AMDG

Name Ms. Kresovic

Adv Geo 9.2: Introduction to Circles Date

Objective

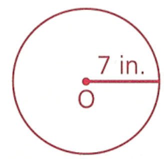
After studying this section, you will be able to

- Begin solving problems involving circles

fence : $C = d\pi$ or $2r\pi$

paint : $A = r^2\pi$

Example 1 *Find the circumference and the area of ⊙O.*



$C = 14\pi$ units

$A = 49\pi$ units^2

The circumference is found with the formula $C = \pi d$, where d is the diameter of the circle.

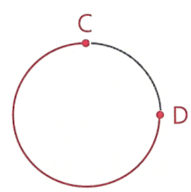
$C = \pi d$
$= 14\pi$

The area is found with the formula $A = \pi r^2$, where r is the circle's radius.

$A = \pi r^2$
$= \pi(7^2)$
$= 49\pi$

The circle's circumference is 14π, or about 43.98, inches and its area is 49π, or about 153.94, square inches.

An *arc* is made up of two points on a circle and all the points of the circle needed to connect those two points by a single path. The blue portion of the figure at the right is called arc CD (symbolized $\overset{\frown}{CD}$).



An exact answer is written in terms of pi (π): 3π is exact.

An estimated value multiplies an estimate of pi: 9.42 is an estimated value of 3π.

exact est.

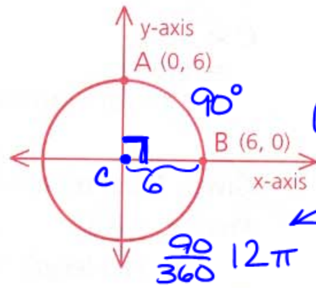
$3\pi \approx 9.42$

The *measure* of an arc is equivalent to the number of degrees it occupies. (A complete circle occupies 360°.) The *length* of an arc is a fraction of a circle's circumference, so it is expressed in linear units, such as feet, centimeters, or inches.

Length is distance (in, ft, mi, cm, m, km)

Example 2 *Find the measure and the length of $\widehat{AB}$.*

↳ degrees = central ∠



$m\widehat{AB} = 90°$

Length $\widehat{AB} = 3\pi$

Length $\frown = \% \odot \cdot C$

$\frac{m\frown}{360} \cdot d\pi$

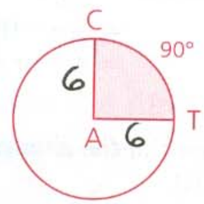
$\frac{90}{360} 12\pi = \frac{1}{4} 12\pi = \boxed{3\pi}$

Since the arc is one fourth of the circle, its measure is $\frac{1}{4}(360)$, or 90. The arc's length (ℓ) can be expressed as a part of the circle's circumference.

$$\begin{aligned}\ell &= \tfrac{90}{360}C \\ &= \tfrac{1}{4}\pi d \\ &= \tfrac{1}{4}(\pi \cdot 12) \\ &= 3\pi, \text{ or } \approx 9.42\end{aligned}$$

pizza slice

A *sector* of a circle is a region bounded by two radii and an arc of the circle. The figure at the right shows sector CAT of ⊙A.



$A_{SECTOR} = (\% \odot)(A_\odot)$

$\frac{90}{360}(\pi 6^2)$

$\frac{1}{4} 36\pi = \boxed{9\pi}$

Since we know that $\widehat{CT}$ has a measure of 90, we can calculate the area of sector CAT as a fraction of the area of ⊙A.

$$\begin{aligned}\text{Area of sector CAT} &= \tfrac{90}{360}(\text{area of } \odot A) \\ &= \tfrac{1}{4}(\pi \cdot 6^2) \\ &= 9\pi, \text{ or } \approx 28.27\end{aligned}$$

What's a central ∠?

∠: vertex = ctr ⊙

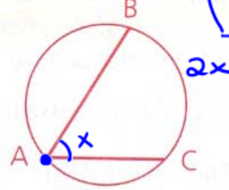
A *chord* is a line segment joining two points on a circle. (A *diameter* is a chord that passes through the center of its circle.) An *inscribed angle* is an angle whose vertex is on a circle and whose sides are determined by two chords of the circle.

Inscribed ∠?

∠: vertex ON ⊙

In the figure at the right, $\overline{AB}$ and $\overline{AC}$ are chords, and ∠BAC is an inscribed angle.

∠BAC is said to *intercept* $\widehat{BC}$. (An intercepted arc is an arc whose endpoints are on the sides of an angle and whose other points all lie within the angle. Although ∠BAC intercepts only one arc, in Chapter 10 you will deal with some angles that intercept two arcs of a circle.)



$\angle A = \frac{\widehat{BC}}{2}$

If ∠A = 45° then $\widehat{BC} = 90°$

If $\widehat{BC} = 100°$ then ∠A = 50°

central ∠W = 40°

$m\widehat{XY} = 40°$

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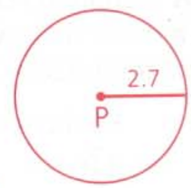
Problem 1 *Find the circumference and the area of ⊙P.*



Solution

$C = \pi d$
$= \pi(5.4)$
$= \underline{5.4\pi}$, or ≈ 16.96
exact est.

$A = \pi r^2$
$= \pi(2.7^2)$
$= \underline{7.29\pi}$, or ≈ 22.90
Exact est

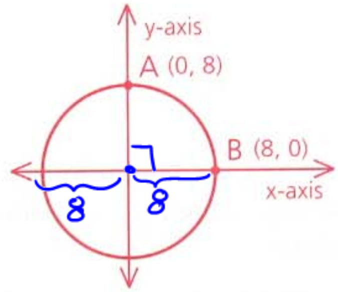
Problem 2 *Given: Diagram as marked*

Find: **a** $m\widehat{AB} = 90°$

b *The length of* $\widehat{AB}$

$= \frac{90}{360}\pi 16$

$= \frac{1}{4} 16\pi = \boxed{4\pi}$



r = 8
d = 16

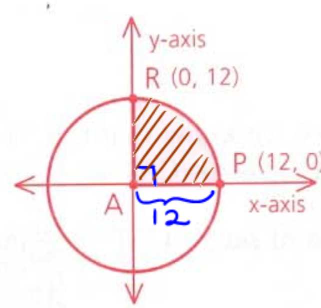
Solution The circle's radius is 8, and $\widehat{AB}$ is one fourth of the circle.

a $m\widehat{AB} = \frac{1}{4}(360)$
$= 90°$

b Length of $\widehat{AB} = \frac{90}{360}C$
$= \frac{1}{4}(\pi \cdot 16)$
$= 4\pi$, or ≈ 12.57

Problem 3 *Find the area of the shaded region (sector PAR).*

$\frac{90}{360}\pi r^2 \Rightarrow \frac{1}{4} \quad \frac{144\pi}{1} \Rightarrow 36\pi$



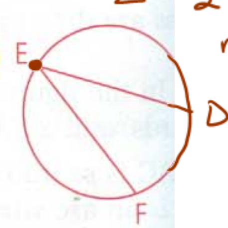
Solution The radius of ⊙A is 12, and $m\widehat{RP} = 90$.

Area of sector PAR $= \frac{90}{360}$(area of ⊙A)
$= \frac{1}{4}(\pi \cdot 12^2)$
$= 36\pi$, or ≈ 113.10

$\angle = \frac{\frown}{2}$

$m\angle E = \frac{m\widehat{DF}}{2}$

Problem 4 *Harry Halph looked ahead to Chapter 10 and discovered that the measure of an inscribed angle is half the measure of its intercepted arc. Use this information to find the measure of inscribed angle DEF.*



Solution $\widehat{DF}$ is the arc intercepted by ∠DEF.

$m\angle DEF = \frac{1}{2}(m\widehat{DF})$
$= \frac{1}{2}(80)$
$= 40$

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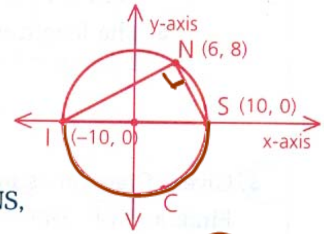
Problem 6 Show that $\triangle INS$ is a right triangle by

a Finding $m\angle INS$

b Finding the slopes of $\overleftrightarrow{IN}$ and $\overleftrightarrow{NS}$



$m\overset{\frown}{ICS} = 180° \Rightarrow \angle N = 90°$

Solution

a $\overset{\frown}{ICS}$ is one-half the circle, so $m\overset{\frown}{ICS} = 180$.

Since $\overset{\frown}{ICS}$ is intercepted by inscribed angle INS,

$$m\angle INS = \tfrac{1}{2}(m\overset{\frown}{ICS})$$
$$= \tfrac{1}{2}(180)$$
$$= 90$$

Therefore, $\angle INS$ is a right angle, and $\triangle INS$ is a right triangle.

b Recall that two lines are perpendicular if their slopes are opposite reciprocals.

Slope of $\overleftrightarrow{IN} = \dfrac{8-0}{6-(-10)} = \dfrac{1}{2}$ $\quad \dfrac{\Delta y}{\Delta x}$

Slope of $\overleftrightarrow{NS} = \dfrac{0-8}{10-6} = -2$ $\quad \left(\tfrac{1}{2}\right)(-2) = -1$

Since $\overleftrightarrow{IN} \perp \overleftrightarrow{NS}$, $\triangle INS$ is a right triangle.

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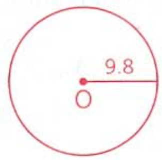
Homework

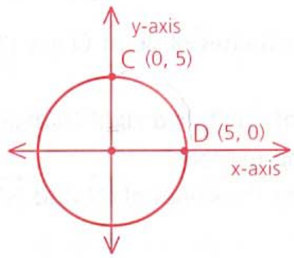
1 Find the circumference and the area of ⊙O. $C = 19.6\pi$, or ≈ 61.58; $A = 96.04\pi$, or ≈ 301.72



2 Given: Diagram as marked

Find: **a** The measure of the arc from C to D ($m\overset{\frown}{CD}$) 90

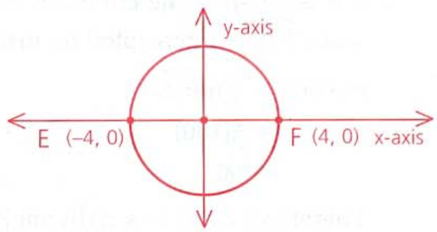
b The length of $\overset{\frown}{CD}$ 2.5π, or ≈ 7.85



3 Given: Diagram as marked

Find: **a** $m\overset{\frown}{EF}$ 180

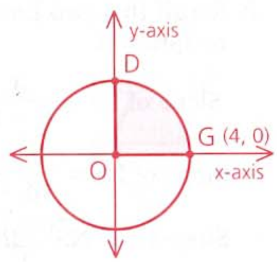
b The length of $\overset{\frown}{EF}$ to the nearest tenth ≈ 12.6



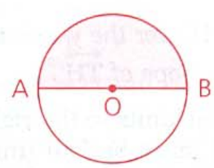
4 Given: Diagram as marked

Find: **a** The coordinates of D (0, 4)

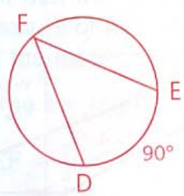
b The area of the shaded region (sector DOG) 4π, or ≈ 12.57



5 If AB = 10, what is the area of the shaded region (sector AOB)? 12.5π, or ≈ 39.27



6 Find $m\angle F$. 45



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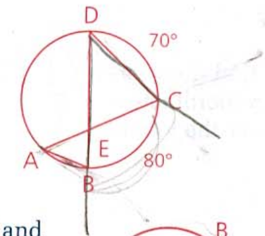
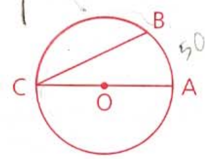
7 Given: Diagram as marked

Find: **a** $m\angle A$ 40

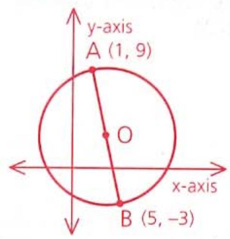
b $m\angle D$ 40



8 In $\odot$O, $m\widehat{AB} = 50$. Find $m\widehat{BC}$ and $m\angle BCA$. 130; 25

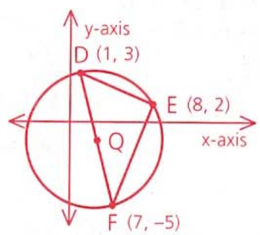


9 In the figure shown, $\overline{AB}$ is a diameter. Find the coordinates of point O, the center of the circle. (3, 3)



10 Find the coordinates of Q, the center of the circle. Then use slopes to show that $\triangle DEF$ is a right triangle.

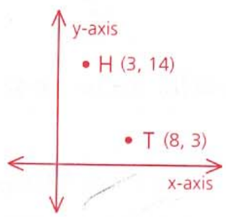
Q = (4, −1); slope of DE = $-\frac{1}{7}$ and slope of EF = 7 (opp. reciprocals), so $\overleftrightarrow{DE} \perp \overleftrightarrow{EF}$ and $\triangle DEF$ is a rt. $\triangle$.



11 Copy the diagram, reflecting H across the y-axis to H′. Then find

a The coordinates of H′ (−3, 14)

b The slope of $\overleftrightarrow{TH'}$ −1



Problem Set B

12 In $\odot$Q, $m\widehat{HJ} = 20$ and $m\widehat{MK} = 40$. The circumference of $\odot$Q is 27π.

a Find $m\widehat{JK}$.

b Find the length of $\widehat{JK}$. 9π

c Find HM (the length of $\overline{HM}$). 27