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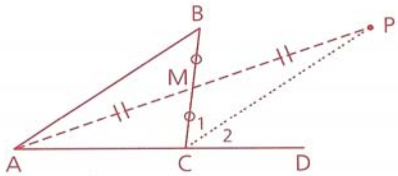
5.2

Theorem 30 ***The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.***

Given: Exterior angle BCD

Prove: m∠BCD > m∠B,
m∠BCD > m∠BAC

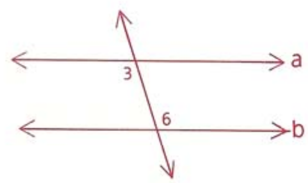


Proof: Locate the midpoint, M, of $\overline{BC}$. Draw $\overline{AP}$ so that AM = MP. Draw $\overline{CP}$. $\overline{MB} \cong \overline{MC}$, $\overline{AM} \cong \overline{MP}$, and vertical angles are congruent. Thus, △ABM ≅ △PCM and∠1 ≅ ∠B. Since m∠BCD > m∠1, we know that m∠BCD > m∠B. The second part of the theorem is proved by extending $\overline{BC}$ to form the other exterior angle, a vertical angle to ∠BCD. The result follows.

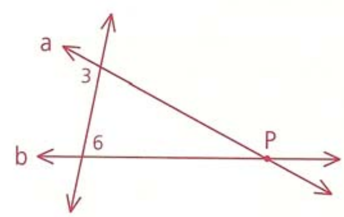
Theorem 31 ***If two lines are cut by a transversal such that two alternate interior angles are congruent, the lines are parallel. (Short form: Alt. int. ∠s ≅ ⇒ ∥ lines)***

Given: ∠3 ≅ ∠6

Prove: a ∥ b



Proof: (Indirect proof) *Assume that the lines are not parallel.* Then a and b must intersect at some point P. ∠3 is an exterior angle of the triangle formed, so by the Exterior Angle Inequality Theorem, m∠3 > m∠6. But this contradicts the given: ∠3 ≅ ∠6. Thus, our assumption was false; the lines *are* parallel.

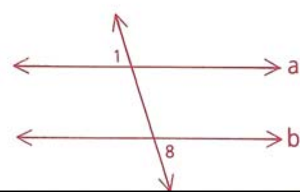


Theorem 32 ***If two lines are cut by a transversal such that two alternate exterior angles are congruent, the lines are parallel. (Alt. ext. ∠s ≅ ⇒ ∥ lines.)***

Given: ∠1 ≅ ∠8

Prove: a ∥ b

This can be proved by use of alt. int. ∠s ≅ ⇒ ∥ lines.



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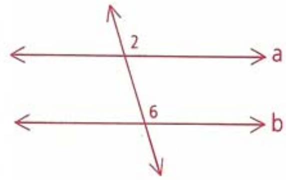
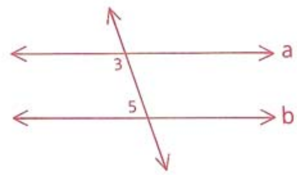
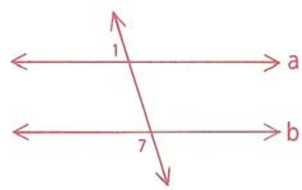
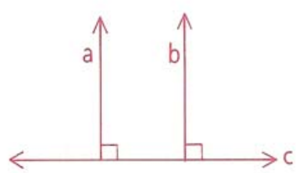
	Theorem 33 ***If two lines are cut by a transversal such that two corresponding angles are congruent, the lines are parallel.*** *(Corr. $\angle$s $\cong \Rightarrow \parallel$ lines)* Given: $\angle 2 \cong \angle 6$ Prove: $a \parallel b$ This can be proved by use of alt. int. $\angle$s $\cong \Rightarrow \parallel$ lines. 
	Theorem 34 ***If two lines are cut by a transversal such that two interior angles on the same side of the transversal are supplementary, the lines are parallel.*** Given: $\angle 3$ supp. $\angle 5$ Prove: $a \parallel b$ This can be proved by use of alt. int. $\angle$s $\cong \Rightarrow \parallel$ lines. 
	Theorem 35 ***If two lines are cut by a transversal such that two exterior angles on the same side of the transversal are supplementary, the lines are parallel.*** Given: $\angle 1$ supp. $\angle 7$ Prove: $a \parallel b$ This can be proved by use of alt. int. $\angle$s $\cong \Rightarrow \parallel$ lines. 
	Theorem 36 ***If two coplanar lines are perpendicular to a third line, they are parallel.*** Given: $a \perp c$ and $b \perp c$ Prove: $a \parallel b$ This can be proved by use of corr. $\angle$s $\cong \Rightarrow \parallel$ lines. 

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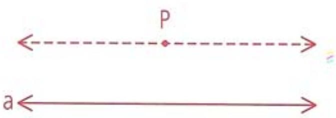
5.3

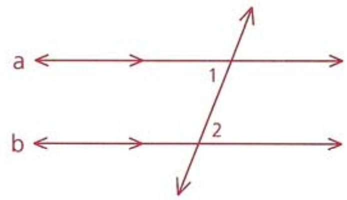
Postulate ***Through a point not on a line there is exactly one parallel to the given line.***



Theorem 37 ***If two parallel lines are cut by a transversal, each pair of alternate interior angles are congruent. (Short form: ∥ lines ⇒ alt. int. ∠s ≅)***

Given: Lines a and b are parallel.

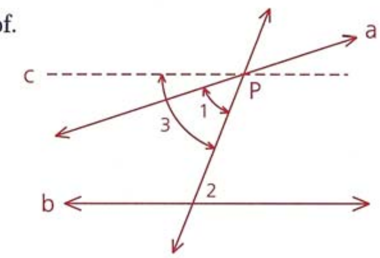
Prove: ∠1 ≅ ∠2



Notice the special tick marks (⇉) used to designate parallel lines.

Proof: This theorem can be verified by indirect proof.

We are given a ∥ b. Assume that ∠1 is not congruent to ∠2. Then there must be another line, c, that intersects the transversal at P to form an angle, ∠3, that is congruent to ∠2. But in Section 5.2 we observed that congruent alternate interior angles lead to parallel lines. Thus, c ∥ b.

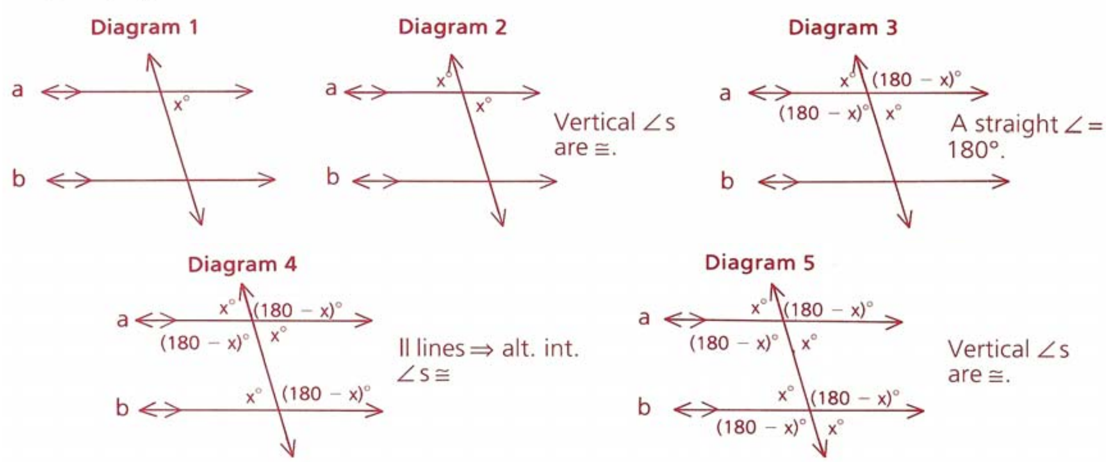


This means that line b is parallel to *two* lines in the plane at point P. This violates the Parallel Postulate. So we can conclude that our assumption is false. Therefore, ∠1 ≅ ∠2.

You may be surprised to learn the following.

Theorem 38 ***If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.***

The proof of this may be developed algebraically by letting x be the measure of any one of the angles. Follow the steps below. In each diagram, a ∥ b.



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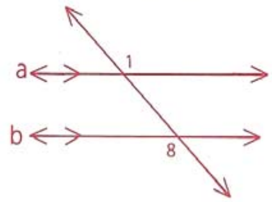
Theorem 39 ***If two parallel lines are cut by a transversal, each pair of alternate exterior angles are congruent.***
(‖ lines ⇒ alt. ext. ∠s ≅)

Given: a ‖ b

Prove: ∠1 ≅ ∠8

Proof: See Diagram 5.

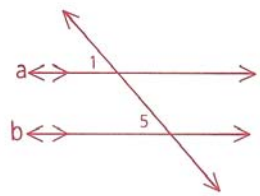


Theorem 40 ***If two parallel lines are cut by a transversal, each pair of corresponding angles are congruent.***
(‖ lines ⇒ corr. ∠s ≅)

Given: a ‖ b

Prove: ∠1 ≅ ∠5

Proof: See Diagram 5.

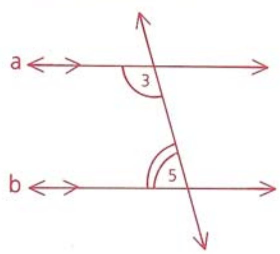


Theorem 41 ***If two parallel lines are cut by a transversal, each pair of interior angles on the same side of the transversal are supplementary.***

Given: a ‖ b

Prove: ∠3 supp. ∠5

Proof: See Diagram 5.

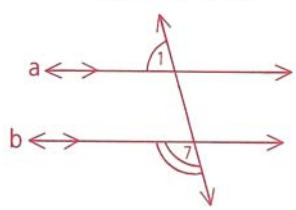


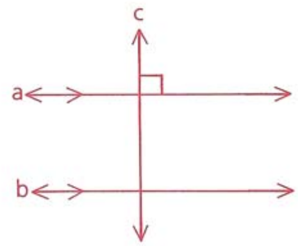
Theorem 42 ***If two parallel lines are cut by a transversal, each pair of exterior angles on the same side of the transversal are supplementary.***

Given: a ‖ b

Prove: ∠1 supp. ∠7

Proof: See Diagram 5.



	Theorem 43 ***In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.*** Given: a ∥ b, c ⊥ a Prove: c ⊥ b  *Proof:* See Diagram 5 and let x = 90.
	In summary, if two parallel lines are cut by a transversal, then ▪ Each pair of alternate interior angles are congruent ▪ Each pair of alternate exterior angles are congruent ▪ Each pair of corresponding angles are congruent ▪ Each pair of interior angles on the same side of the transversal are supplementary ▪ Each pair of exterior angles on the same side of the transversal are supplementary
5.4	**Definition** A ***convex polygon*** is a polygon in which each interior angle has a measure less than 180.
	Definition A ***diagonal*** of a polygon is any segment that connects two nonconsecutive (nonadjacent) vertices of the polygon.

Quadrilaterals

A ***quadrilateral*** is a four-sided polygon.

The following are special quadrilaterals.

A ***parallelogram*** is a quadrilateral in which both pairs of opposite sides are parallel.

A ***rectangle*** is a parallelogram in which at least one angle is a right angle.

A ***rhombus*** is a parallelogram in which at least two consecutive sides are congruent.

A ***kite*** is a quadrilateral in which two *disjoint* pairs of consecutive sides are congruent.

A ***square*** is a parallelogram that is both a rectangle and a rhombus.

A ***trapezoid*** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases* of the trapezoid.

An ***isosceles trapezoid*** is a trapezoid in which the nonparallel sides (*legs*) are congruent. In the figure, $\angle A$ and $\angle B$ are called the ***lower base angles,*** and $\angle C$ and $\angle D$ are called the ***upper base angles***.

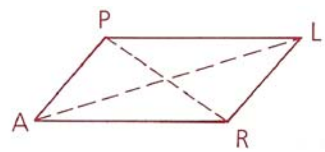
5.5

Properties of Parallelograms

In this section, we will list some of the properties of special quadrilaterals, beginning with parallelograms. (You should be able to prove many of these properties.) Read the properties carefully and learn them. They will be used often in the sections to follow.

Learning so many properties may seem overwhelming at first, but most are concepts that you already know or that you discovered in Section 5.4. With some effort you will soon learn them all.

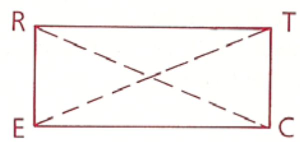
In a parallelogram,



1. The opposite sides are parallel by definition ($\overline{PL} \parallel \overline{AR}$, $\overline{AP} \parallel \overline{RL}$)
2. The opposite sides are congruent ($\overline{PL} \cong \overline{AR}$, $\overline{AP} \cong \overline{RL}$)
3. The opposite angles are congruent ($\angle PAR \cong \angle PLR$, $\angle ARL \cong \angle APL$)
4. The diagonals bisect each other ($\overline{AL}$ bis. $\overline{PR}$, $\overline{PR}$ bis. $\overline{AL}$)
5. Any pair of consecutive angles are supplementary ($\angle PAR$ supp. $\angle ARL$, etc.)

Properties of Rectangles

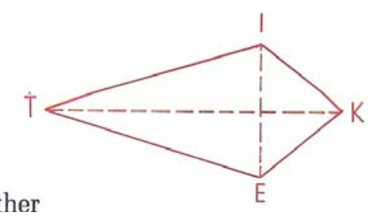
In a rectangle,



1. All the properties of a parallelogram apply by definition
2. All angles are right angles ($\angle REC$ is a right angle, etc.)
3. The diagonals are congruent ($\overline{ET} \cong \overline{RC}$)

Properties of Kites

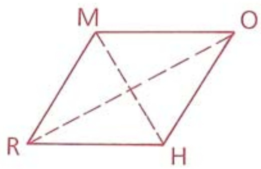
In a kite,



1. Two disjoint pairs of consecutive sides are congruent by definition ($\overline{IT} \cong \overline{ET}$, $\overline{IK} \cong \overline{EK}$)
2. The diagonals are perpendicular ($\overline{TK} \perp \overline{IE}$)
3. One diagonal is the perpendicular bisector of the other ($\overline{TK} \perp$ bis. $\overline{IE}$)
4. One of the diagonals bisects a pair of opposite angles ($\overrightarrow{TK}$ bis. $\angle ITE$, $\overrightarrow{TK}$ bis. $\angle IKE$)
5. One pair of opposite angles are congruent ($\angle TIK \cong \angle TEK$)

Properties 3–5 are sometimes called the *half properties* of kites.

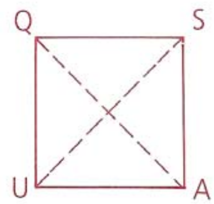
Properties of Rhombuses



In a rhombus,

1 All the properties of a parallelogram apply by definition
2 All the properties of a kite apply (In fact, the half properties become full properties)
3 All sides are congruent—that is, a rhombus is equilateral ($\overline{RH} \cong \overline{HO} \cong \overline{OM} \cong \overline{MR}$)
4 The diagonals bisect the angles ($\overrightarrow{RO}$ bis. $\angle MRH$, $\overrightarrow{RO}$ bis. $\angle MOH$, etc.)
5 The diagonals are perpendicular bisectors of each other ($\overline{RO} \perp$ bis. $\overline{MH}$, $\overline{MH} \perp$ bis. $\overline{RO}$)
6 The diagonals divide the rhombus into four congruent right triangles

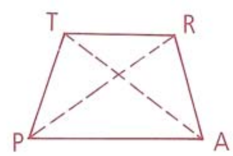
Properties of Squares



In a square,

1 All the properties of a rectangle apply by definition
2 All the properties of a rhombus apply by definition
3 The diagonals form four isosceles right triangles ($45°$-$45°$-$90°$ triangles)

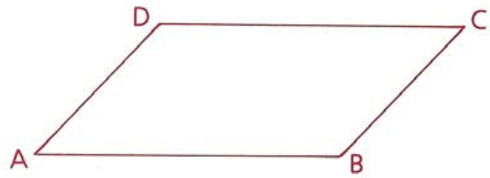
Properties of Isosceles Trapezoids



In an isosceles trapezoid,

1 The legs are congruent by definition ($\overline{TP} \cong \overline{RA}$)
2 The bases are parallel (by definition of *trapezoid*) ($\overline{TR} \parallel \overline{PA}$)
3 The lower base angles are congruent ($\angle RAP \cong \angle TPA$)
4 The upper base angles are congruent ($\angle PTR \cong \angle ART$)
5 The diagonals are congruent ($\overline{PR} \cong \overline{AT}$)
6 Any lower base angle is supplementary to any upper base angle ($\angle PAR$ supp. $\angle PTR$, etc.)

5.6



Any one of the following methods might be used to prove that quadrilateral ABCD is a parallelogram.

1 If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram (reverse of the definition).

2 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram (converse of a property).

3 If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

4 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram (converse of a property).

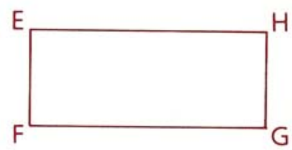
5 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram (converse of a property).

5.7

Proving That a Quadrilateral Is a Rectangle

When you want to prove that a figure is one of the special quadrilaterals, you must be sure that you prove sufficient properties to establish the quadrilateral's identity.

You can prove that quadrilateral EFGH is a rectangle by first showing that the quadrilateral is a parallelogram and then using either of the following methods to complete the proof.



1 If a parallelogram contains at least one right angle, then it is a rectangle (reverse of the definition).

2 If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

You can also prove that a quadrilateral is a rectangle without first showing that it is a parallelogram.

3 If all four angles of a quadrilateral are right angles, then it is a rectangle.

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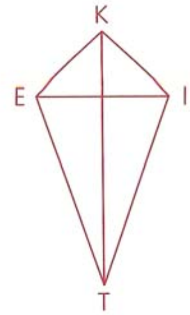
Proving That a Quadrilateral Is a Kite

To prove that a quadrilateral is a kite, either of the following methods can be used.

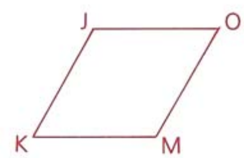


1 If two disjoint pairs of consecutive sides of a quadrilateral are congruent, then it is a kite (reverse of the definition).

2 If one of the diagonals of a quadrilateral is the perpendicular bisector of the other diagonal, then the quadrilateral is a kite.

Proving That a Quadrilateral Is a Rhombus

To prove that quadrilateral KMOJ is a rhombus, you may first show that it is a parallelogram and then apply either of the following methods.



1 If a parallelogram contains a pair of consecutive sides that are congruent, then it is a rhombus (reverse of the definition).

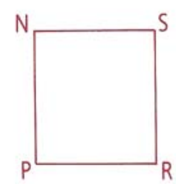
2 If either diagonal of a parallelogram bisects two angles of the parallelogram, then it is a rhombus.

You can also prove that a quadrilateral is a rhombus without first showing that it is a parallelogram.

3 If the diagonals of a quadrilateral are perpendicular bisectors of each other, then the quadrilateral is a rhombus.

Proving That a Quadrilateral is a Square

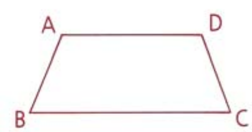
The following method can be used to prove that NPRS is a square:



- If a quadrilateral is both a rectangle and a rhombus, then it is a square (reverse of the definition).

Proving That a Trapezoid Is Isosceles

Any one of the following methods can be used to prove that a trapezoid is isosceles.



1 If the nonparallel sides of a trapezoid are congruent, then it is isosceles (reverse of the definition).

2 If the lower or the upper base angles of a trapezoid are congruent, then it is isosceles.

3 If the diagonals of a trapezoid are congruent, then it is isosceles.