

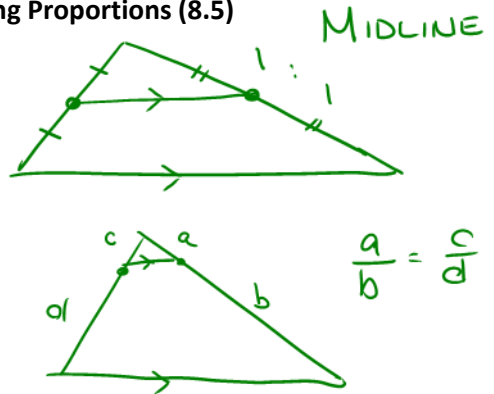
**Similar Polygons: Three Theorems Involving Proportions (8.5)**

**Objective**

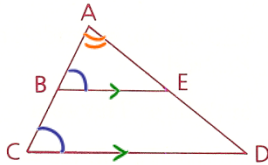
After studying this section, you will be able to

- Apply three theorems frequently used to establish proportionality

**Theorem 65** *If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally. (Side-Splitter Theorem)*



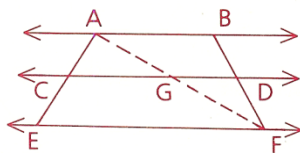
Given:  $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$   
 Prove:  $\frac{AB}{BC} = \frac{AE}{ED}$



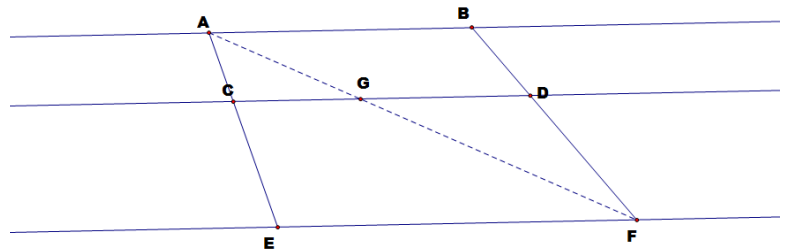
Statements	Reasons
1. $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$	1. GIVEN
2. $\angle ABE \cong \angle C$	2. $\parallel \Rightarrow$ CORR $\angle$ s $\cong$
3. $\angle A \cong \angle A$	3. REF
4. $\triangle ABE \sim \triangle ACD$	4. AA $\sim$
5. $\frac{AB}{BC} = \frac{AE}{ED}$	5. $\sim \triangle \Rightarrow$ CORR. S.D.S. PROP.

**Theorem 66** *If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.*

Given:  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$   
 Conclusion:  $\frac{AC}{CE} = \frac{BD}{DF}$



GSP Demo:



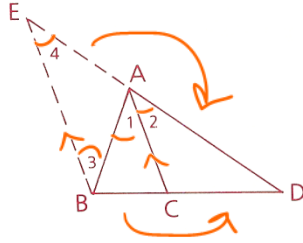
TOP	AC = 2.19 cm	AG = 4.96 cm	BD = 2.69 cm
BOTTOM	CE = 3.99 cm	GF = 9.04 cm	DF = 4.91 cm
	$\frac{AC}{CE} = 0.55$	$\frac{AG}{GF} = 0.55$	$\frac{BD}{DF} = 0.55$

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**Theorem 67** If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides. (Angle Bisector Theorem)

Given:  $\triangle ABD$ ;  
 $\overrightarrow{AC}$  bisects  $\angle BAD$ .

Prove:  $\frac{BC}{CD} = \frac{AB}{AD}$



Proof:

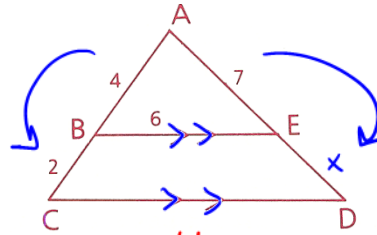
1 $\triangle ABD$	1 GIVEN
2 $\overrightarrow{AC}$ bisects $\angle BAD$ .	2 GIVEN
3 $\angle 1 \cong \angle 2$	3 $\rightarrow$ BIS $\angle \Rightarrow \cong \angle$ s
4 Draw through B the line that is $\parallel$ to $\overrightarrow{AC}$ .	4 parallel post
5 Extend $\overrightarrow{DA}$ to intersect the $\parallel$ line at some point E.	5 def line
6 $\frac{BC}{CD} = \frac{EA}{AD}$	6 sidesplitter
7 $\angle 1 \cong \angle 3$	7 $\parallel \Rightarrow$ alt int $\angle$ s $\cong$
8 $\angle 2 \cong \angle 4$	8 $\parallel \Rightarrow$ corr $\angle$ s $\cong$
9 $\angle 3 \cong \angle 4$	9 trans
10 $\overline{EA} \cong \overline{AB}$	10 $\triangle \Rightarrow \cong$
11 $\frac{BC}{CD} = \frac{AB}{AD}$	11 substitute

Practice Problems

**Problem 1**

Given:  $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$ ,  
 lengths as shown

Find: a ED  
 b CD



*\* mistake*

a) SIDE SPLITTER:  $\frac{4}{2} = \frac{7}{x}$ ,  $\frac{2}{1} = \frac{7}{x}$ ,  $2x = 7$ ,  $x = \frac{7}{2}$

b)  $\sim \triangle \Rightarrow$  corr. sds. prop  $\frac{AB}{BE} = \frac{AC}{CD}$

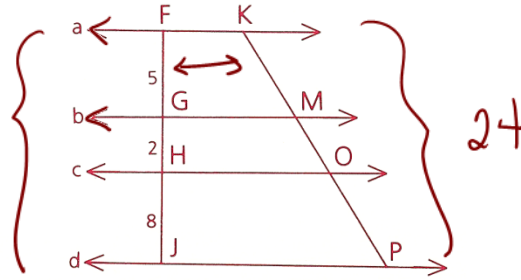
$\frac{4}{6} = \frac{6}{y}$ ,  $4y = 36$ ,  $y = 9$

NAME \_\_\_\_\_  
 Adv Geo - \_\_\_\_\_

Ms. Kresovic  
 Date \_\_\_\_\_

**Problem 2** Given:  $a \parallel b \parallel c \parallel d$ ,  
 lengths as shown,  
 $KP = 24$   
 Find:  $KM$

15

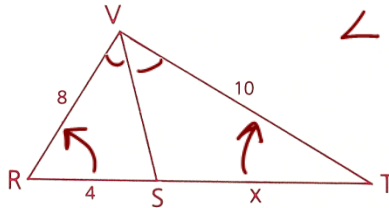


$$\frac{KM}{24} = \frac{5}{15}, \quad \frac{KM}{24} = \frac{1}{3},$$

$$KM = 8$$

**Problem 3** Given:  $\angle RVS \cong \angle SVT$ ,  
 lengths as shown  
 Find:  $ST$

$\angle$  BIS THM



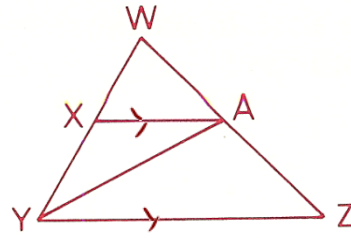
$$\frac{4}{8} = \frac{x}{10}, \quad \frac{1}{2} = \frac{x}{10}, \quad x = 5$$

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**Problem 4**

Given:  $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$ ,  
 $\angle XAY \cong \angle XYA$

Conclusion:  $\frac{WX}{XA} = \frac{WA}{AZ}$



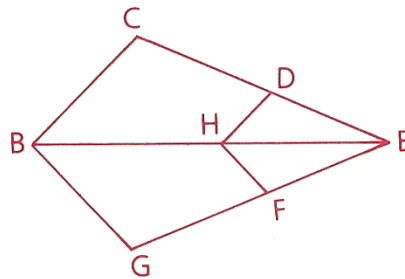
**Proof**

1 $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$	1
2 $\frac{WX}{XY} = \frac{WA}{AZ}$	2
3 $\angle XAY \cong \angle XYA$	3
4 $\overline{XA} \cong \overline{XY}$	4
5 $\frac{WX}{XA} = \frac{WA}{AZ}$	5

**Problem 5**

Given:  $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$ ,  
 $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$

Prove:  $\frac{CD}{DE} = \frac{GF}{FE}$



**Proof**

1 $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$	1
2 $\frac{CD}{DE} = \frac{BH}{HE}$	2
3 $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$	3
4 $\frac{BH}{HE} = \frac{GF}{FE}$	4
5 $\frac{CD}{DE} = \frac{GF}{FE}$	5

10 Given:  $\overleftrightarrow{SV} \parallel \overleftrightarrow{RW}$ ,  
 $RW = 15$ ,  $RS = 10$ ,  
 $ST = 3$ ,  $WV = 8$

Find:  $SV$  and  $VT$

