AMDG

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Adv Geo –
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Special Right Triangles (9.7) Notes

Objectives

After studying this section, you will be able to

- Identify the ratio of side lengths in a 30°-60°-90° triangle
- Identify the ratio of side lengths in a 45°-45°-90° triangle

Theorem 72 ***In a triangle whose angles have the measures 30, 60, and 90, the lengths of the sides opposite these angles can be represented by x, $x\sqrt{3}$, and 2x respectively. (30°-60°-90°-Triangle Theorem)***

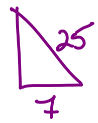
$\boxed{(3,4,5)}$

$3^2+4^2=5^2$

$9+16=25$

$\boxed{(5,12,13)}$

$\boxed{(8,15,17)}$



$7^2+x^2=25^2$

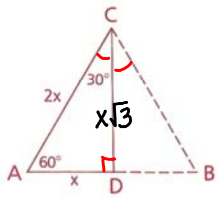
$x^2=625-49$

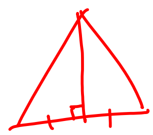
$x^2=576$

$x=24$

$\boxed{(7,24,25)}$

Given: △ABC is equilateral.
$\overrightarrow{CD}$ bisects ∠ACB.
Prove: AD:DC:AC = x:$x\sqrt{3}$:2x





Proof: Since △ABC is equilateral, ∠ACD = 30°, ∠A = 60°, ∠ADC = 90°, and AD = $\frac{1}{2}$(AC).
By the Pythagorean Theorem, in △ADC,

$x^2 + (DC)^2 = (2x)^2$

$x^2 + DC^2 = 4x^2$

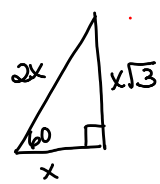
$-x^2 \qquad -x^2$

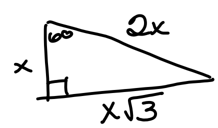
$\sqrt{DC^2} = \sqrt{3x^2}$

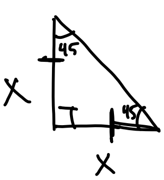
$DC = x\sqrt{3}$

$$\boxed{\begin{matrix}(30°, 60°, 90°)\\(x, x\sqrt{3}, 2x)\end{matrix}}$$

work through the rest now.







(45°, 45°, 90°)

x , x , ?

$x^2+x^2=?^2$

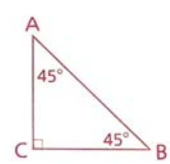
$\sqrt{2x^2}=\sqrt{?^2}$

$x\sqrt{2}=?$

$$\boxed{\begin{matrix}(45°, 45°, 90°)\\(x, x, x\sqrt{2})\end{matrix}}$$

Theorem 73 ***In a triangle whose angles have the measures 45, 45, and 90, the lengths of the sides opposite these angles can be represented by x, x, and $x\sqrt{2}$, respectively. (45°-45°-90°-Triangle Theorem)***

Given: △ACB, with ∠A = 45° and ∠B = 45°.
Prove: AC:CB:AB = x:x:$x\sqrt{2}$
The proof of this theorem is left to you.

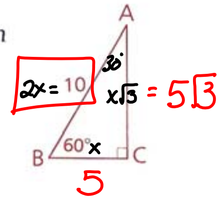


You will see 30°-60°-90° and 45°-45°-90° triangles frequently in this book and in other mathematics courses. Their ratios are worth memorizing now.

Six Common Families of Right Triangles	
* 30°-60°-90° ⇔ (x, $x\sqrt{3}$, 2x)	(5, 12, 13)
* 45°-45°-90° ⇔ (x, x, $x\sqrt{2}$)	(7, 24, 25)
(3, 4, 5)	(8, 15, 17)

Examples

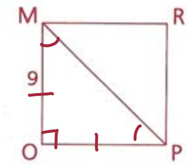
Problem 1 *Type: Hypotenuse (2x) known*
Find BC and AC.



$2x = 10$

$x = 5$

Problem 3 *Type: Leg (x) known*
MOPR is a square.
Find MP.

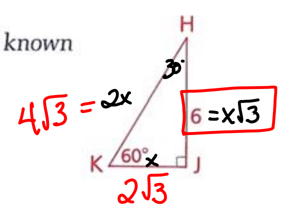


$(45°, 45°, 90°)$

$(x, x, x\sqrt{2})$

$9, 9, 9\sqrt{2}$

Problem 2 *Type: Longer leg $(x\sqrt{3})$ known*
Find JK and HK.



$$\frac{6}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$$

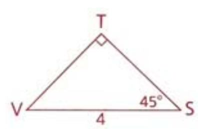
$$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{6}{\sqrt{3}} = x$$

rationalize the denominator

$$\frac{\cancel{6}\sqrt{3}}{\cancel{3}} = x$$

$2\sqrt{3} = x$

Problem 4 *Type: Hypotenuse $(x\sqrt{2})$ known*
Find ST and TV.



45°	45°	90°
x	x	$x\sqrt{2}$
$2\sqrt{2}$	$2\sqrt{2}$	4

$$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{4}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$$

$$\frac{\cancel{4}\sqrt{2}}{\cancel{2}} = x$$

$2\sqrt{2}$