

NAME

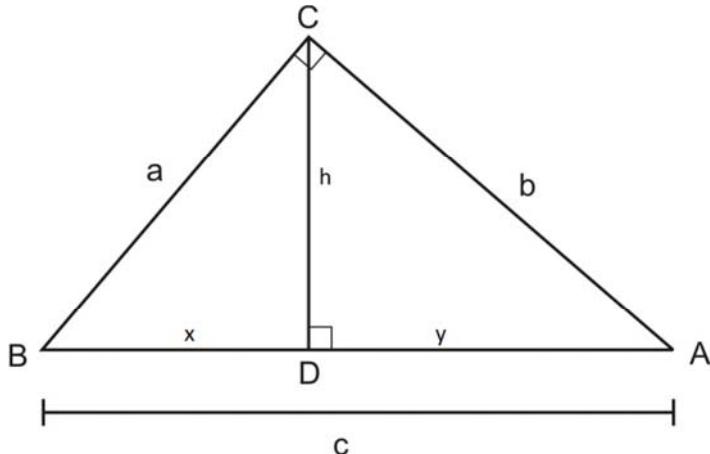
Ms. Kresovic

Adv. Geo. -

Thurs 28 February 2013

## 9.3. Altitude Hypotenuse Theorems

Objective: After studying this section, you will be able to identify the relationships between the parts of a right triangle when an altitude is drawn to the hypotenuse.



Prior Knowledge: Pythagorean Theorem, as  $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$  where  $a$  &  $b$  are legs and  $c$  is the hypotenuse. In our worksheet, we used similar triangles to observe that the altitude is the geometric mean of the hypotenuse parts, that is  $h^2 = xy$ . Some of those exercises were leading us to observe two more theorems:  $a^2 = xc$  and  $b^2 = yc$ .

Compare this diagram to the one in our book (below) and see how the formulas are similar. Can you come up with a more generalized (verbal) formula?

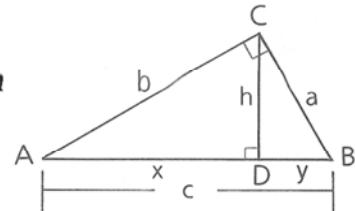
**Theorem 68** *If an altitude is drawn to the hypotenuse of a right triangle, then*

- a *The two triangles formed are similar to the given right triangle and to each other*  
 $\Delta ADC \sim \Delta ACB \sim \Delta CDB$
- b *The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse*

$$\frac{x}{h} = \frac{h}{y}, \text{ or } h^2 = xy$$

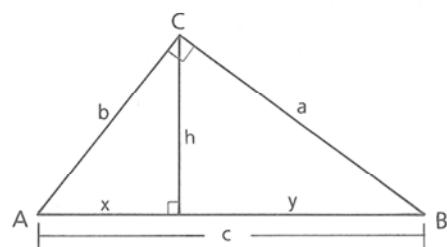
- c *Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hypotenuse)*

$$\frac{y}{a} = \frac{a}{c}, \text{ or } a^2 = yc; \text{ and } \frac{x}{b} = \frac{b}{c}, \text{ or } b^2 = xc$$



Parts b and c of Theorem 68 can be summarized as follows.

$$\begin{aligned} h^2 &= x \cdot y \\ b^2 &= x \cdot c \\ a^2 &= y \cdot c \end{aligned}$$



NAME

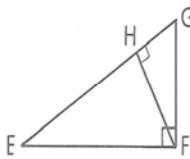
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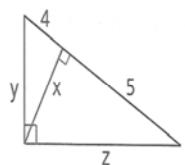
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## 9.3. Altitude Hypotenuse Theorems

1 a If  $EH = 7$  and  $HG = 3$ , find  $HF$ .  
 b If  $EH = 7$  and  $HG = 4$ , find  $EF$ .  
 c If  $GF = 6$  and  $EG = 9$ , find  $HG$ .



2 a Find  $2x$ .    b Find  $\frac{1}{2}y$ .    c Find  $z + 8$ .



1a

2a

1b

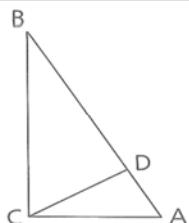
2b

1c

2c

3 Given:  $\overline{AC} \perp \overline{CB}$ ,  $\overline{CD} \perp \overline{AB}$

a If  $AD = 4$  and  $BD = 9$ , find  $CD$ .  
 b If  $AD = 4$  and  $AB = 16$ , find  $AC$ .  
 c If  $BD = 6$  and  $AB = 8$ , find  $BC$ .  
 d If  $CD = 8$  and  $BD = 16$ , find  $AD$ .  
 e If  $AD = 3$  and  $BD = 24$ , find  $AC$ .  
 f If  $BC = 8$  and  $BD = 20$ , find  $AB$ .

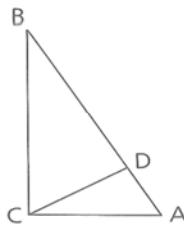


3a

3b

3 Given:  $\overline{AC} \perp \overline{CB}$ ,  $\overline{CD} \perp \overline{AB}$

- If  $AD = 4$  and  $BD = 9$ , find  $CD$ .
- If  $AD = 4$  and  $AB = 16$ , find  $AC$ .
- If  $BD = 6$  and  $AB = 8$ , find  $BC$ .
- If  $CD = 8$  and  $BD = 16$ , find  $AD$ .
- If  $AD = 3$  and  $BD = 24$ , find  $AC$ .
- If  $BC = 8$  and  $BD = 20$ , find  $AB$ .



3c

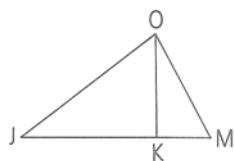
3d

3e

3f

4 Given:  $\angle JOM = 90^\circ$ ;  $\overline{OK}$  is an altitude.

- If  $JK = 12$  and  $KM = 5$ , find  $OK$ .
- If  $OK = 3\sqrt{5}$  and  $JK = 9$ , find  $KM$ .
- If  $JO = 3\sqrt{2}$  and  $JK = 3$ , find  $JM$ .
- If  $KM = 5$  and  $JK = 6$ , find  $OM$ .

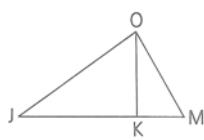


4a

4b

4 Given:  $\angle JOM = 90^\circ$ ;  $\overline{OK}$  is an altitude.

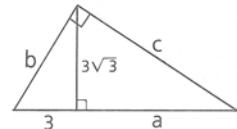
- If  $JK = 12$  and  $KM = 5$ , find  $OK$ .
- If  $OK = 3\sqrt{5}$  and  $JK = 9$ , find  $KM$ .
- If  $JO = 3\sqrt{2}$  and  $JK = 3$ , find  $JM$ .
- If  $KM = 5$  and  $JK = 6$ , find  $OM$ .



4c

4d

5 a Find  $a$ .  
b Find  $ab$ .  
c Find  $a + b + c$ .



5a

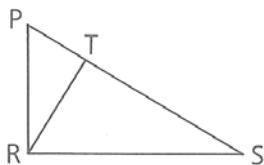
5b

5c

5d

6 Given:  $\overline{RT}$  is an altitude.  $\angle PRS$  is a right  $\angle$ .

Conclusion:  $\frac{PR}{RS} = \frac{RT}{ST}$

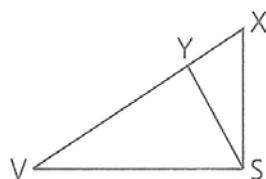


Statements

Reasons

7 Given:  $\overline{SY}$  is an altitude.  $\angle VSX$  is a right  $\angle$ .

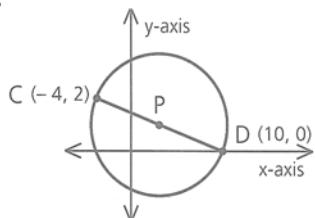
Prove:  $XY \cdot SV = XS \cdot YS$



Statements

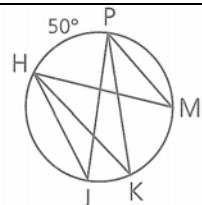
Reasons

8 Find the coordinates of P, the center of the circle.

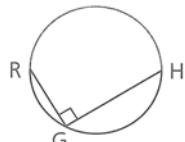


9 Given: Diagram as marked

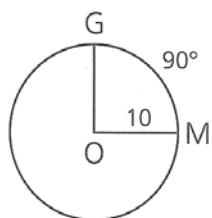
Find:  $m\angle HJP$ ,  $m\angle HKP$ , and  $m\angle HMP$



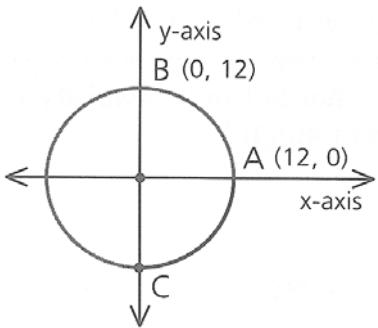
10 Find the measure of  $\widehat{RH}$ .



11 Find the area of sector MOG.



**12** **a** Find the coordinates of point C.  
**b** Find the measure of the arc from A to B to C ( $m\widehat{ABC}$ ).  
**c** Find the length of  $\widehat{ABC}$ .

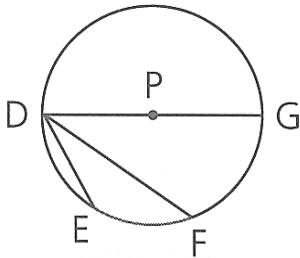


12a

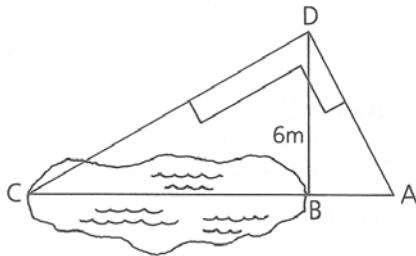
12b

12c

**13** In  $\odot P$ ,  $m\widehat{FG} = 80$  and  $m\widehat{DE} = 40$ . Find  $m\widehat{EF}$  and  $m\angle EDF$ .



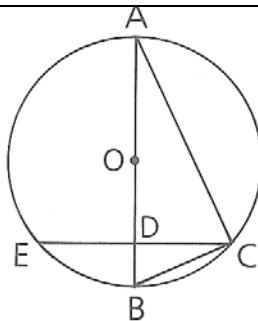
**14** As Slarpy stood at B, the foot of a 6-m pole, he asked Carpy how far it was across the pond from B to C. Carpy got his carpenter's square and climbed the pole. Using his lines of sight, he set up the figure shown. When Slarpy found that  $AB = 3$  m, Carpy knew the answer. What was it?



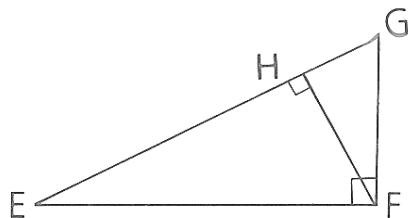
15 Given:  $\odot O$ ,  $\overline{CD} \perp \overline{AB}$ ;  
 $\angle ACB$  is a right  $\angle$ .

Conclusions: a  $\frac{AD}{CD} = \frac{CD}{BD}$

b  $\frac{AD}{ED} = \frac{ED}{BD}$



16 a If  $HG = 4$  and  $EF = 3\sqrt{5}$ , find  $EH$ .  
b If  $GF = 6$  and  $EH = 9$ , find  $EG$ .

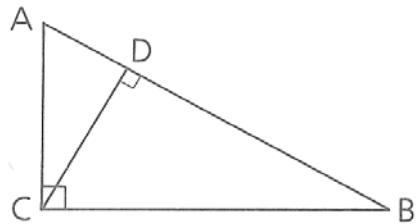


16a

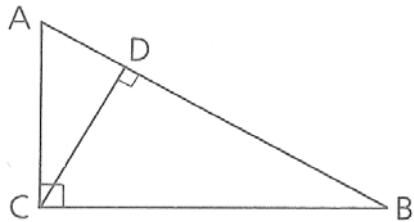
16b

17 a If  $AD = 7$  and  $AB = 11$ , find  $CD$ .  
b If  $CD = 8$  and  $AD = 6$ , find  $AB$ .  
c If  $AB = 12$  and  $AD = 4$ , find  $BC$ .  
d If  $AC = 7$  and  $AB = 12$ , find  $BD$ .

17a



17 a If  $AD = 7$  and  $AB = 11$ , find  $CD$ .  
 b If  $CD = 8$  and  $AD = 6$ , find  $AB$ .  
 c If  $AB = 12$  and  $AD = 4$ , find  $BC$ .  
 d If  $AC = 7$  and  $AB = 12$ , find  $BD$ .



17c

17b

21 Given:  $\overline{AD} \perp \overline{CD}$ ,  
 $\overline{BD} \perp \overline{AC}$ ,  
 $BC = 5$ ,  $AD = 6$

Find:  $BD$

