

**Similar Polygons: Three Theorems Involving Proportions (8.5)**

**Objective**

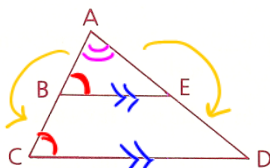
After studying this section, you will be able to

- Apply three theorems frequently used to establish proportionality



**Theorem 65** *If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally. (Side-Splitter Theorem)*

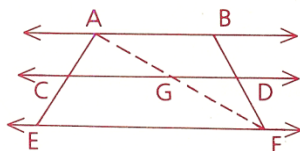
Given:  $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$   
 Prove:  $\frac{AB}{BC} = \frac{AE}{ED}$



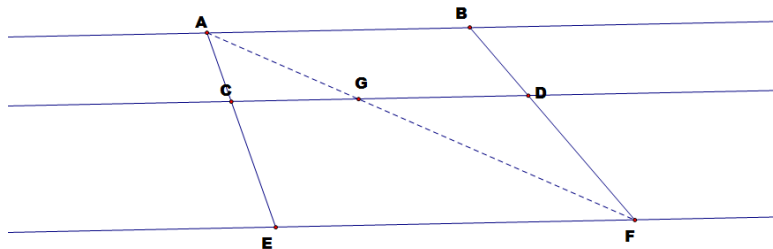
Statements	Reasons
1. $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$	1. GIVEN
2. $\angle ABE \cong \angle C$	2. $\parallel \Rightarrow$ CORR. $\angle$ s $\cong$
3. $\angle A \cong \angle A$	3. REF
4. $\triangle ABE \sim \triangle ACD$	4. AA $\sim$
5. $\frac{AB}{BC} = \frac{AE}{ED}$	5. $\sim \triangle \Rightarrow$ CORR SDS PROP

**Theorem 66** *If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.*

Given:  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$   
 Conclusion:  $\frac{AC}{CE} = \frac{BD}{DF}$



GSP Demo:

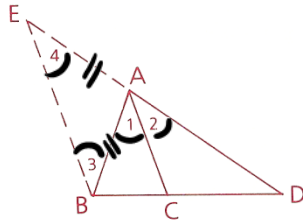


TOP	AC = 2.19 cm	AG = 4.96 cm	BD = 2.69 cm
BOTTOM	CE = 3.99 cm	GF = 9.04 cm	DF = 4.91 cm
	$\frac{AC}{CE} = 0.55$	$\frac{AG}{GF} = 0.55$	$\frac{BD}{DF} = 0.55$

**Theorem 67** If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides. (Angle Bisector Theorem)

Given:  $\triangle ABD$ ;  
 $\overrightarrow{AC}$  bisects  $\angle BAD$ .

Prove:  $\frac{BC}{CD} = \frac{AB}{AD}$



Proof:

- 1  $\triangle ABD$
- 2  $\overrightarrow{AC}$  bisects  $\angle BAD$ .
- 3  $\angle 1 \cong \angle 2$
- 4 Draw through B the line that is  $\parallel$  to  $\overrightarrow{AC}$ .
- 5 Extend  $\overrightarrow{DA}$  to intersect the  $\parallel$  line at some point E.
- 6  $\frac{BC}{CD} = \frac{EA}{AD}$
- 7  $\angle 1 \cong \angle 3$
- 8  $\angle 2 \cong \angle 4$
- 9  $\angle 3 \cong \angle 4$
- 10  $\overline{EA} \cong \overline{AB}$
- 11  $\frac{BC}{CD} = \frac{AB}{AD}$

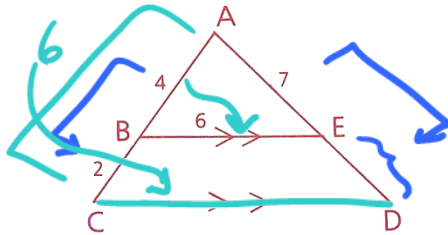
- 1 GIVEN
- 2 GIVEN
- 3 BIS  $\Rightarrow$   $\cong$   $\angle$  S
- 4 PARALLEL POSTULATE
- 5 DEF OF LINE
- 6 SIDE-SPLITTER
- 7  $\parallel \Rightarrow$  ALT INT  $\angle$  S  $\cong$
- 8  $\parallel \Rightarrow$  CORR  $\angle$  S  $\cong$
- 9 Trans
- 10  $\triangle \Rightarrow \triangle$
- 11 Substitute

Practice Problems

**Problem 1**

Given:  $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$ ,  
 lengths as shown

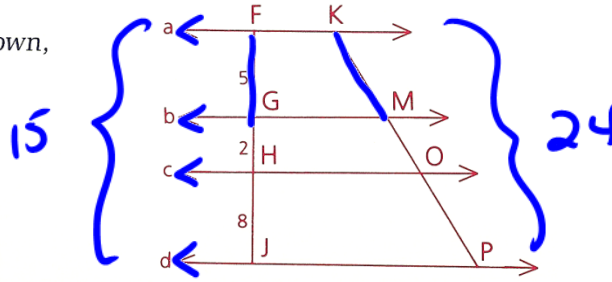
Find: a ED  
 b CD



a) Side Splitter :  $\frac{4}{2} = \frac{7}{ED}$  ,  $4ED = 14$  ,  $ED = \frac{7}{2}$

b)  $\sim \triangle \Rightarrow$  corr sds prop  $\frac{4}{6} = \frac{6}{CD}$  ,  $4CD = 36$  ,  $CD = 9$

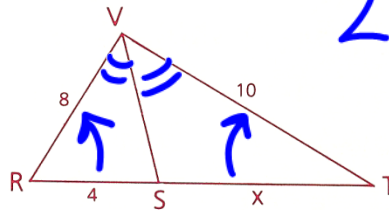
**Problem 2** Given:  $a \parallel b \parallel c \parallel d$ ,  
 lengths as shown,  
 $KP = 24$   
 Find:  $KM$



$$\frac{5}{15} = \frac{KM}{24}$$

$$\frac{1}{3} = \frac{KM}{24}, \quad 3KM = 24, \quad KM = 8$$

**Problem 3** Given:  $\angle RVS \cong \angle SVT$ ,  
 lengths as shown  
 Find:  $ST$



$\angle$  BIS. THM.

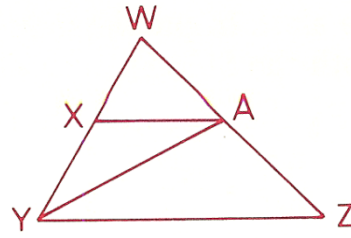
$$\frac{8}{8} = \frac{x}{10}, \quad \frac{1}{1} = \frac{x}{10}, \quad x = 5$$

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**Problem 4**

Given:  $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$ ,  
 $\angle XAY \cong \angle XYA$

Conclusion:  $\frac{WX}{XA} = \frac{WA}{AZ}$



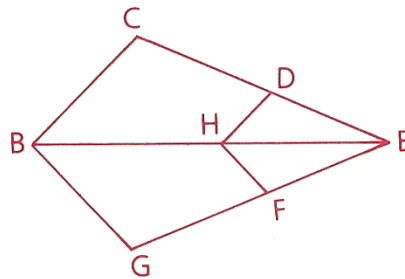
**Proof**

1 $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$		1
2 $\frac{WX}{XY} = \frac{WA}{AZ}$		2
3 $\angle XAY \cong \angle XYA$		3
4 $\overline{XA} \cong \overline{XY}$		4
5 $\frac{WX}{XA} = \frac{WA}{AZ}$		5

**Problem 5**

Given:  $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$ ,  
 $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$

Prove:  $\frac{CD}{DE} = \frac{GF}{FE}$



**Proof**

1 $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$		1
2 $\frac{CD}{DE} = \frac{BH}{HE}$		2
3 $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$		3
4 $\frac{BH}{HE} = \frac{GF}{FE}$		4
5 $\frac{CD}{DE} = \frac{GF}{FE}$		5

10 Given:  $\overleftrightarrow{SV} \parallel \overleftrightarrow{RW}$ ,  
 $RW = 15$ ,  $RS = 10$ ,  
 $ST = 3$ ,  $WV = 8$

Find:  $SV$  and  $VT$

