Notes

Objectives

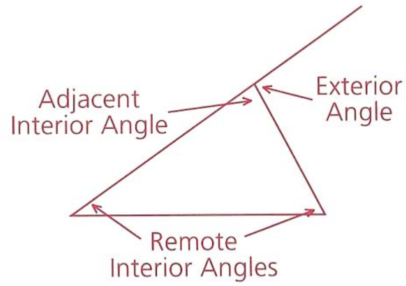
After studying this section, you will be able to

- Apply the Exterior Angle Inequality Theorem
- Use various methods to prove lines parallel

Part One: Introduction

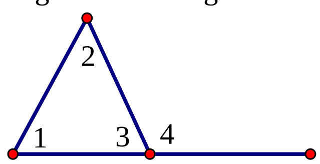
The Exterior Angle Inequality Theorem

An exterior angle of a triangle is formed whenever a side of the triangle is extended to form an angle supplementary to the adjacent interior angle.



Theorem 30 ***The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.***

Consider this: Angles of a triangle sum to 180°. Supplementary angles sum to 180°.



In the GSP lab, we observed:

Theorem 31 ***If two lines are cut by a transversal such that two alternate interior angles are congruent, the lines are parallel.*** (*Short form: Alt. int.* $\angle s \cong \Rightarrow \parallel$ *lines*)

Theorem 32 ***If two lines are cut by a transversal such that two alternate exterior angles are congruent, the lines are parallel.*** (*Alt. ext.* $\angle s \cong \Rightarrow \parallel$ *lines.*)

Theorem 33 ***If two lines are cut by a transversal such that two corresponding angles are congruent, the lines are parallel.*** (*Corr.* $\angle s \cong \Rightarrow \parallel$ *lines*)

Theorem 34 ***If two lines are cut by a transversal such that two interior angles on the same side of the transversal are supplementary, the lines are parallel.***

Theorem 35 ***If two lines are cut by a transversal such that two exterior angles on the same side of the transversal are supplementary, the lines are parallel.***

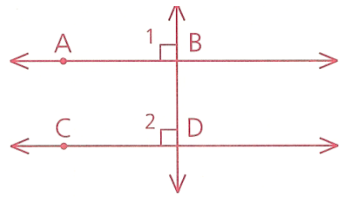
Theorem 36 ***If two coplanar lines are perpendicular to a third line, they are parallel.***

Class Examples

Problem 1 *Prove Theorem 36.*

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$ *and* $\overleftrightarrow{CD} \perp \overleftrightarrow{BD}$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



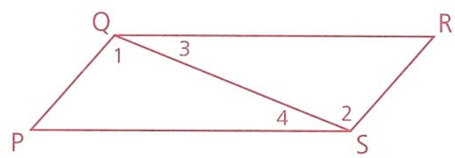
Proof

1 $\overleftrightarrow{BD} \perp \overleftrightarrow{AB}$	1
2 $\angle 1$ is a right $\angle$.	2
3 $\overleftrightarrow{BD} \perp \overleftrightarrow{CD}$	3
4 $\angle 2$ is a right $\angle$.	4
5 $\angle 1 \cong \angle 2$	5
6 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	6

Problem 2 *A parallelogram is a four-sided figure with* both *pairs of opposite sides parallel.*

Given: $\angle 1 \cong \angle 2$,
$\angle PQR \cong \angle RSP$

Prove: PQRS *is a parallelogram.*



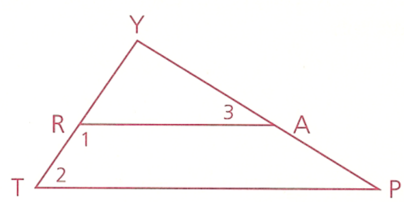
Proof

1 $\angle 1 \cong \angle 2$	1
2 $\overline{PQ} \parallel \overline{RS}$	2
3 $\angle PQR \cong \angle RSP$	3
4 $\angle 3 \cong \angle 4$	4
5 $\overline{QR} \parallel \overline{PS}$	5
6 PQRS is a parallelogram.	6

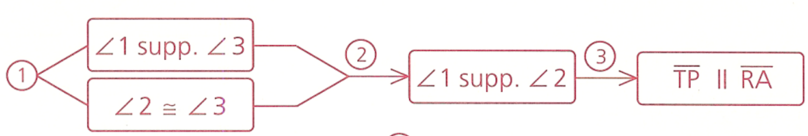
Problem 3 *A trapezoid is a four-sided figure with exactly one pair of parallel sides.*

Given: $\angle 1$ *supp.* $\angle 3$,
$\angle 2 \cong \angle 3$

Prove: TRAP *is a trapezoid.*



Proof We can use a flow diagram.



Reasons ① Given
② Substitution
③ Int. ∠s on same side supp. ⇒ ∥ lines

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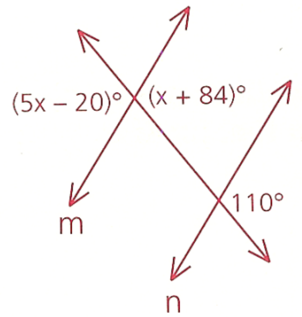
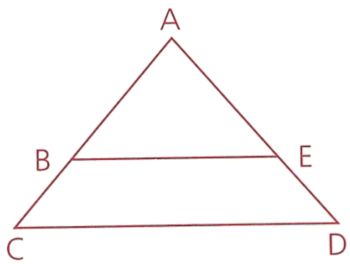
Homework

16 Solve for x and justify that m ∥ n.



19 Given: ∠D ≅ ∠ABE, $\overline{BE} \nparallel \overline{CD}$

Prove: $\overline{AC} \ncong \overline{AD}$



Either ________________ or _________________.

Assume ____________________________________.

If sides then angles, so ______________________ .

We are given that ∠D ≅ ∠ABE.

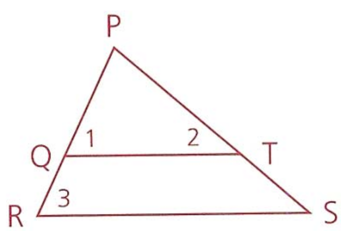
By the transitive property_____________________.

If corr. ∠s ≅ then ||, so ______________. But this is impossible as it contradicts the given information ____________.

Consequently the assumption is false and ______________ is the only possibility. *QED*

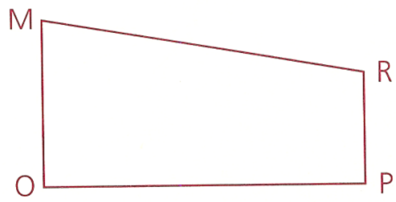
20 Given: ∠1 comp. ∠2, ∠3 comp. ∠2

Prove: $\overline{QT} \parallel \overline{RS}$



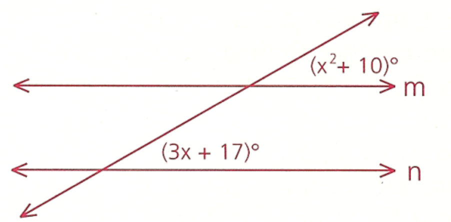
Statements	Reasons
1. ∠1 comp ∠2 & ∠3 comp ∠2	1. Given
2. ∠1 ≅ ∠3	2.
3.	3.

21 Given: $\angle MOP$ is a right angle.
$\overline{RP} \perp \overline{OP}$
Prove: $\overline{MO} \parallel \overline{RP}$



Statements	Reasons
1. $\angle MOP$ rt $\angle$	1. Given
2. $\overline{RP} \perp \overline{OP}$	2. Given
3.	3.
4. $\angle MOP$ supp $\angle OPR$	4.
5. $\overline{MO} \parallel \overline{RP}$	5.

26 Find the value(s) of x (to the nearest tenth) that will allow you to prove that m ∥ n. (Hint: You may wish to review the quadratic formula.)



Quadratic Formula: For any quadratic in standard form, that is $ax^2 + bx + c = 0$, the solutions are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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In Class Work

The following exercises need to be handed in before you leave class. You may work alone, with a partner, or in a small group. Please feel free to move desks but return them to their original location before the bell rings. The key for these exercises will be posted by the end of the school day so that you may refer to it when you complete the homework (which will be collected).

I worked with:

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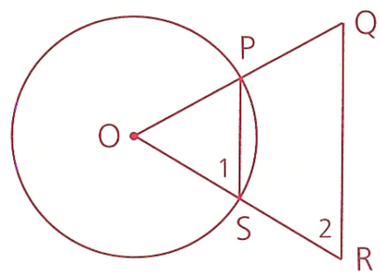
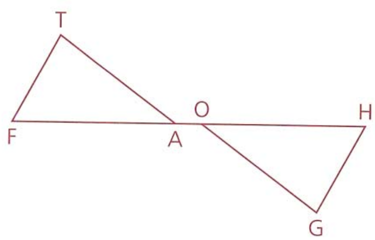
12 Given: ⊙O,
$\angle 1 \cong \angle 2$
Prove: $\overline{PS} \parallel \overline{QR}$

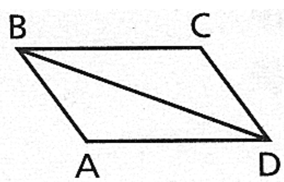


Statements	Reasons
$\angle 1 \cong \angle 2$	
$\overline{PS} \parallel \overline{QR}$	

13 Given: $\angle FAT \cong \angle HOG$
Prove: $\overline{AT} \parallel \overline{GO}$



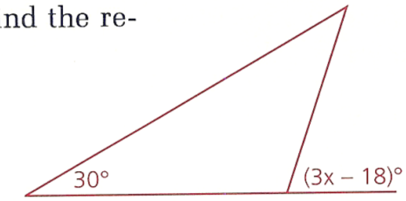
14 **Given:** $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{AD}$
Prove: $\overline{AB} \parallel \overline{CD}$



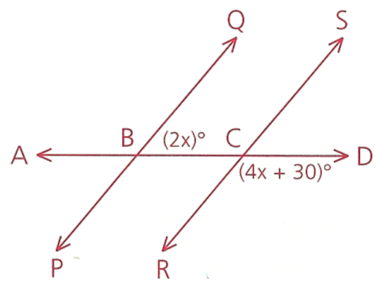
1 $\overline{AB} \cong \overline{CD}$	1
2 $\overline{BC} \cong \overline{AD}$	2
3 $\overline{BD} \cong \overline{BD}$	3
4 $\triangle BAD \cong \triangle DCB$	4
5 $\angle ABD \cong \angle BDC$	5
6 $\overline{AB} \parallel \overline{CD}$	6

AMDG

17 Write a valid inequality and find the restrictions on x.



23 If $\overleftrightarrow{PQ} \nparallel \overleftrightarrow{RS}$, can x be 25? Explain.



In a problem like this, include reasons (even though it's not a proof).

Remember that geometry is about "explaining why something is true".

Either PQ || RS or PQ is not || RS. Assume PQ || RS. || $\Rightarrow$ int. $\angle$s same side supp, so $\angle$QBC supp $\angle$SCA.

We know that vertical angles are congruent so $\angle SCA \cong \angle DCR$.

Hence, 2x + 4x+30 = 180. (You need to finish this problem.)