10$

$V_{cyl} = 360\pi$

$TA_{cyl} = LA + 2(A_{base})$

$TA = Ch + 2(\pi r^2)$

$= 2\pi rh + 2(\pi r^2)$

$= 2\pi(6)(10) + 2\pi(6^2$

$= 120\pi + 2(36\pi)$

$TA = 120\pi + 72\pi = 192$

b The base of the prism is a rt $\triangle$ and $x = 12$ because 3-4 is a Pythagorean Triple.

$V_{prism} = Bh$

$V_{prism} = \frac{1}{2}bh(h)$

$V_{prism} = \frac{1}{2}(9)(12)(10)$

$V_{prism} = 540$

$TA_{prism} = LA + 2(A_{base})$

$TA = bh + bh + bh + 2(\frac{1}{2}bh)$

$= 12(10) + 9(10) + 15(10) + 9(12)$

$TA = 120 + 90 + 150 + 108 = 468$

c Radius = 5 because 5-12-13 is a Pythagorean Triple.

$V_{cone} = \frac{1}{3}Bh$

$V_{cone} = \frac{1}{3}\pi r^2 h$

$V_{cone} = \frac{1}{3}\pi(5)^2(12)$

$V_{cone} = \frac{1}{3}(25\pi)(12)$

$V_{cone} = 100\pi$

$TA_{cone} = LA + A_{base}$

$TA_{cone} = \frac{1}{2}C\ell + \pi r^2$

$= \pi r\ell + \pi r^2$

$= \pi(5)13 + \pi(5)^2$

$TA = 65\pi + 25\pi = 90\pi$

4 $V_{rect} = B \cdot h$

$V_{rect} = (9)(5)2 = 90$

5 a $V_{rect\ box} = \ell wh$

$100 = 15(1\frac{1}{3})h$

$100 = 20h$

$5 = h$

b $V_{cube} = x^3$

$216 = x^3$

$6 = x$

$h = 6$

6 By using the Pythagorean triple 8-15-17, the base diameter is 8.

$V_{cyl} = B \cdot h$

$V_{cyl} = 16\pi(15) = 240\pi$

7 Use the "Divide and Conquer" method.

$V_{rect\ box} = \ell wh$

$V = (100)(25)(15)$

$V = 37{,}500$ cu cm

$V_{rect\ box} = \ell wh$

$V = 100(25)(15 + 15)$

$V = 100(25)(30)$

$V = 75{,}000$ cu cm

$V_{rect\ box} = \ell wh$

$V = (100)(100)(15 + 15 + 15)$

$V = 100(100)(45)$

$V = 450{,}000$ cu cm

$V_{concrete} = 37{,}500 + 75{,}000 + 450{,}000 = 562{,}500$ cu cm

8 a $LA = \frac{1}{2}(10)(16)(4) = 320$

b $TA = LA + A_{base}$

$= 320 + 256 = 576$

c $V = \frac{1}{3}(256)(6) = 512$

9 $TA_{sphere} = 4\pi r^2$ $\quad V_{sphere} = \frac{4}{3}\pi r^3$

$36\pi = 4\pi r^2$ $\quad V_{sphere} = \frac{4}{3}\pi(3)^3$

$9 = r^2, r = 3$ $\quad V_{sphere} = \frac{4}{3}\pi(27) = 36\pi$

10 a Slant height = 12

$LA = \frac{1}{2}(10)(12)(3) = 180$

$A_{base} = \frac{1}{2}(10)(\sqrt{75})$

$= 25\sqrt{3}$

$TA = 180 + 25\sqrt{3}$

b $LA = \pi r\ell$

$= \pi(6)(10)$

$= 60\pi$

$TA = LA + A_{base}$

$= 60\pi + 36\pi$

$= 96\pi$

c $LA = 2\pi rh$

$= 2\pi(6.5)(15)$

$= 195\pi$

$TA = LA + A_{base}$

$= 195\pi + (2)(\pi)(6.5)^2$

$= 195\pi + 84\frac{1}{2}\pi$

$= 279\frac{1}{2}\pi$

11 a $A = \frac{1}{2}\cdot 4\pi(5)^2 + \pi(5)^2$

$= 50\pi + 25\pi$

$= 75\pi$

b $A_{half\ cone} = \frac{1}{2}(\pi\cdot 5\cdot 13 + \pi 5^2)$

$= \frac{1}{2}(65\pi + 25\pi)$

$= 45\pi$

$A\Delta = \frac{1}{2}(10)(12) = 60$

$TA = 45\pi + 60$

12 $V_{cyl} = Bh = 16\pi h$

Since A base $= \pi r^2$, $r = 4$ and diameter $= 8$. $C = \pi d = 8\pi$

Therefore the length of the rectangle $= 8\pi$. The height

must $= 3$, because $(3)(8\pi) = 24\pi$, the area of the rectan

$V_{cyl} = 16\pi(3) = 48\pi$

13 $A_{rhombus} = \frac{1}{2}(d_1d_2)$ $\quad V_{pyramid} = \frac{1}{3}Bh$

$A_{rhombus} = \frac{1}{2}(7)(6)$ $\quad V_{pyramid} = \frac{1}{3}(21)(5)$

$A_{rhombus} = \frac{1}{2}(42) = 21$ $\quad V_{pyramid} = 35$

14 $A_{bases} = (\frac{1}{2}ap)(2)$

$= \frac{1}{2}(6\sqrt{3})(72)(2)$

$= 432\sqrt{3}$

$LA = (12)(20)(6)$

$= 1440$

$TA = 1440 + 432\sqrt{3}$

$V = 216\sqrt{3}(20)$

$= 4320\sqrt{3}$ cu cm

15 Draw the alt to the base of the isosceles Δ, bisecting

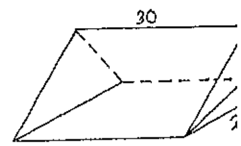
7-24-25 is a Pythagorean Triple, so the height = 24.

$V_{prism} = Bh$

$V_{prism} = (\frac{1}{2}bh)h$

$V_{prism} = (\frac{1}{2}(14)(24))(30)$

$V_{prism} = (168)(30) = 5040$



16 $V_{castle\ rect} = Bh = (50)(100)30 = 150{,}000$

$V_{rectangular\ tower} = Bh = (9)(50) = 450$

$V_{pyramid\ on\ tower} = \frac{1}{3}Bh = \frac{1}{3}(9)(3) = 9$

$V_{cylinder\ tower} = Bh = 9\pi(50) = 450\pi$

$V_{hemisphere} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi(27) = 18\pi$

$V_{trianglular\ tower} = Bh = \frac{9}{4}\sqrt{3}(50) = \frac{450}{4}\sqrt{3}$

$V_{pyramid\ on\ tower} = \frac{1}{3}Bh = \frac{1}{3}(\frac{9}{4}\sqrt{3})(3) = \frac{9}{4}\sqrt{3}$

$V_{cylinder\ tower} = Bh = 9\pi(50) = 450\pi$

$V_{cone\ on\ tower} = \frac{1}{3}Bh = \frac{1}{3}(9\pi)3 = 9\pi$

Total Volume $= 150{,}459 + 927\pi + \frac{459}{4}\sqrt{3}$

17 $V_{rect} = Bh$ $\quad V_{cyl} = Bh$

$V_{rect} = 30(8)$ $\quad V_{cyl} = (1^2)\pi(8)$

$V_{rect} = 240$ $\quad V_{cyl} = 8\pi$

$V_{cyl} \approx 8(3.14) \approx 25.12$

Remaining Volume $\approx 240 - 25.12 \approx 215$

18 $V_{prism} = Bh = A_{\triangle}h$ $\qquad s = \frac{5+9+6}{2} = 10$

$V_{prism} = (\sqrt{s(s-a)(s-b)(s-c)})h$

$V_{prism} = (\sqrt{10(10-5)(10-9)(10-6)})h$

$V_{prism} = (\sqrt{200})h$

$V_{prism} = (10\sqrt{2})7 = 70\sqrt{2}$

19 $A = \frac{1}{4}(12\pi \cdot 10) + 2(6 \cdot 10) + 2(\frac{\pi}{4} \cdot 36)$

$= 120 + 48\pi$

20 To find B, find the area of the shaded segment. The inscribed $\triangle$ will be equilateral, so the radii will be 10 because all sides of an equilateral $\triangle$ are $\cong$.

$B = A_{sector} - A_{eq\triangle}$ $\qquad V_{prism} = Bh$

$B = \left(\frac{m\ arc}{360}\right)\pi r^2 - \frac{s^2}{4}\sqrt{3}$ $\qquad = (\frac{50}{3}\pi - 25\sqrt{3})30$

$B = (\frac{60}{360})\pi(10)^2 - \frac{10^2}{4}\sqrt{3}$ $\qquad V_{prism} = 500\pi - 750\sqrt{3}$

$B = \frac{1}{6}(100\pi) - 25\sqrt{3}$

$B = \frac{50}{3}\pi - 25\sqrt{3}$

$TA = 2(B) + A_{rect} + \frac{m\ arc}{360}(LA_{cyl})$

$TA = 2(\frac{50}{3}\pi - 25\sqrt{3}) + bh + \frac{60}{360}Ch$

$TA = \frac{100}{3}\pi - 50\sqrt{3} + 10(30) + (\frac{1}{6})2\pi(10)(30)$

$TA = \frac{100}{3}\pi - 50\sqrt{3} + 300 + 100\pi$

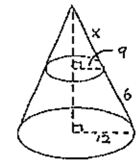
$TA = 300 + \frac{400}{3}\pi - 50\sqrt{3}$

21 $\frac{x}{x+6} = \frac{9}{12}$

$12x = 9x + 54$

$3x = 54$

$x = 18$



Using the Pythagorean Theorem, the height of the sma cone $= 9\sqrt{3}$ the height of the large cone $= 12\sqrt{3}$

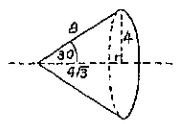
$V_{frustum} = V_{lg\ cone} - V_{sm\ cone}$

$V_{lg\ cone} = \frac{1}{3}Bh = \frac{1}{3}(144\pi)(12\sqrt{3}) = 576\pi\sqrt{3}$

$V_{sm\ cone} = \frac{1}{3}Bh = \frac{1}{3}(81\pi)(9\sqrt{3}) = 243\pi\sqrt{3}$

$V_{frustum} = 576\pi\sqrt{3} - 243\pi\sqrt{3} = 333\pi\sqrt{3}$

22 a The $\triangle$ is a 30°60°90° $\triangle$, so the radius of cone $= \frac{1}{2}(8) = 4$ and the height $= 4\sqrt{3}$.



$V_{cone} = \frac{1}{3}Bh$

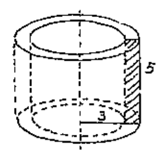
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4)^2(4\sqrt{3})$

$V = \frac{1}{3}(16\pi)4\sqrt{3} = \frac{64\pi\sqrt{3}}{3}$

b Radius small cyl = 3

radius large cyl = 3 + 1 = 4



$V = Bh$

$V = (\pi r^2 - \pi r^2)5$

$V = (\pi(4)^2 - \pi(3)^2)5$

$V = (16\pi - 9\pi)5 = 35\pi$

Pages 598–603 Chapters 1-12 Cumulative Review

1 $9x + x = 90$

$10x = 90, x = 9$

The measure of the larger acute $\angle$ is $9(9) = 81°$.

2 $2x + 3 + 4x - 5 + 8x - 19 = 28$

$14x - 21 = 28$

$14x = 49, x = \frac{7}{2}$

$AB = 2x + 3$	$BC = 4x - 5$	$CA = 8x - 19$
$AB = 2(\frac{7}{2}) + 3$	$BC = 4(\frac{7}{2}) - 5$	$CA = 8(\frac{7}{2}) - 19$
$AB = \frac{14}{2} + 3$	$BC = \frac{28}{2} - 5$	$CA = \frac{56}{2} - 19$
$AB = 10$	$BC = 9$	$CA = 9$

$\triangle ABC$ is isosceles.