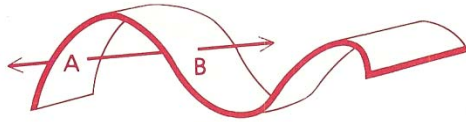


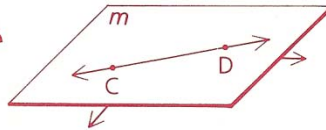
Objectives

After studying this chapter, you will be able to

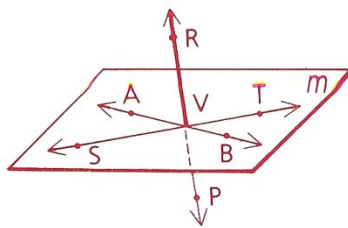
- Understand basic concepts relating to planes
- Identify four methods of determining a plane
- Apply two postulates concerning lines and planes



A surface that is not a plane



Plane surface



A, B, S, T, and V are *coplanar* points.

\overleftrightarrow{AB} and \overleftrightarrow{ST} are *coplanar* lines.

\overline{AB} and \overline{ST} are *coplanar* segments.

A, B, S, T, and R are *noncoplanar* points.

\overleftrightarrow{AB} , \overleftrightarrow{ST} , and \overleftrightarrow{RP} are *noncoplanar* lines.


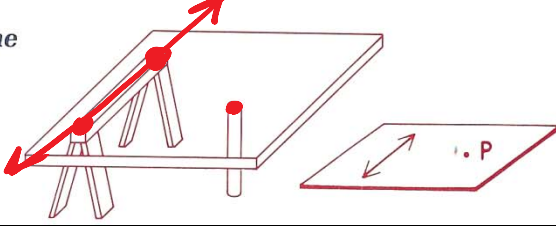
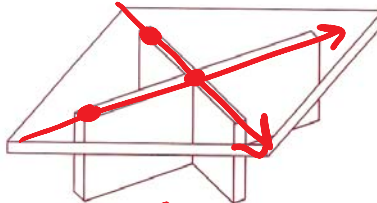
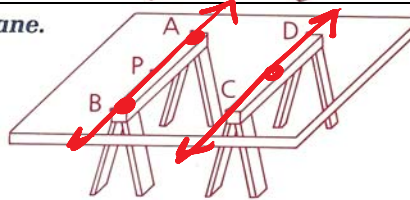
\overline{AB} , \overline{ST} , and \overline{RP} are *noncoplanar* segments.

Definition

The point of intersection of a line and a plane is called the **foot** of the line.

Four Ways to Determine a Plane

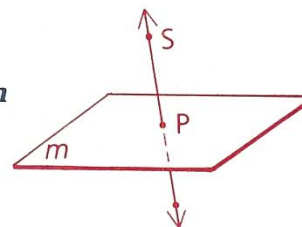
Prior knowledge: Any two points determine a line.

1.	<p>Postulate <i>Three noncollinear points determine a plane.</i></p> 
2.	<p>Theorem 45 <i>A line and a point not on the line determine a plane.</i></p> 
3.	<p>Theorem 46 <i>Two intersecting lines determine a plane.</i></p> 
4.	<p>Theorem 47 <i>Two parallel lines determine a plane.</i></p>  <p><i>Proof:</i> If \overleftrightarrow{AB} and \overleftrightarrow{CD} are parallel, then according to the definition of parallel lines, they lie in a plane. We need to show that they lie in <i>only one</i> plane. If P is any point on \overleftrightarrow{AB}, then according to Theorem 45, there is only one plane containing P and \overleftrightarrow{CD}. Thus, there is only one plane that contains \overleftrightarrow{AB} and \overleftrightarrow{CD}, because every plane containing \overleftrightarrow{AB} contains P.</p>

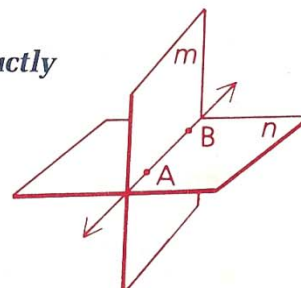
Two Postulates Concerning Lines and Planes

We shall assume the following two statements.

Postulate *If a line intersects a plane not containing it, then the intersection is exactly one point.*



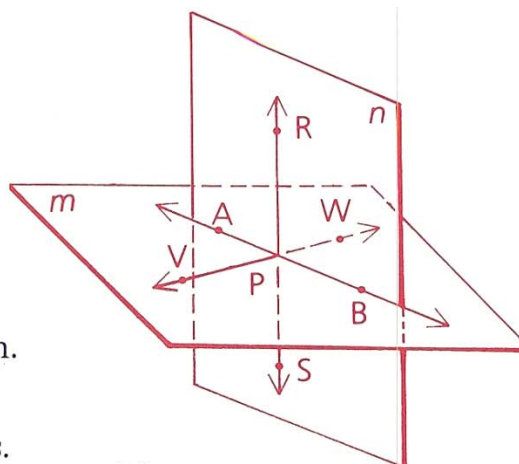
Postulate *If two planes intersect, their intersection is exactly one line.*



In Class Examples

Problem 1

- a $m \cap n = \underline{\quad? \quad}$
- b A, B, and V determine plane $\underline{\quad? \quad}$.
- c Name the foot of \overleftrightarrow{RS} in m.
- d \overleftrightarrow{AB} and \overleftrightarrow{RS} determine plane $\underline{\quad? \quad}$.
- e \overleftrightarrow{AB} and point $\underline{\quad? \quad}$ determine plane n.
- f Does W lie in plane n?
- g Line AB and line $\underline{\quad? \quad}$ determine plane m.
- h A, B, V, and $\underline{\quad? \quad}$ are coplanar points.
- i A, B, V, and $\underline{\quad? \quad}$ are noncoplanar points.
- j If R and S lie in plane n, what can be said about \overleftrightarrow{RS} ?



A	\overleftrightarrow{AB}	C	P	E	R or S	G	\overleftrightarrow{VW}	I	Line S
B	m	D	n	F	No	H	W or P	J	\overleftrightarrow{RS} lies in n

Problem 2

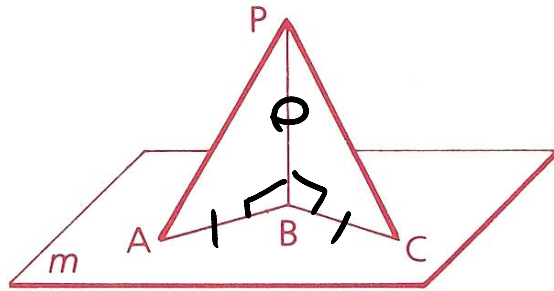
Given: A, B, and C lie in plane m.

$$\overline{PB} \perp \overline{AB},$$

$$\overline{PB} \perp \overline{BC},$$

$$\overline{AB} \cong \overline{BC}$$

Prove: $\angle APB \cong \angle CPB$

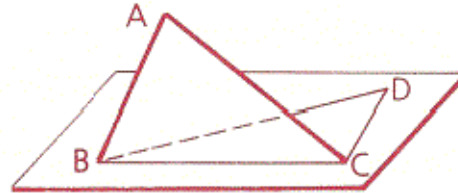


- 1 $\overline{PB} \perp \overline{AB}, \overline{PB} \perp \overline{BC}$
- 2 $\angle PBA$ and $\angle PBC$ are right \angle s.
- 3 $\angle PBA \cong \angle PBC$
- 4 $\overline{AB} \cong \overline{BC}$
- 5 $\overline{PB} \cong \overline{PB}$
- 6 $\triangle PBA \cong \triangle PBC$
- 7 $\angle APB \cong \angle CPB$

- 1 Given
- 2 $\perp \Rightarrow \text{RTLS}$
- 3 $\text{RTLS} \Rightarrow \cong \angle \text{S}$
- 4 Given
- 5 Ref
- 6 SAS
- 7 CPCTC

12 Cut a quadrilateral out of paper and fold it along a diagonal as shown in the figure. Is every four-sided figure a plane figure?

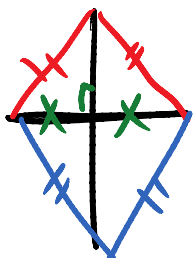
No



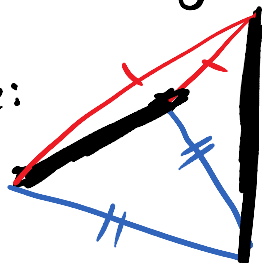
13 If two points in space are equidistant from the endpoints of a segment, will the line joining them be the perpendicular bisector of the segment? Explain.

If pts are in space + not in a plane, then the line joining them may not intersect the seg.

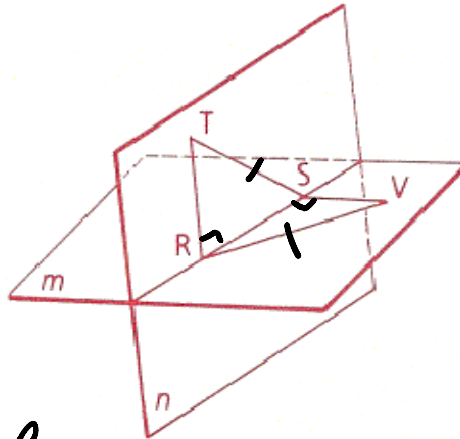
In a plane



In space:



14 Given: Planes m and n
intersect in \overleftrightarrow{RS} .
 m contains $R, S,$ and V .
 n contains $R, S,$ and T .
 $\overline{TS} \cong \overline{VR}$,
 $\overline{TR} \perp \overline{RS}$,
 $\overline{VS} \perp \overline{RS}$
Prove: $\overline{TR} \cong \overline{VS}$



1. Planes m & $n \cap \overleftrightarrow{RS}$ $\overset{S}{\parallel}$ $\overset{R}{\parallel}$ given
 $\overline{TS} \cong \overline{VR}$
 $\overline{TR} \perp \overline{RS}, \overline{VS} \perp \overline{RS}$
2. $\angle TRS$ & $\angle VSR$ $\angle \perp \Rightarrow \text{rt } \angle$
 $\text{rt } \angle$
3. $\overline{RS} \cong \overline{SR}$ 3 Ref
4. $\triangle TRS \cong \triangle VSR$ $\text{4. HL}(2, 1, 3)$
5. $\overline{TR} \cong \overline{VS}$ 5. CPCTC

$\cong \triangle$

SSS
SAS
ASA
HL

16 Given: $A, B, C,$ and D lie in plane m .

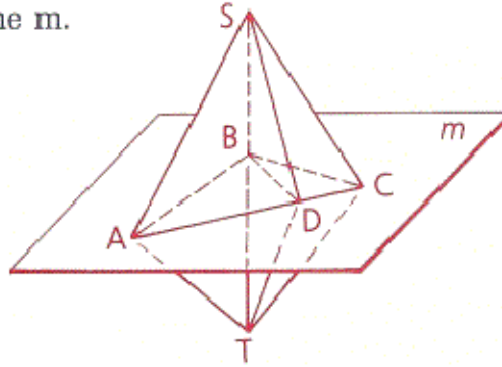
\overleftrightarrow{ST} intersects m at B .

D is any point on \overline{AC} .

$\overleftrightarrow{ST} \perp \overleftrightarrow{AB}, \overleftrightarrow{ST} \perp \overleftrightarrow{BC},$

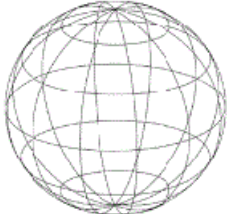
$\overline{SB} \cong \overline{TB}$

Prove: $\overleftrightarrow{ST} \perp \overleftrightarrow{BD}$



Homework

- 1 Consider a spherical object, such as an orange or a globe. If two points are marked on it and a straight line is drawn through the two points, does the line lie on the surface? Is it possible to draw a straight line that will lie entirely on the surface?



- 3 Consider two points on a cylindrical surface, such as the curved surface of a tin can. Does the line connecting two such points *always* lie on the surface? Does it *ever* lie on the surface?

J

I



- 5 A three-legged stool will not rock, even if the legs are of different lengths. Many four-legged stools wobble. Explain.

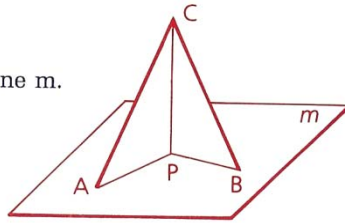
AMDG

7 Given: A, P, and B lie in plane m.

$$\overleftrightarrow{CP} \perp \overleftrightarrow{AP}, \overleftrightarrow{CP} \perp \overleftrightarrow{PB},$$

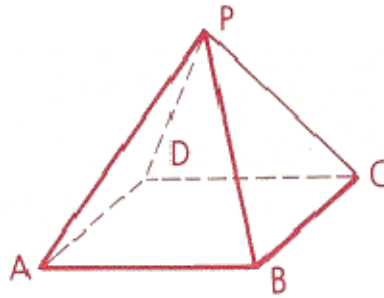
$$\overline{PA} \cong \overline{PB}$$

Prove: $\overline{CA} \cong \overline{CB}$



Statements	Reasons
1. A, P, and B lie in plane as shown.	1. Given
2.	2. Given
3.	3. Given
4.	4. $\perp \Rightarrow$ rt \angle s
5.	5. rt \angle s \Rightarrow cong. \angle s
6.	5. Reflexive
7.	6. SAS
8.	7. CPCTC

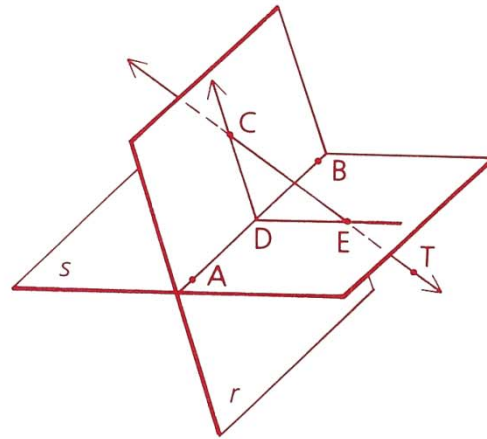
15 The figure at the right is a square pyramid. How many planes are determined by its vertices? (There are more than five.) Name them. Create a list.



Class Work

This work is required, and due by the end of class. You are encouraged to work together.

- 2 a $r \cap s = \underline{\quad? \quad}$
- b $\overleftrightarrow{AB} \cap s = \underline{\quad? \quad}$
- c Name three collinear points.
- d Name four noncoplanar points.
- e What plane do points A, B, and E determine?
- f What plane do \overleftrightarrow{AB} and \overleftrightarrow{ED} determine?
- g Name the foot of \overleftrightarrow{TC} in plane s.
- h Name the foot of \overleftrightarrow{TC} in plane r.
- i Do \overleftrightarrow{CD} and \overleftrightarrow{ED} determine a plane?
- j If $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$, name the right angles formed.

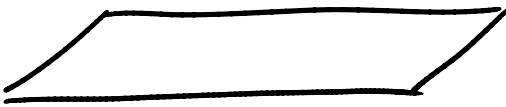


A	C	E	G	I
B	D	F	H	J

4. Make freehand sketches of

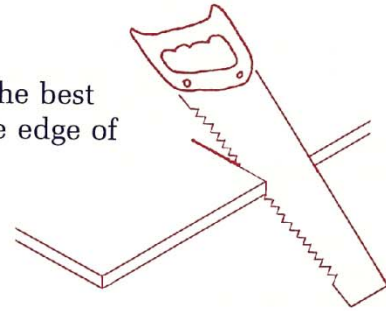
(a) a horizontal plane,

(b) a vertical plane, and



(c) two intersecting planes.

- 6 What theorem or assumption in this chapter provides the best explanation for the fact that when you saw a board, the edge of the cut is a straight line?



- 8 Given: $\odot O$ lies in plane p .

$$\overleftrightarrow{VO} \perp \overleftrightarrow{OS},$$

$$\overleftrightarrow{VO} \perp \overleftrightarrow{OT}$$

Prove: $\angle VSO \cong \angle VTO$

