Notes

Objective

After studying this section, you will be able to

- Write indirect proofs

An indirect proof may be useful in a problem where a direct proof would be difficult to apply. Study the following example of an indirect proof.

Example

Given: $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \ncong \overline{DF}$

Prove: $\angle B \ncong \angle E$

Proof: Either $\angle B \cong \angle E$ or $\angle B \ncong \angle E$.
Assume $\angle B \cong \angle E$.
From the given information, $\angle A \cong \angle D$ and $\overline{AB} \cong \overline{DE}$.
Thus, $\triangle ABC \cong \triangle DEF$ by ASA.
$\therefore \overline{AC} \cong \overline{DF}$

But this is impossible, since $\overline{AC} \ncong \overline{DF}$ is given.
Thus, our assumption was false and $\angle B \ncong \angle E$, because this is the only other possibility.

The following procedure will help you to write indirect proofs.

Indirect-Proof Procedure

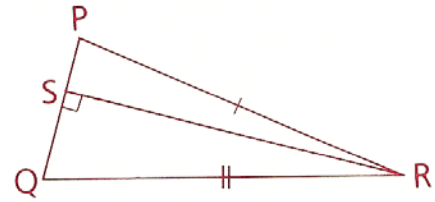
1 List the possibilities for the conclusion.
2 Assume that the *negation* of the desired conclusion is correct.
3 Write a chain of reasons until you reach an impossibility. This will be a contradiction of either
(a) given information or
(b) a theorem, definition, or other known fact.
4 State the remaining possibility as the desired conclusion.

Class Examples

Note Remember to start by looking at the conclusion.

Problem 1

Given: $\overline{RS} \perp \overline{PQ}$,
$\overline{PR} \not\cong \overline{QR}$
Prove: $\overrightarrow{RS}$ *does not bisect* $\angle PRQ$.



Proof

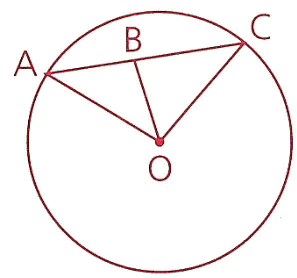
Either or

Assume
Then we can say that

But this is impossible because it contradicts the given fact that $\overline{PR} \not\cong \overline{QR}$. Consequently, the assumption must be false. $\therefore \overrightarrow{RS}$ does not bisect $\angle PRQ$, the only other possibility.

Problem 2

Given: $\odot O$, $\overline{AB} \not\cong \overline{BC}$
Prove: $\angle AOB \not\cong \angle COB$



Proof

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Adv Geo - | ***5.1: Indirect Proof*** | Date:

In Class Work

The following exercises need to be handed in before you leave class. You may work alone, with a partner, or in a small group. Please feel free to move desks but return them to their original location before the bell rings. The key for these exercises will be posted by the end of the school day so that you may refer to it when you complete the homework (which will be collected).

I worked with:

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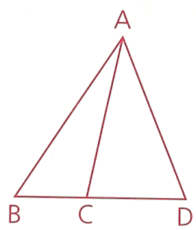
1 Given: $\overline{AB} \cong \overline{AD}$, $\angle BAC \ncong \angle DAC$
Prove: $\overline{BC} \ncong \overline{DC}$



Proof:

Either ______________________ or ______________________.

Assume ____.

We are given that $\overline{AB} \cong \overline{AD}$.

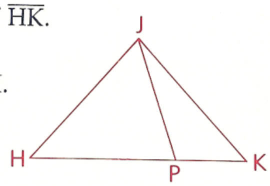
By the Reflexive Property ___________________________.

Then by SSS, ________________________________, and by CPCTC ____.

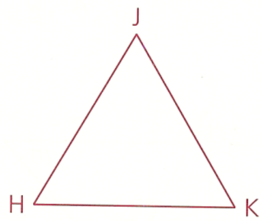
But this is impossible as it contradicts the given information $\angle BAC \ncong \angle DAC$.

Consequently the assumption is false and _______________ is the only possibility. *QED*

2 Given: P is not the midpoint of $\overline{HK}$.
$\overline{HJ} \cong \overline{JK}$
Prove: $\overrightarrow{JP}$ does not bisect $\angle HJK$.



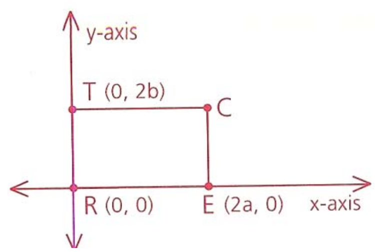
4 Given: $\angle H \not\cong \angle K$
Prove: $\overline{JH} \not\cong \overline{JK}$



7 RECT is a rectangle.

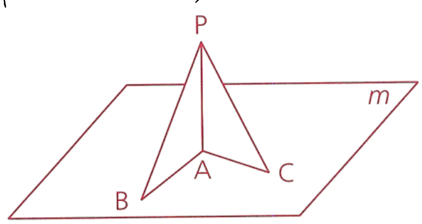
a In terms of a and b, find the coordinates of C.

b Does $\overline{RC}$ appear to be congruent to $\overline{ET}$?



**Regardless of format – two column or paragraph – proof is <u>supporting</u> a specific case with a trustworthy generalization (or axiom from our ASN).*

10 Given: $\overline{PA} \perp \overline{AB}$,
$\overline{PA} \perp \overline{AC}$,
$\angle B \not\cong \angle C$
Prove: $\overline{AB} \not\cong \overline{AC}$



Either ____________________ or __________________________. Assume ____.

We are given $\overline{PA} \perp \overline{AB}$ and $\overline{PA} \perp \overline{AC}$.

If two segments are perpendicular then they form right angles, thus ____ .

All right angles are congruent, so ____ .

Any segment is congruent to itself (aka the Reflexive Property), so ____ .

Hence, ____ by SAS.

Then by CPTPC, ____ .

But this is impossible as it contradicts the given information ____

Consequently the assumption is false and ________________ is the only possibility. QED

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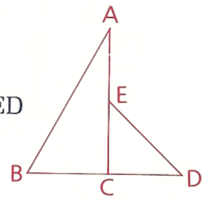
Adv Geo - ***5.1: Indirect Proof*** Date:

Homework

3 Given: $\overline{AC} \perp \overline{BD}$,
$\overline{BC} \cong \overline{EC}$,
$\overline{AB} \not\cong \overline{ED}$

Prove: $\angle B \not\cong \angle CED$



Either ________________________ or ________________________.

Assume ________________________________.

We are given $\overline{BC} \cong \overline{EC}$ & $AC \perp \overline{BD}$.

If two segments are perpendicular then they form right angles, thus ________________________ .

Hence, ____ by HL.

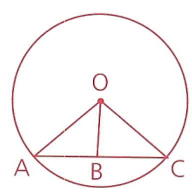
Then by CPTPC, ____ .

But this is impossible as it contradicts the given information ________________________

Consequently the assumption is false and ________ is the only possibility. QED

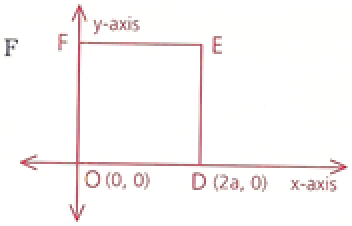
5 Given: $\odot O$;
$\overline{OB}$ is not an altitude.

Prove: $\overrightarrow{OB}$ does not bisect $\angle AOC$.



6 ODEF is a square.
In terms of a, find

a The coordinates of points E and F



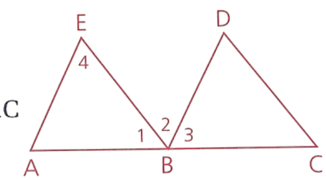
b The area of the square

c The midpoint of $\overline{FD}$

d The midpoint of $\overline{OE}$

9 Identify each of the following pairs of angles as alternate interior, alternate exterior, or corresponding.

a For $\overleftrightarrow{BE}$ and $\overleftrightarrow{CD}$ with transversal $\overleftrightarrow{BC}$, $\angle 1$ and $\angle C$



b For $\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$ with transversal $\overleftrightarrow{BE}$, $\angle 2$ and $\angle 4$

12 Given: $\odot O$; $\overline{HE}$ is not the perpendicular bisector of $\overline{DF}$.

Prove: $\overline{DE} \ncong \overline{EF}$