NAME_____ Ms. Kresovic

Adv Geo – _____ Date_____

Similar Polygons: Three Theorems Involving Proportions (8.5)

Objective

After studying this section, you will be able to

- Apply three theorems frequently used to establish proportionality

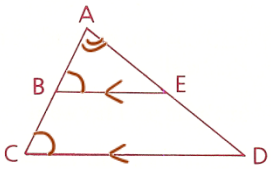
Midline: connect midpts ÷ sides =

$\frac{a}{b} = \frac{c}{d}$

Theorem 65 ***If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally.* (*Side-Splitter Theorem*)**

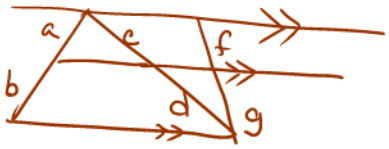
Given: $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$

Prove: $\frac{AB}{BC} = \frac{AE}{ED}$



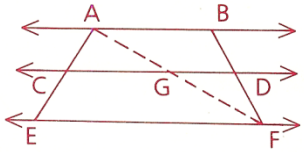
Statements	Reasons
1. $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$	1. GIVEN
2. $\angle ABE \cong \angle C$	2. $\parallel \Rightarrow$ CORR $\angle$S $\cong$ (1)
3. $\angle A \cong \angle A$	3. REF
4. $\Delta ABE \sim \Delta ACD$	4. AA$\sim$
5. $\frac{AB}{BC} = \frac{AE}{ED}$	5. $\sim$ (S) $\Rightarrow$ CORR SDS PROP

Theorem 66 ***If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.***

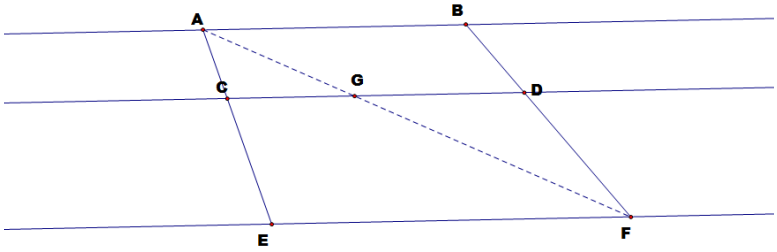


Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

Conclusion: $\frac{AC}{CE} = \frac{BD}{DF}$



GSP Demo:

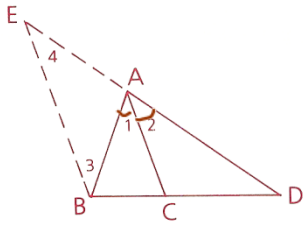


TOP	AC = 2.19 cm	AG = 4.96 cm	BD = 2.69 cm
BOTTOM	CE = 3.99 cm	GF = 9.04 cm	DF = 4.91 cm
	$\frac{AC}{CE}$ = 0.55	$\frac{AG}{GF}$ = 0.55	$\frac{BD}{DF}$ = 0.55

Theorem 67 ***If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides.*** **(*Angle Bisector Theorem*)**

Given: $\triangle ABD$;
$\overrightarrow{AC}$ bisects $\angle BAD$.

Prove: $\frac{BC}{CD} = \frac{AB}{AD}$



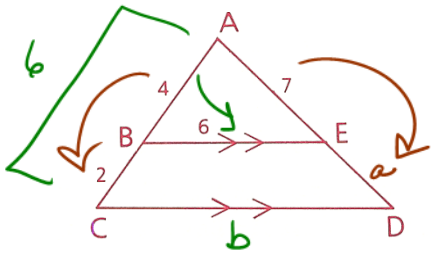
Proof:

1 $\triangle ABD$	1 GIVEN
2 $\overrightarrow{AC}$ bisects $\angle BAD$.	2 GIVEN
3 $\angle 1 \cong \angle 2$	3 → bis ∠ ⇒ 2 ≅ ∠s
4 Draw through B the line that is $\parallel$ to $\overline{AC}$.	4 parallel post.
5 Extend $\overleftrightarrow{DA}$ to intersect the $\parallel$ line at some point E.	5 def line
6 $\frac{BC}{CD} = \frac{EA}{AD}$	6 side splitter
7 $\angle 1 \cong \angle 3$	7 $\parallel \Rightarrow$ alt int ∠s ≅
8 $\angle 2 \cong \angle 4$	8 $\parallel \Rightarrow$ corr ∠s ≅
9 $\angle 3 \cong \angle 4$	9 trans
10 $\overline{EA} \cong \overline{AB}$	10 △ ⇒ ◺
11 $\frac{BC}{CD} = \frac{AB}{AD}$	11 substitute

Practice Problems

Problem 1 Given: $\overleftrightarrow{BE} \parallel \overleftrightarrow{CD}$, *lengths as shown*

Find: **a** ED
b CD



a) use side splitter $\frac{4}{2} = \frac{7}{a}$, $4a = 14$, $a = 7/2$

b) $\sim \triangle$s: $\frac{AB}{BE} = \frac{AC}{CD}$, $\frac{4}{6} = \frac{6}{CD}$, $4CD = 36$, $CD = 9$

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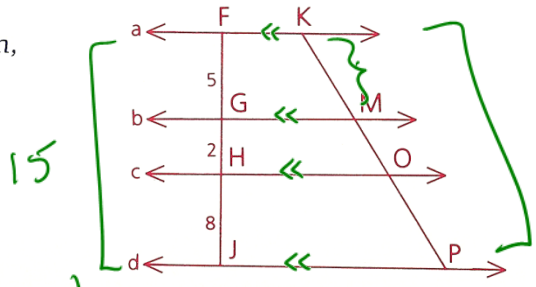
Problem 2 *Given:* a ∥ b ∥ c ∥ d, *lengths as shown*, KP = 24

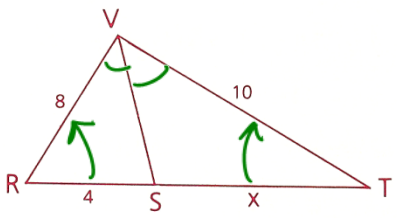
Find: KM



$\frac{KM}{24} = \frac{5}{15}$, $\frac{KM}{24} = \frac{1}{3}$, $3KM = 24$, $KM = 8$

Problem 3 *Given:* ∠RVS ≅ ∠SVT, *lengths as shown*

Find: ST



$\frac{4}{8} = \frac{x}{10}$

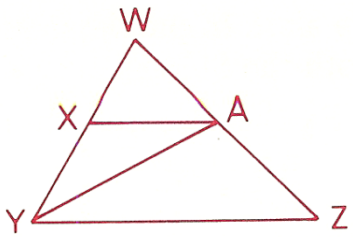
$\frac{1}{2} = \frac{x}{10}$

$2x = 10$

$x = 5$

Problem 4 Given: $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$, $\angle XAY \cong \angle XYA$

Conclusion: $\frac{WX}{XA} = \frac{WA}{AZ}$

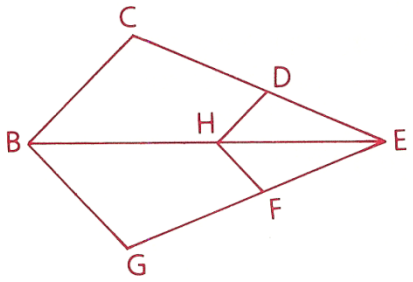


Proof

1 $\overleftrightarrow{XA} \parallel \overleftrightarrow{YZ}$	1
2 $\frac{WX}{XY} = \frac{WA}{AZ}$	2
3 $\angle XAY \cong \angle XYA$	3
4 $\overline{XA} \cong \overline{XY}$	4
5 $\frac{WX}{XA} = \frac{WA}{AZ}$	5

Problem 5 Given: $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$

Prove: $\frac{CD}{DE} = \frac{GF}{FE}$



Proof

1 $\overleftrightarrow{DH} \parallel \overleftrightarrow{BC}$	1
2 $\frac{CD}{DE} = \frac{BH}{HE}$	2
3 $\overleftrightarrow{HF} \parallel \overleftrightarrow{BG}$	3
4 $\frac{BH}{HE} = \frac{GF}{FE}$	4
5 $\frac{CD}{DE} = \frac{GF}{FE}$	5

10 Given: $\overleftrightarrow{SV} \parallel \overleftrightarrow{RW}$, RW = 15, RS = 10, ST = 3, WV = 8

Find: SV and VT