AMDG

Name — Ms. Kresovic

Adv Geo - — 5.3: Congruent angles associated with parallel lines — M 25 Nov 2013

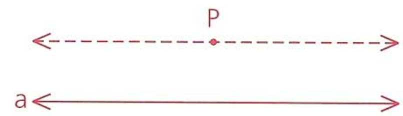
Objectives

After studying this section, you will be able to

- Apply the Parallel Postulate
- Identify the pairs of angles formed by a transversal cutting parallel lines
- Apply six theorems about parallel lines

Notes

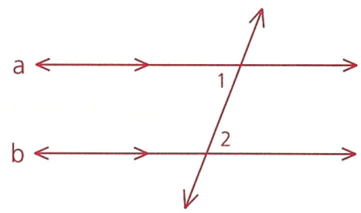
Postulate ***Through a point not on a line there is exactly one parallel to the given line.***



Theorem 37 ***If two parallel lines are cut by a transversal, each pair of alternate interior angles are congruent. (Short form: ∥ lines ⇒ alt. int. ∠s ≅)***

Given: Lines a and b are parallel.

Prove: $\angle 1 \cong \angle 2$



Notice the special tick marks (⇉) used to designate parallel lines.

Theorem 38 ***If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.***

The proof of this may be developed algebraically by letting x be the measure of any one of the angles. Follow the steps below. In each diagram, a ∥ b.

Diagram 1

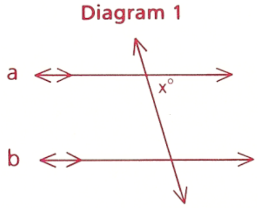


Diagram 2

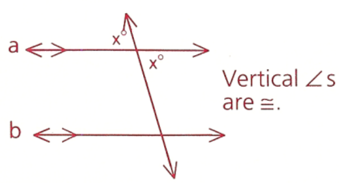


Diagram 3

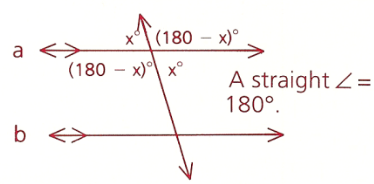


Diagram 4

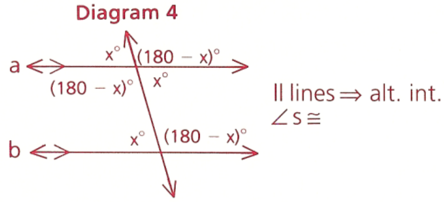
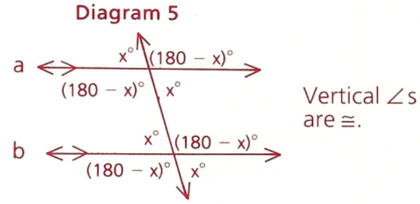


Diagram 5



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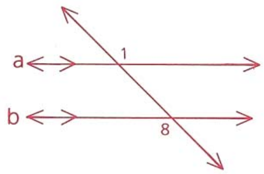
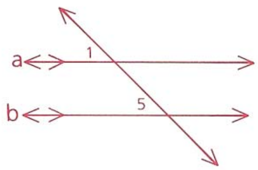
Theorem 39 ***If two parallel lines are cut by a transversal, each pair of alternate exterior angles are congruent.*** **(∥ *lines* ⇒ *alt. ext.* ∠*s* ≅)**

Given: a ∥ b
Prove: ∠1 ≅ ∠8
Proof: See Diagram 5.



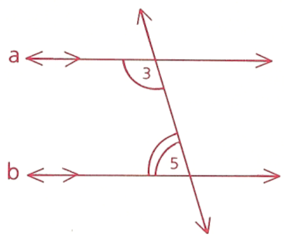
Theorem 40 ***If two parallel lines are cut by a transversal, each pair of corresponding angles are congruent.*** **(∥ *lines* ⇒ *corr.* ∠*s* ≅)**

Given: a ∥ b
Prove: ∠1 ≅ ∠5
Proof: See Diagram 5.



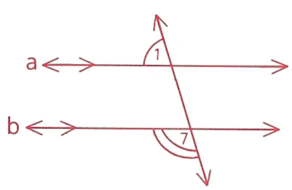
Theorem 41 ***If two parallel lines are cut by a transversal, each pair of interior angles on the same side of the transversal are supplementary.***

Given: a ∥ b
Prove: ∠3 supp. ∠5
Proof: See Diagram 5.



Theorem 42 ***If two parallel lines are cut by a transversal, each pair of exterior angles on the same side of the transversal are supplementary.***

Given: a ∥ b
Prove: ∠1 supp. ∠7
Proof: See Diagram 5.



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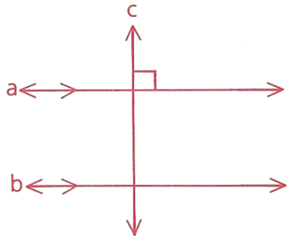
Theorem 43 ***In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.***

Given: $a \parallel b$,
$c \perp a$

Prove: $c \perp b$



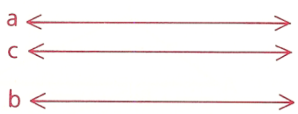
Proof: See Diagram 5 and let $x = 90$.

The following is another useful theorem about parallel lines.

Theorem 44 ***If two lines are parallel to a third line, they are parallel to each other. (Transitive Property of Parallel Lines)***

Given: $a \parallel b$, $b \parallel c$

Prove: $a \parallel c$



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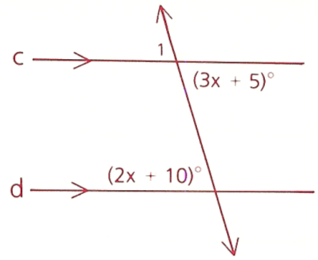
Class Examples

Problem 1 *If* c ∥ d, *find* m∠1.

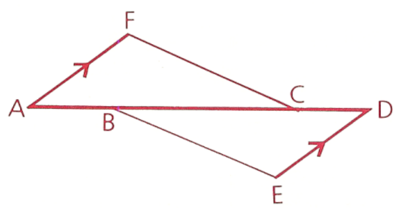


Solution Since alt. int. ∠s are ≅,

Because vertical angles are ≅,

Problem 2 *Given:* $\overline{FA} \parallel \overline{DE}$,
$\overline{FA} \cong \overline{DE}$,
$\overline{AB} \cong \overline{CD}$

Prove: ∠F ≅ ∠E

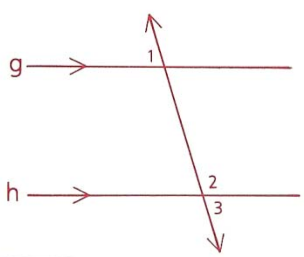


Proof

1 $\overline{FA} \parallel \overline{DE}$	1
2 ∠A ≅ ∠D	2
3 $\overline{FA} \cong \overline{DE}$	3
4 $\overline{AB} \cong \overline{CD}$	4
5 $\overline{AC} \cong \overline{BD}$	5
6 △FAC ≅ △EDB	6
7 ∠F ≅ ∠E	7

Problem 3 *Given:* g ∥ h

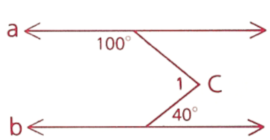
Prove: ∠1 supp. ∠2



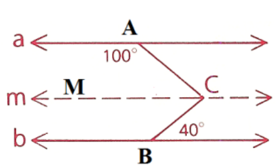
Proof

1 g ∥ h	1
2 ∠2 supp. ∠3	2
3 ∠1 ≅ ∠3	3
4 ∠1 supp. ∠2	4

Problem 4 (*A crook problem*)
If a ∥ b, *find* m∠1.



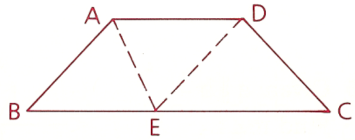
Solution Using the Parallel Postulate, draw m parallel to a.



Problem 5

Given: *Figure* ABCD, *with* $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \cong \overline{DC}$, *and* $\overline{AB} \nparallel \overline{DC}$

Prove: $\angle B \cong \angle C$



Note Figure ABCD is called an isosceles trapezoid.

Proof

1 Figure ABCD, with $\overline{AD} \parallel \overline{BC}$	1 Given
2 $\overline{AB} \nparallel \overline{DC}$	2 Given
3 Draw $\overline{DE} \parallel \overline{AB}$.	3
4 Draw $\overline{AE}$.	4 Two points determine a line.
5 $\angle DAE \cong \angle BEA$	5
6 $\angle BAE \cong \angle DEA$	6 Same as 5
7 $\overline{AE} \cong \overline{AE}$	7 Reflexive Property
8 $\triangle AEB \cong \triangle EAD$	8 ASA (5, 7, 6)
9 $\overline{AB} \cong \overline{DE}$	9 CPCTC
10 $\overline{AB} \cong \overline{DC}$	10 Given
11 $\overline{DE} \cong \overline{DC}$	11 Transitive Property
12 $\angle DEC \cong \angle C$	12
13 $\angle B \cong \angle DEC$	13
14 $\angle B \cong \angle C$	14 Transitive Property

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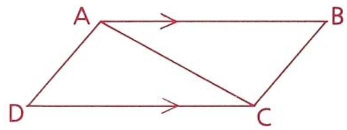
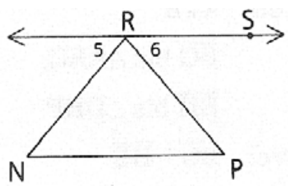
Homework

1 Given: $\overline{AB} \cong \overline{DC}$,
$\overline{AB} \parallel \overline{DC}$
Conclusion: $\overline{AD} \cong \overline{BC}$

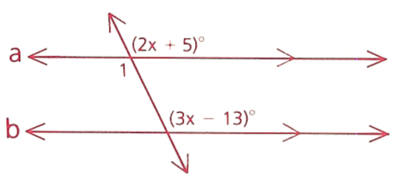


4 Given: $\angle 5 \cong \angle 6$
$\overline{RS} \parallel \overline{NP}$
Prove: $\triangle NPR$ is isos.



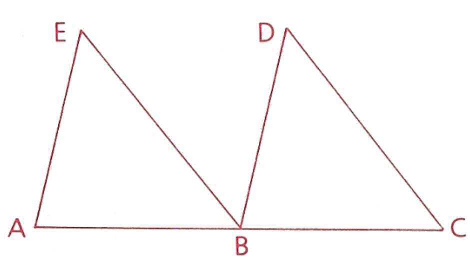
1 $\angle 5 \cong \angle 6$	1 Given
2 $\overline{RS} \parallel \overline{NP}$	2 Given
3 $\angle 6 \cong \angle P$	3
4 $\angle 5 \cong \angle N$	4
5 $\angle P \cong \angle N$	5
6 $\overline{NR} \cong \overline{PR}$	6
7 $\triangle NPR$ is isos.	7

5 Given: $a \parallel b$
Find $m\angle 1$.



Include the reason(s).

9 Given: $\overline{EA} \parallel \overline{DB}$ and $\overline{EA} \cong \overline{DB}$;
B is the midpt. of $\overline{AC}$.
Prove: $\overline{EB} \parallel \overline{DC}$



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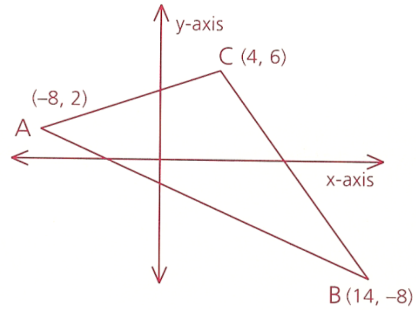
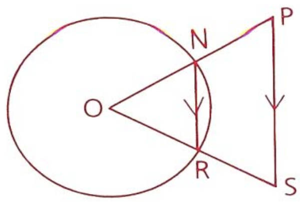
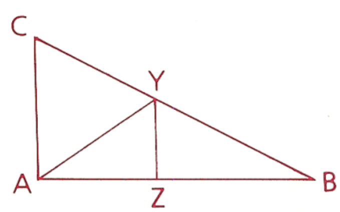
12 One of the sides of $\triangle ABC$ has a midpoint whose x-coordinate is negative. Find the coordinates of that midpoint.



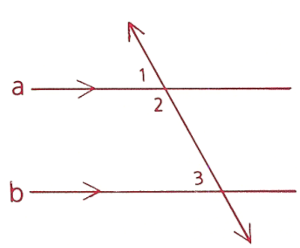
16 Given: $\odot O$,
$\overline{NR} \parallel \overline{PS}$
Prove: $\triangle OSP$ is isosceles.



20 Given: $\overline{CY} \cong \overline{AY}$,
$\overline{YZ} \parallel \overline{CA}$
Prove: $\overrightarrow{YZ}$ bisects $\angle AYB$.



22 Given: $a \parallel b$,
$\angle 1 = (x + 3y)°$,
$\angle 2 = (2x + 30)°$,
$\angle 3 = (5y + 20)°$
Find: $m\angle 1$



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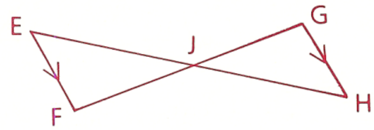
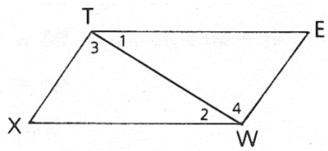
Classwork

2 Given: $\overline{EF} \parallel \overline{GH}$,
$\overline{EF} \cong \overline{GH}$
Conclusion: $\overline{EJ} \cong \overline{JH}$

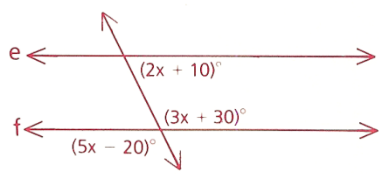


6 Given: $\overline{TE} \parallel \overline{XW}$
$\overline{TE} \cong \overline{XW}$
Concl: $\overline{TX} \parallel \overline{EW}$



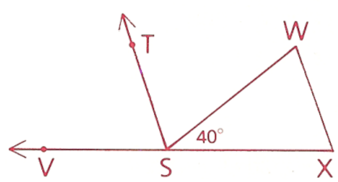
1 $\overline{TE} \parallel \overline{XW}$	1 Given
2 $\overline{TE} \cong \overline{XW}$	2 Given
3 Draw $\overline{TW}$	3
4 $\overline{TW} \cong \overline{TW}$	4 Reflexive prop
5 $\angle 1 \cong \angle 2$	5
6 $\triangle TEW \cong \triangle WXT$	6
7 $\angle 3 \cong \angle 4$	7
8 $\overline{TX} \parallel \overline{EW}$	8

7 Are e and f parallel?
include reason(s)



Vertical ∠s ⇒ ≅ ∠s: **$3x + 30 = 5x - 20$**

8 Given: $\overline{ST} \parallel \overline{XW}$;
$\overrightarrow{ST}$ bisects $\angle VSW$.
Find: $m\angle X$ and
$m\angle W$
What do you notice about $\triangle WSX$?



SHOW WORK and PROVIDE REASON:

∠VSW supp ∠WSX (∵ st ∠ ⇒ supp).
∠VSW + ∠WSX = 180 (∵supp ∠s sum to 180°).
∠VSW + 40° = 180 (∵substitution).
∠VSW = 140 (∵subtraction).
$\overrightarrow{ST}$ bis ∠VSW (given).

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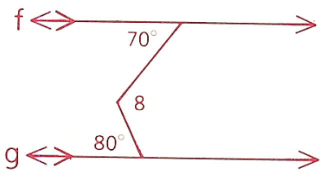
10: crook problems

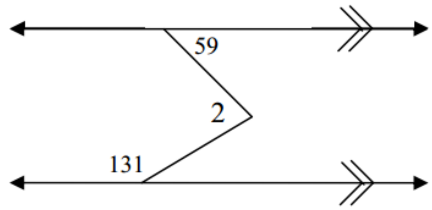
10a

If f || g, find m∠8: __________



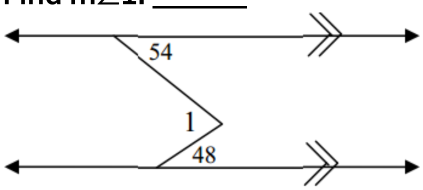
10c

Find m∠2: ______



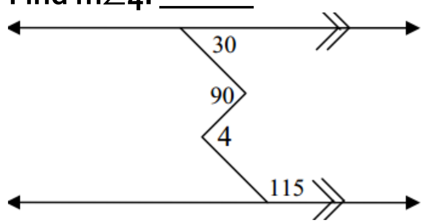
10b

Find m∠1: ______



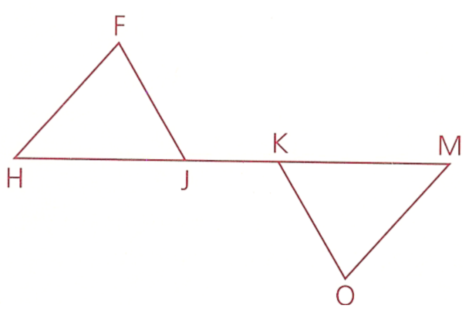
10d

Find m∠4: ______



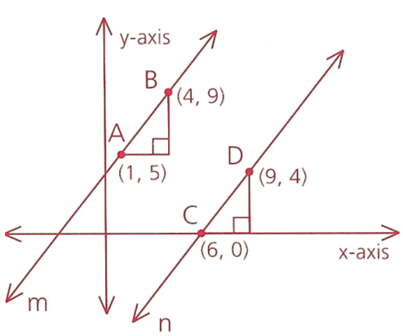
14 Given: $\overline{FJ} \nparallel \overline{KO}$,
$\overline{FH} \parallel \overline{MO}$,
$\overline{HK} \cong \overline{MJ}$

Prove: $\overline{FH} \ncong \overline{MO}$



Either ______________ or _______________. Assume ___________________. It's given FH || MO. || ⇒ _______________________ so _________________________. It's also given that HK ≅ MJ. Then $\overline{HJ} \cong \overline{KM}$ by _____________________. Thus ΔHJF ≅ _______ by _______. Then ∠FJH ≅ ____________ by ____________. ____, so _____________________. But this is impossible because it contradicts the given ________________. Consequently the assumption is false and ______________ is the only other possibility.

18 Explain why lines m and n are parallel.



19 Given: ∠C supp. ∠D

Prove: ∠A supp. ∠B