AMDG

Name | **7: Polygons** | Ms. Kresovic

Adv Geo - | *7.3: Formulas involving polygons* | Date

Objective

After studying this section, you will be able to

- Use some important formulas that apply to polygons

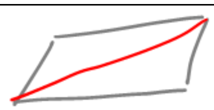
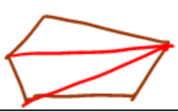
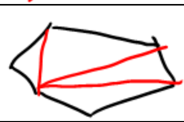
A polygon with three sides can be called a 3-gon. Similarly, a polygon with seven sides can be called a 7-gon. Most of the polygons you will encounter have special names, like those given in the following chart.

No. of Sides (or Vertices)	Polygon	No. of Sides (or Vertices)	Polygon
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Duo → Dodecagon
7	Heptagon	15	Pentadecagon
		n	n-gon

memorize

Yes, you do need to know these names, and how to spell them correctly.

Let's look back at 5.4: 11. *Find the sum of the measure of the angles of a polygon.*

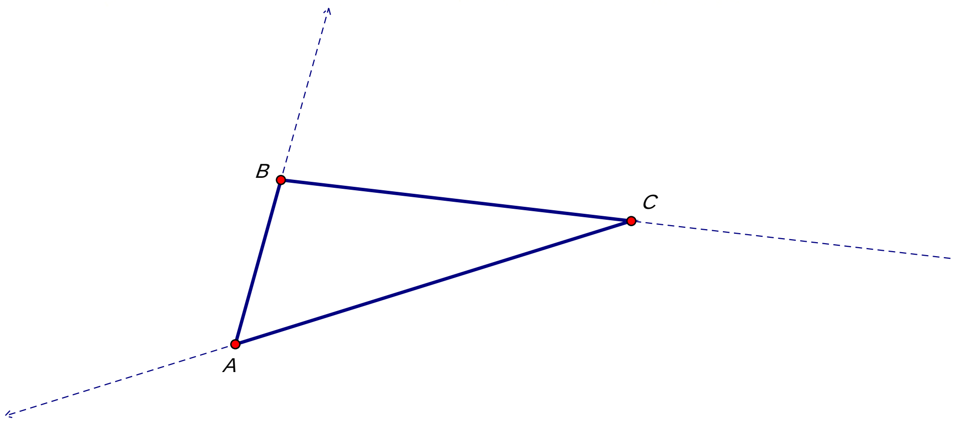
Polygon Drawing	Number of sides = n	Number of triangles that can be drawn from $\boxed{\text{ONE}}$ vertex.	Sum of angles of the polygon
	3 →	1	180
	4 →	2	2(180) = 360
	5 →	3	3(180) = 540
	6 → −2	4	4(180) = 720
	7		
	8		
	n	n−2	(n−2)(180)

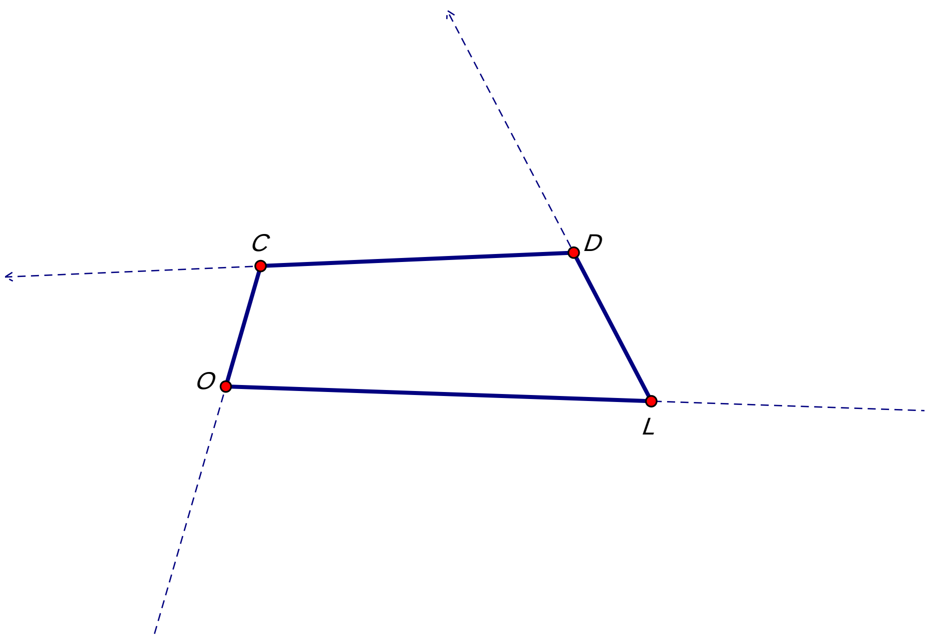
(n−2)(180)

Theorem 55 ***The sum S_i of the measures of the angles of a polygon with n sides is given by the formula $S_i = (n - 2)180$.***

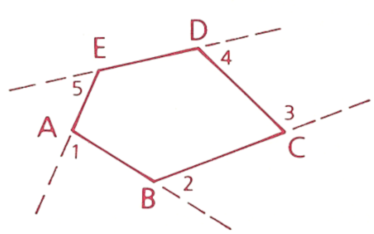
On occasion, we may refer to the angles of a polygon as the ***interior angles*** of the polygon.

In the following diagram, we have formed an exterior angle at each vertex by extending one of the sides of the polygon.





In the following diagram, we have formed an exterior angle at each vertex by extending one of the sides of the polygon.



At vertex A, m∠1 + m∠EAB = 180. In a similar manner, we can add each exterior angle to its adjacent interior angle, getting a sum of 180 at each vertex. Since there are five vertices, the total is 5(180), or 900.

According to Theorem 55, the sum of the measures of the angles of polygon ABCDE is 540. Since 900 − 540 = 360, we may conclude that m∠1 + m∠2 + m∠3 + m∠4 + m∠5 = 360.

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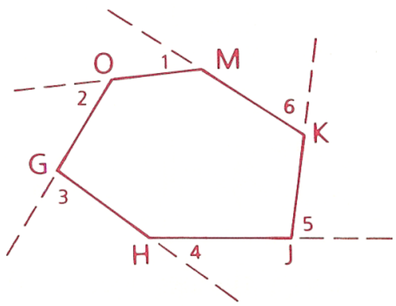
What is the sum of the measures of exterior angles 1, 2, 3, 4, 5, and 6 in this figure?

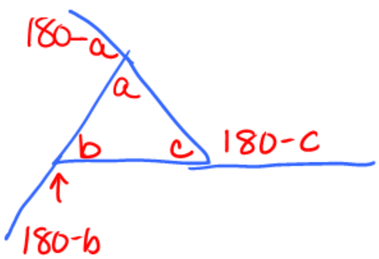
Again, the sum of the interior and the exterior angle is 180 at each of the six vertices, for a total measure of 6(180), or 1080. Moreover, according to Theorem 55, the sum of the measures of the angles of polygon GHJKMO is 720.



Because 1080 − 720 = 360, we may conclude that in this figure, too, m∠1 + m∠2 + m∠3 + m∠4 + m∠5 + m∠6 = 360.

These examples suggest the next theorem, which we present without formal proof.

Theorem 56 ***If one exterior angle is taken at each vertex, the sum S_e of the measures of the exterior angles of a polygon is given by the formula $S_e = 360$.***



$a+b+c=180$

$S_{e\triangle} = 180-a+180-b+180-c$

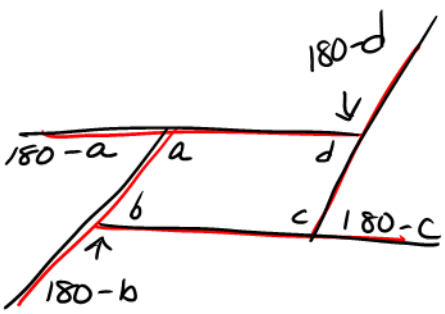
$= 3(180)-a-b-c$

$= 3(180)-1(a+b+c)$ substitute

$= 3(180)-1(180)$

$= 2(180)$

$= 360$



$a+b+c+d=360$

$S_{e(quad)} = 180-a+180-b+180-c+180-d$

$= 4(180)-(a+b+c+d)$
substitute

$= 4(180)-2(180)$

$= 2(180$

$= 360$

If $n = 10$

$$S_{e\,(\text{decagon})} =$$

$$S_{\text{int}\angle} = (10-2)180)$$
$$8\ (180)$$
$$= 1440$$

a b c d e f g h i j

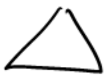
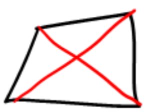
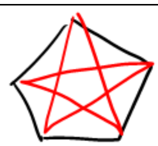
$$S_e = (180-a) + (180-b) + (180-c) + (180-d) + (180-e)$$
$$(180-f) + (180-g) + (180-h) + (180-i) + (180-j)$$

$$S_e = 10(180) - \underbrace{(a+b+c+d+e+f+g+h+i+j)}$$

$$S_e = 10(180) - 1440$$
$$= 10(180) - 8(180)$$
$$= 2(180)$$
$$= 360$$

$$S_e = n(\underline{180}) - (n-2)(\underline{180})$$
$$= 180\left[n - (n-2)\right]$$
$$= 180\left[n - n + 2\right]$$
$$= 180(2)$$
$$= 360$$

Looking back at our work in 5.4: How many diagonals does a n-sided polygon have?

Polygon Drawing	Number of sides = n	Number of diagonals	1^{st} change: Difference in diagonal count	2^{nd} change: Difference in the difference	Attempted formulas:	How many vertices does the polygon have?	How many diagonals meet at one vertex of the polygon
	3	0	- -	- -			
	4	2	+2	- -	$\frac{n(n-3)}{2} = \frac{4}{2} = 2$		
	5	5	+3	+1	$5(5-3)/2 = \frac{5(2)}{2} = \frac{10}{2} = 5$		
	6	9	+4	+1	$\frac{6(3)}{2} = \frac{18}{2} = 9$		
	7	14	+5	+1	$\frac{7(4)}{2} = 14$		
	8						
	n				$\frac{n(n-3)}{2}$		

Theorem 57 ***The number d of diagonals that can be drawn in a polygon of n sides is given by the formula***

$$d = \frac{n(n-3)}{2}$$

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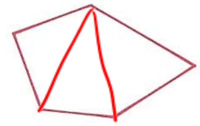
Class Examples

implied S_i

Problem 1 Find the sum of the measures of the angles of the figure to the right.



Solution

$S_i = (n-2)180, \; n=5$

$3(180) = 3(100+80)$

$300 + 240 = \boxed{540}$

Problem 2 Find the number of diagonals that can be drawn in a pentadecagon.

Solution

$n = 15$

$d = \frac{n(n-3)}{2}$

$= \frac{15(15-3)}{2} = \frac{15(\cancel{12}^{6})}{\cancel{2}} = 15(6)$ or $6(10+5) =$

$60 + 30 = 90$

Problem 3 What is the name of a polygon if the sum of the measures of its angles is 1080?

Solution

$S_i = (n-2)180$

$1080 = (n-2)180$

$6 = n-2$

$8 = n \Rightarrow$ Octagon

4	Find the sums of the measures of the angles of a…			
	Name	*Number of sides*	*work*	*Solution*
A	Quadrilateral			
B	Heptagon			
C	Octagon			
D	dodecagon			
E	93- gon			

Homework:

7.3: 307: 1-8, 10-14, 16, 17, 19 & 20

* Don't forget to write up the axioms for the next section to hand in as well!