AMDG

Name | *Ch 5: Parallel Lines and Related Figures* | Ms. Kresovic

Adv Geo - | ***5.1: Indirect Proof*** | Date:

Notes

Objective

After studying this section, you will be able to

- Write indirect proofs

An indirect proof may be useful in a problem where a direct proof would be difficult to apply. Study the following example of an indirect proof.

Example

Given: $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \ncong \overline{DF}$

Prove: $\angle B \ncong \angle E$

Proof: Either $\angle B \cong \angle E$ or $\angle B \ncong \angle E$.
Assume $\angle B \cong \angle E$.
From the given information, $\angle A \cong \angle D$ and $\overline{AB} \cong \overline{DE}$.
Thus, $\triangle ABC \cong \triangle DEF$ by ASA.
$\therefore \overline{AC} \cong \overline{DF}$

But this is impossible, since $\overline{AC} \ncong \overline{DF}$ is given.
Thus, our assumption was false and $\angle B \ncong \angle E$, because this is the only other possibility.

The following procedure will help you to write indirect proofs.

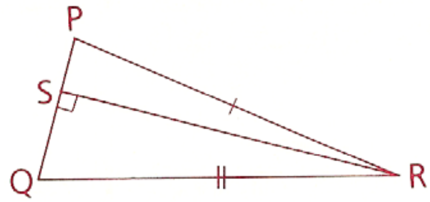
Indirect-Proof Procedure P: $\angle 1 \ncong \angle 2$

1 List the possibilities for the conclusion. $\angle 1 \cong \angle 2$ or $\angle 1 \ncong \angle 2$
2 Assume that the *negation* of the desired conclusion is correct. Assume $\angle 1 \cong \angle 2$
3 Write a chain of reasons until you reach an impossibility.
This will be a contradiction of either
(a) given information or
(b) a theorem, definition, or other known fact.
4 State the remaining possibility as the desired conclusion.

Class Examples

Note Remember to start by looking at the conclusion.

Problem 1 *Given:* $\overline{RS} \perp \overline{PQ}$,
$\overline{PR} \not\cong \overline{QR}$
Prove: $\overrightarrow{RS}$ *does not bisect* $\angle PRQ$.

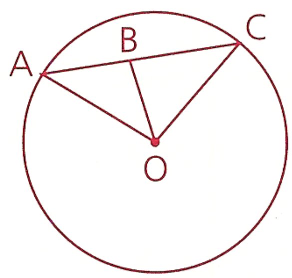


Proof Either or

Assume
Then we can say that

But this is impossible because it contradicts the given fact that $\overline{PR} \not\cong \overline{QR}$. Consequently, the assumption must be false. $\therefore$ $\overrightarrow{RS}$ does not bisect $\angle PRQ$, the only other possibility.

Problem 2 *Given:* $\odot O$, $\overline{AB} \not\cong \overline{BC}$
Prove: $\angle AOB \not\cong \angle COB$



Proof

In Class Work

The following exercises need to be handed in before you leave class. You may work alone, with a partner, or in a small group. Please feel free to move desks but return them to their original location before the bell rings. The key for these exercises will be posted by the end of the school day so that you may refer to it when you complete the homework (which will be collected).

I worked with:

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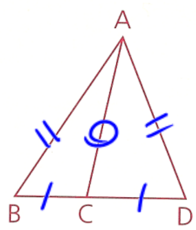
1 Given: $\overline{AB} \cong \overline{AD}$, $\angle BAC \ncong \angle DAC$
Prove: $\overline{BC} \ncong \overline{DC}$



Proof:

Either $\overline{BC} \cong \overline{DC}$ or $\overline{BC} \ncong \overline{DC}$.

Assume $\overline{BC} \cong \overline{DC}$.

We are given that $\overline{AB} \cong \overline{AD}$.

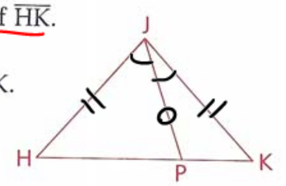
By the Reflexive Property $\overline{AC} \cong \overline{AC}$.

Then by SSS, $\triangle BCA \cong \triangle DCA$, and by CPCTC $\angle BAC \cong \angle DAC$.

But this is impossible as it contradicts the given information $\angle BAC \ncong \angle DAC$.

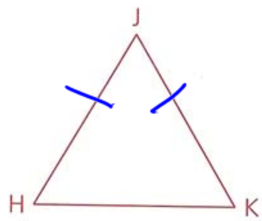
Consequently the assumption is false and $\overline{BC} \ncong \overline{DC}$ is the only possibility. *QED*

2 Given: P is not the midpoint of $\overline{HK}$.
$\overline{HJ} \cong \overline{JK}$
Prove: $\overrightarrow{JP}$ does not bisect $\angle HJK$.



EITHER $\overrightarrow{JP}$ DOES BIS $\angle HJK$ OR $\overrightarrow{JP}$ DOES NOT BIS $\angle HJK$.
ASSUME $\overrightarrow{JP}$ DOES BIS $\angle HJK$.
IF BIS THEN $\cong \angle$S, SO $\angle HJP \cong \angle KJP$.
BY REF, $\overline{JP} \cong \overline{JP}$. IT'S GIVEN $\overline{HJ} \cong \overline{JK}$.
BY SAS, $\triangle HJP \cong \triangle KJP$.
BY CPCTC, $\overline{HP} \cong \overline{PK}$. THEN $\cong$ SEGS $\Rightarrow$ MDPT; P MDPT $\overline{HK}$.
THIS IS IMPOSSIBLE $\because$ IT'S GIVEN P NOT MDPT $\overline{HK}$.
SO OUR ASSUMPTION IS FALSE & $\overrightarrow{JP}$ DOES NOT BIS $\angle HJK$ IS THE ONLY OTHER POSSIBILITY.

4 Given: $\angle H \ncong \angle K$
Prove: $\overline{JH} \ncong \overline{JK}$

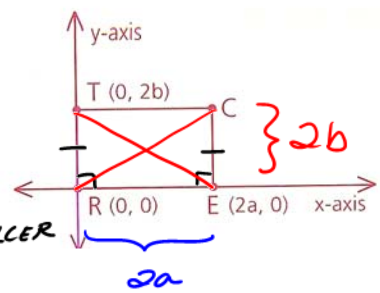


EITHER $\overline{JH} \ncong \overline{JK}$ OR $\overline{JH} \cong \overline{JK}$. ASSUME $\overline{JH} \cong \overline{JK}$. IF △ THEN △ SO $\angle H \cong \angle K$. BUT THIS IS IMPOSSIBLE ∵ IT'S GIVEN $\angle H \ncong \angle K$. THEREFORE OUR ASSUMPTION IS FALSE & $\overline{JH} \ncong \overline{JK}$ IS THE ONLY OTHER POSSIBILITY.

7 RECT is a rectangle.

a In terms of a and b, find the coordinates of C. $(2a, 2b)$

b Does $\overline{RC}$ appear to be congruent to $\overline{ET}$?

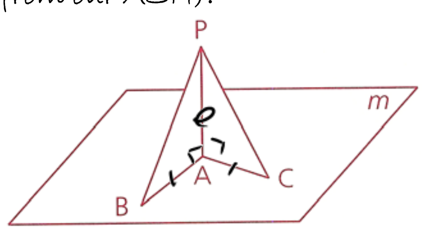


$TR = 2b$ & $CE = 2b$ ∴ $\overline{TR} \cong \overline{CE}$
$\angle TRE$ & $\angle CER$ rt∠s ⇒ $\angle TRE \cong \angle CER$
$\overline{RE} \cong \overline{RE}$
∴ $\triangle TRE \cong \triangle CER$ by SAS
& $\overline{TE} \cong \overline{CR}$ by CPCTC.

*Regardless of format – two column or paragraph – proof is <u>supporting</u> a specific case with a trustworthy generalization (or axiom from our ASN).

10 Given: $\overline{PA} \perp \overline{AB}$,
$\overline{PA} \perp \overline{AC}$,
$\angle B \ncong \angle C$
Prove: $\overline{AB} \ncong \overline{AC}$

OPT.



Either $\overline{AB} \cong \overline{AC}$ or $\overline{AB} \ncong \overline{AC}$. Assume $\overline{AB} \cong \overline{AC}$.

OPT 2 COL

We are given $\overline{PA} \perp \overline{AB}$ and $\overline{PA} \perp \overline{AC}$.

If two segments are perpendicular then they form right angles, thus $\angle PAC$ & $\angle PAB$ rt∠s.

All right angles are congruent, so $\angle PAB \cong \angle PAC$.

Any segment is congruent to itself (aka the Reflexive Property), so $\overline{PA} \cong \overline{PA}$.

Hence, $\triangle PAB \cong \triangle PAC$ by SAS.

Then by CPCTC, $\angle B \cong \angle C$.

But this is impossible as it contradicts the given information $\angle B \ncong \angle C$

Consequently the assumption is false and $\overline{AB} \ncong \overline{AC}$ is the only possibility. QED

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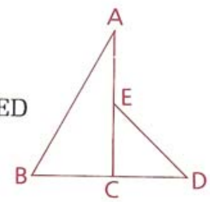
Homework

3 Given: $\overline{AC} \perp \overline{BD}$,
$\overline{BC} \cong \overline{EC}$,
$\overline{AB} \ncong \overline{ED}$

Prove: $\angle B \ncong \angle CED$



Either ______________________________ or ______________________________.

Assume ____.

We are given $\overline{BC} \cong \overline{EC}$ & $AC \perp \overline{BD}$.

If two segments are perpendicular then they form right angles, thus ______________________________ .

——____ by —— .

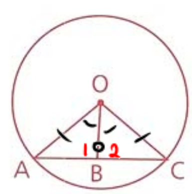
Then by CPTPC, ____ .

But this is impossible as it contradicts the given information ______________________________

Consequently the assumption is false and __________ is the only possibility. QED

5 Given: $\odot O$;
$\overline{OB}$ is not an altitude.

Prove: $\overrightarrow{OB}$ does not bisect $\angle AOC$.



EITHER $\overrightarrow{OB}$ DOES NOT BIS $\angle AOC$ OR $\overrightarrow{OB}$ BIS $\angle AOC$.

ASSUME $\overrightarrow{OB}$ BIS $\angle AOC$.

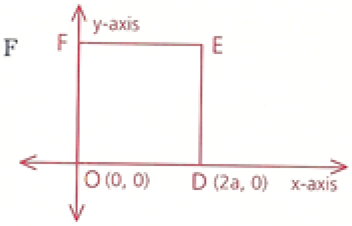
S	R
1. $\overrightarrow{OB}$ BIS $\angle AOC$	1. GIVEN
2. $\angle AOB \cong \angle COB$	2. $\rightarrow$ BIS $\angle \Rightarrow \cong \angle$
3. $\odot O$	3. GIVEN
4. $\overline{OA} \cong \overline{OC}$	4. $\odot \Rightarrow \cong$ RAD
5. $\overline{OB} \cong \overline{OB}$	5. Ref
6 $\triangle AOB \cong \triangle COB$	6. SAS
7. $\angle 1 \cong \angle 2$	7. CPCTC
8. $\angle 1$ supp $\angle 2$	8. st $\angle \Rightarrow$ supp $\angle$
9. $\angle 1$ & $\angle 2$ rt $\angle$s	9. $\cong$ & supp $\angle$s $\Rightarrow$ rt $\angle$
10. $\overline{OB}$ alt $\overline{AC}$	10. rt $\angle \Rightarrow$ alt

BUT THIS IS IMPOSSIBLE $\because$ $\overline{OB}$ is not alt $\overline{AC}$ is given.

$\therefore$ OUR ASSUMPTION IS FALSE & $\overrightarrow{OB}$ DOES NOT BIS $\angle AOC$ IS THE ONLY OTHER POSSIBILITY.

6 ODEF is a square.
In terms of a, find

a The coordinates of points E and F



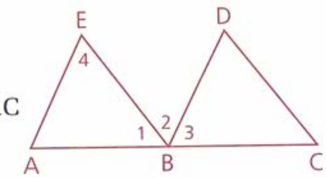
b The area of the square

c The midpoint of $\overline{FD}$

d The midpoint of $\overline{OE}$

9 Identify each of the following pairs of angles as alternate interior, alternate exterior, or corresponding.

a For $\overleftrightarrow{BE}$ and $\overleftrightarrow{CD}$ with transversal $\overleftrightarrow{BC}$, $\angle 1$ and $\angle C$



b For $\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$ with transversal $\overleftrightarrow{BE}$, $\angle 2$ and $\angle 4$

12 Given: $\odot$O; $\overline{HE}$ is not the perpendicular bisector of $\overline{DF}$.

Prove: $\overline{DE} \not\cong \overline{EF}$