Name NAME
Course - period ADV GEO - 7
Ms. Kresovic
Date 5 Jan 16

Objective

Course Goal: EXTERNAL → EMPIRICAL → ANALYTICAL
(outside) (Examples) (Generalizations: Proof
memorization - - - - - - - - - Formula

After studying this section, you will be able to

- Apply theorems about the interior angles, the exterior angles, and the midlines of triangles.

Thurs' GSP lab.

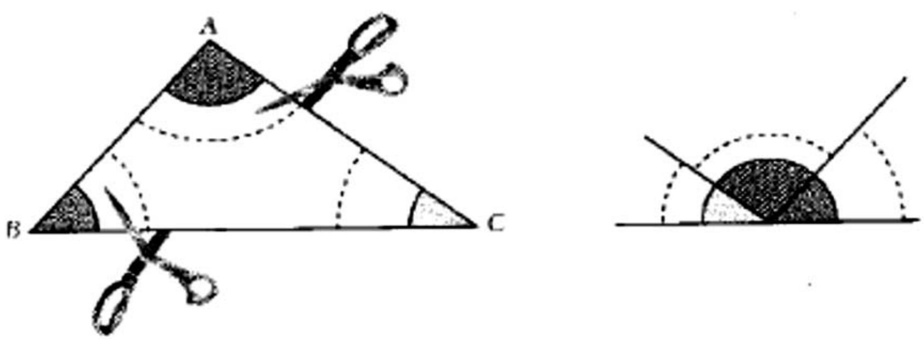
Theorem 50 ***The sum of the measures of the three angles of a triangle is*** **180.**

External reasoning:
An authority (e.g., the book, teacher) told me, so it's true.

Empirical: single example ||

① En découpant

a. Trace sur papier blanc un triangle *ABC* comme celui dessiné ci-dessous.
b. Découpe chacun de ses angles, puis « regroupe »-les comme l'indique le dessin ci-dessous.



def post

c. Quelle semble être la valeur de la somme des angles du triangle ? 180°

Empirical: 3 examples

② En mesurant avec ton rapporteur

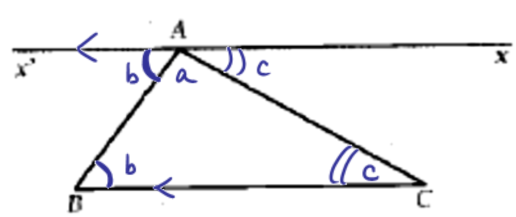
a. Trace trois triangles.
b. Mesure les angles de chacun de ces triangles à l'aide d'un rapporteur, puis calcule la somme des angles de chaque triangle.
c. Quelle semble être la valeur de cette somme ? 180°

Analytic: Axiomatic

thms

③ Une démonstration à présent

La droite $(x'x)$ est parallèle à la droite (BC) et passe par *A*.

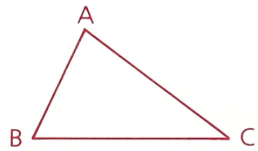


|| ⇒ ALT INT ∠s ≅

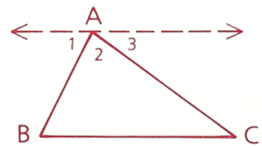
a. Compare les angles $\widehat{ABC}$ et $\widehat{BAx'}$, puis $\widehat{ACB}$ et $\widehat{CAx}$.
b. Explique alors pourquoi la somme des trois angles du triangle *ABC* est égale à 180°.

Given: $\triangle ABC$

Prove: $m\angle A + m\angle B + m\angle C = 180$

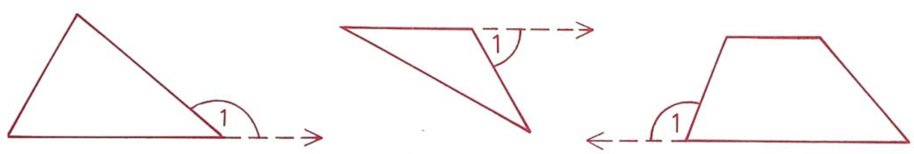


Proof: According to the Parallel Postulate, there exists exactly one line through point A parallel to $\overleftrightarrow{BC}$, so the figure at the right can be drawn.



Because of the straight angle, we know that $\angle 1 + \angle 2 + \angle 3 = 180°$. Since $\angle 1 \cong \angle B$ and $\angle 3 \cong \angle C$ (by $\parallel$ lines $\Rightarrow$ alt. int. $\angle$s $\cong$), we may substitute to obtain $\angle B + \angle 2 + \angle C = 180°$. Hence, $m\angle A + m\angle B + m\angle C = 180$.

Definition An ***exterior angle*** of a polygon is an angle that is adjacent to and supplementary to an interior angle of the polygon.

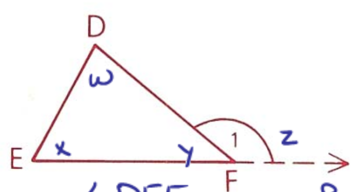


Theorem 51 ***The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.***

Never Use a Thm to prove itself (circular logic & incorrect to do so.)

Given: $\triangle DEF$, with exterior angle 1 at F

Prove: $m\angle 1 = m\angle D + m\angle E$



Let $m\angle D = w°$, $m\angle E = x$, $m\angle DFE = y$, & $m\angle 1 = z$

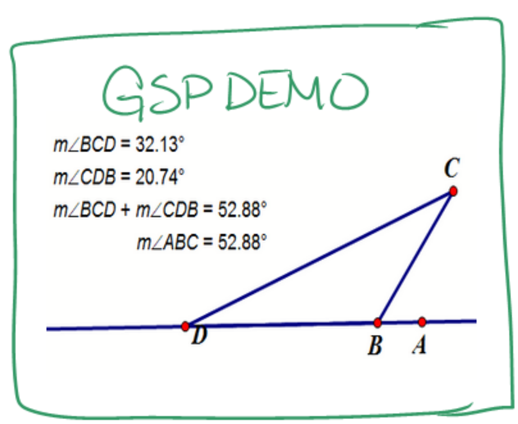
Then $w + x + y = 180°$ (Sum $\angle$s in $\triangle = 180°$)

$y + z = 180°$ (st$\angle = 180°$)

Hence $w + x + y = y + z$ (Substitution)

$w + x = z$ (subtraction)

$\therefore m\angle D + m\angle E = m\angle 1$ (Substitution)

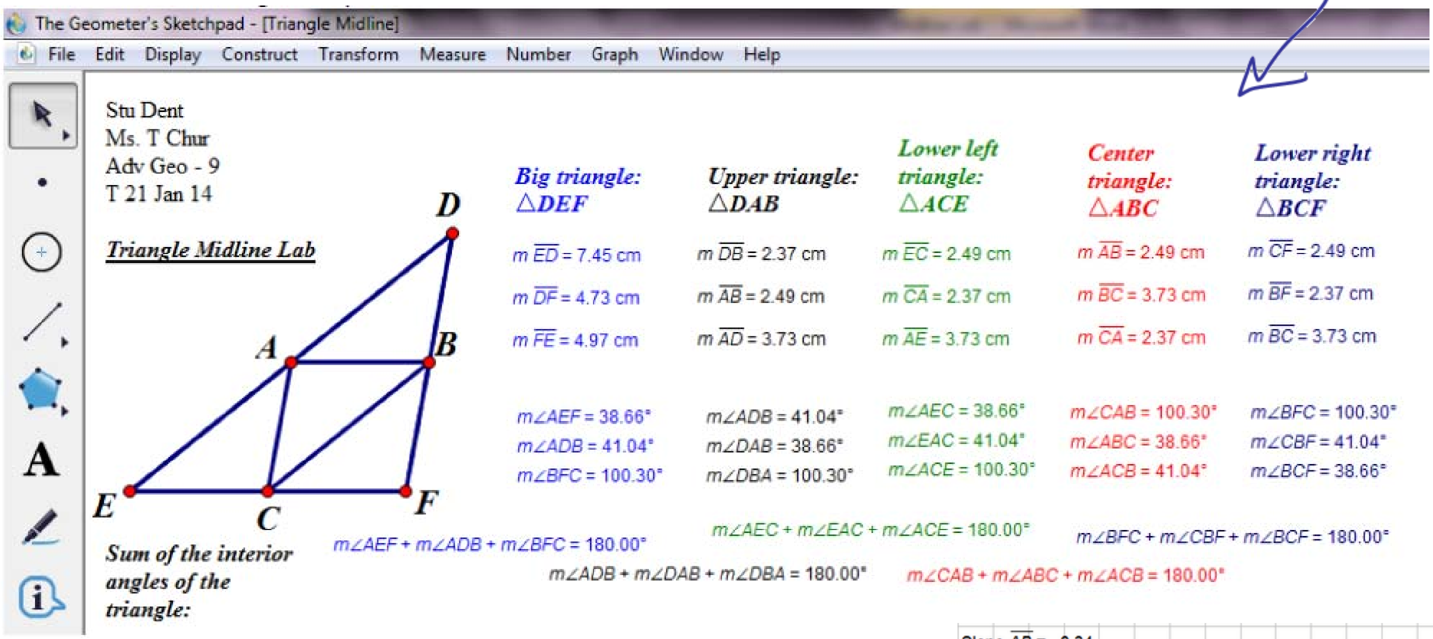


Name | **7: Polygons** | Ms. Kresovic

Course - period | *7.1:* Triangle Application Theorems | Date

Theorem 52 ***A segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is one-half the length of the third side. (Midline Theorem)***

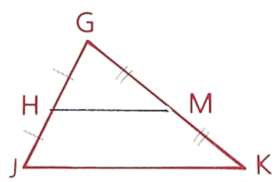
GSP thurs



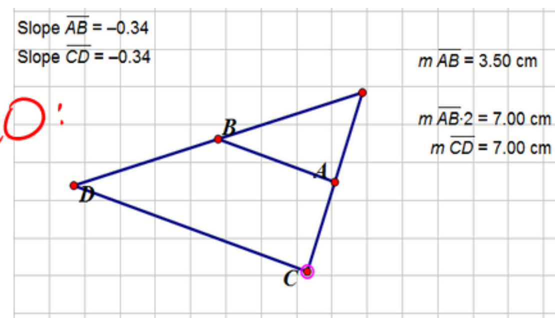
Given: H is a midpoint.
M is a midpoint.

Prove: **a** $\overline{HM} \parallel \overline{JK}$

b $HM = \frac{1}{2}(JK)$

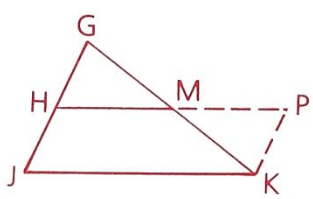


DEMO:



Proof: Extend $\overleftrightarrow{HM}$ through M to a point P so that $\overline{MP} \cong \overline{HM}$. P is now established, so P and K determine $\overleftrightarrow{PK}$.

We know that $\overline{GM} \cong \overline{KM}$ (*by the definition of midpoint*) and that $\angle GMH \cong \angle KMP$ (vertical $\angle$s are $\cong$). Thus, $\triangle GMH \cong \triangle KMP$ by SAS.



Since $\angle G = \angle PKM$ by CPCTC, $\overleftrightarrow{PK} \parallel \overleftrightarrow{HJ}$ by alt. int. $\angle$s $\Rightarrow \parallel$ lines. Also, $\overline{GH} \cong \overline{PK}$ by CPCTC, and $\overline{GH} \cong \overline{HJ}$ (by the definition of *midpoint*). By transitivity, then, $\overline{PK} \cong \overline{HJ}$.

Two sides, $\overline{PK}$ and $\overline{HJ}$, are parallel and congruent, so PKJH is a parallelogram. Therefore, $\overleftrightarrow{HP} \parallel \overleftrightarrow{JK}$.

Opposite sides of a parallelogram are congruent, so HP = JK.

Also, since we made MP = HM, $HM = \frac{1}{2}(HP)$ and, by substitution, $HM = \frac{1}{2}(JK)$.

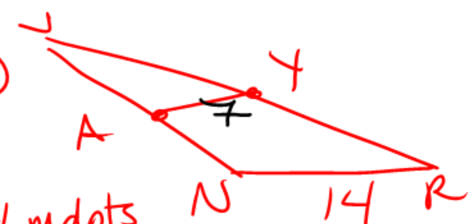
PRACTICE

I & R mdpts

IR = 10

NT = 20

① 

② 

A + Y mdpts

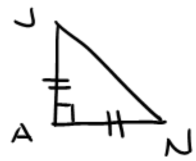
midline

ONE MORE EXAMPLE:

#14 from 7.1 asks to Prove

If a rt$\triangle$ is isos, then it's 45-45-90.

G: $\triangle JAN$ is isos rt $\triangle$ with
$\angle A$ rt$\angle$
$\overline{AJ} \cong \overline{AN}$



P: $m\angle J = m\angle N = 45°$

PROOF: WE ARE GIVEN $\triangle JAN$ with $\angle A$ rt$\angle$ & $\overline{AJ} \cong \overline{AN}$.

$m\angle A = 90°$ (rt$\angle \Rightarrow 90°$)

$m\angle J = m\angle N$ ($\triangle \Rightarrow \triangle$)

Let $m\angle J = x°$

$90 + x + x = 180°$ (Sum $\angle$s in $\triangle = 180°$)

$2x = 90°$ (Arithmetic)

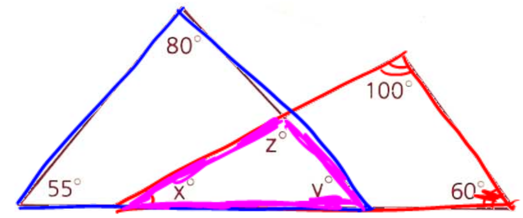
$x = 45°$ (Division)

Hence $m\angle J$ & $m\angle N$ is $45°$

Therefore the isos rt $\triangle$ is a $45°$-$45°$-$90°$.

Class Examples

Problem 1 *Given: Diagram as marked*
Find: x, y, and z



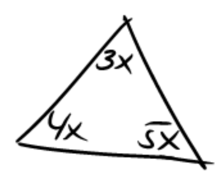
Solution Since the sum of the measures of the angles of a triangle is 180,

POSITION:

Right triangle	Left triangle	Center triangle
$x + 100 + 60 = 180$	$55 + 80 + y = 180$	$x + y + z = 180$
$-160 \quad -160$	$-135 \quad -135$	$20° + 45 + z = 180$
$x = 20°$	$y = 45°$	$-65 \quad -65$
		$z = 115°$

Problem 2 *The measures of the three angles of a triangle are in the ratio 3:4:5. Find the measure of the largest angle.*

RATIOS ⇒
$3x : 4x : 5x$

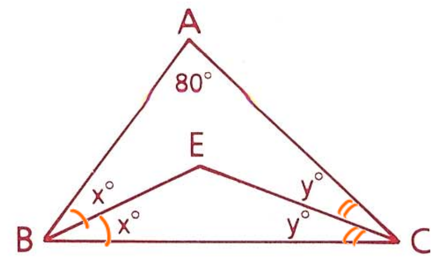


mental math
$12x = 180$
$4 \cdot 3 \cdot x = 3 \cdot 4 \cdot 15$
$x = 15$

Can use calc.

larg ∠ = $5x$
$5(10+5) =$
$50 + 25 =$
$\boxed{75°}$

Problem 3 *If one of the angles of a triangle is 80°, find the measure of the angle formed by the bisectors of the other two angles.*



big △:

$80° + 2x + 2y = 180°$

$2x + 2y = 100°$

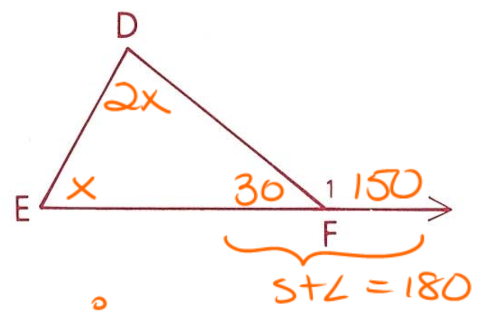
$x + y = 50°$

then $\angle E + x + y = 180°$ ← little △

$\angle E + 50 = 180°$

$\angle E = 130°$

Problem 4 $\angle 1 = 150°$, *and the measure of* $\angle D$ *is twice that of* $\angle E$*. Find the measure of each angle of the triangle.*



$3x + 30 = 180$

$3x = 150$

$x = 50$

$m\angle D = 100°$

$m\angle E = 50°$

$m\angle DFE = 30°$

Name **7: Polygons** Ms. Kresovic

Course - period *7.1:* Triangle Application Theorems Date

Homework

9 Tell whether each statement is true Always, Sometimes, or Never (A, S, or N).

a The acute angles of a right triangle are complementary.

b The supplement of one of the angles of a triangle is equal in measure to the sum of the other two angles of the triangle.

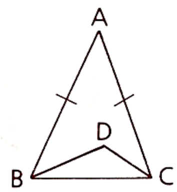
c A triangle contains two obtuse angles.

d If one of the angles of an isosceles triangle is 60°, the triangle is equilateral.

e If the sides of one triangle are doubled to form another triangle, each angle of the second triangle is twice as large as the corresponding angle of the first triangle.

9a	9b	9c	9d	9e

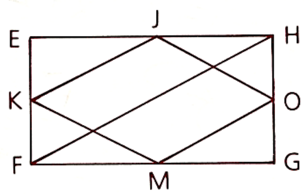
16 Given: $\angle A = 30°$, $\overline{AB} \cong \overline{AC}$;
$\overrightarrow{CD}$ bisects $\angle ACB$.
$\overrightarrow{BD}$ is one of the trisectors of $\angle ABC$.
Find: $m\angle D$



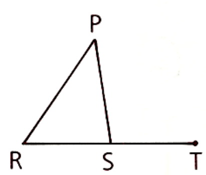
17 Given: EFGH is a rectangle.
FH = 20;
J, K, M, and O are midpoints.

a Find the perimeter of JKMO.

b What is the most descriptive name for JKMO?



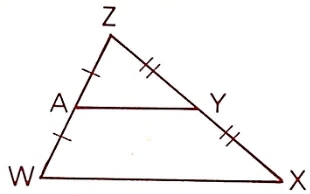
18 Given: $\angle PST = (x + 3y)°$,
$\angle P = 45°$, $\angle R = (2y)°$,
$\angle PSR = (5x)°$
Find: $m\angle PST$



Classwork

All of the following exercises must be completed and handed in before the class ends.

7 In the diagram as marked, if $WX = 18$, find AY.



12 In $\triangle DEF$, the sum of the measures of $\angle D$ and $\angle E$ is 110. The sum of the measures of $\angle E$ and $\angle F$ is 150. Find the sum of the measures of $\angle D$ and $\angle F$.