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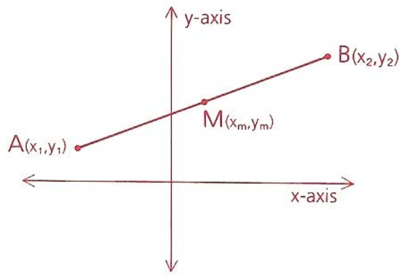
Adv Geo period ___ | W 30 October 2013

Lines in the Plane – Chapter 4 – ASN

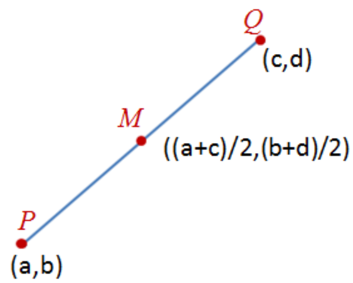
4.1

Theorem 22 ***If A = (x_1, y_1) and B = (x_2, y_2), then the midpoint M = (x_m, y_m) of $\overline{AB}$ can be found by using the midpoint formula:***

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Coordinate Proof: Let (a,b) be the coordinates of P and (c,d) be the coordinates of Q. In showing that M with coordinates $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$ is the midpoint of PQ, we have to show that (1) PM = MQ, and (2) M is on PQ. (Why?)



For (1), we show that the distance of PM is equal to the distance of MQ (using the Pythagorean Theorem).

Distance of PM:

$$|PM| = \sqrt{\left(\frac{a+c}{2} - a\right)^2 + \left(\frac{b+d}{2} - b\right)^2}$$

$$|PM| = \sqrt{\left(\frac{a+c-2a}{2}\right)^2 + \left(\frac{b+d-2b}{2}\right)^2}$$

$$|PM| = \sqrt{\left(\frac{c-a}{2}\right)^2 + \left(\frac{d-b}{2}\right)^2}$$

Distance of MQ:

$$|MQ| = \sqrt{\left(c - \frac{a+c}{2}\right)^2 + \left(d - \frac{b+d}{2}\right)^2}$$

$$|MQ| = \sqrt{\left(\frac{2c-a-c}{2}\right)^2 + \left(\frac{2d-b-d}{2}\right)^2}$$

$$|MQ| = \sqrt{\left(\frac{c-a}{2}\right)^2 + \left(\frac{d-b}{2}\right)^2}$$

Hence, PM = MQ. If two segments have the same measure, then they are congruent. Therefore, $\overline{PM} \cong \overline{MQ}$

For (2), we have to show that the slope of PM is equal to the slope of MQ.
Slope of PM:

$$\frac{\frac{b+d}{2} - b}{\frac{a+c}{2} - a} = \frac{\frac{b+d-2b}{2}}{\frac{a+c-2a}{2}} = \frac{d-b}{c-a}$$

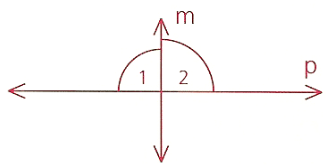
Slope of MQ:

$$\frac{d - \frac{b+d}{2}}{c - \frac{a+c}{2}} = \frac{\frac{2d-b-d}{2}}{\frac{2c-a-c}{2}} = \frac{d-b}{c-a}$$

Since m(PQ)=m(MQ) and $\overline{PM} \cong \overline{MQ}$, we may conclude that M is the midpoint of PQ. *QED*

4.3 **Theorem 23** ***If two angles are both supplementary and congruent, then they are right angles.***

Given: $\angle 1 \cong \angle 2$
Prove: $\angle 1$ and $\angle 2$ are right angles.



Two-column proof:

Statements	*Reasons*
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. If two angles are congruent, then they have the same measure.
3. $\angle 1$ is supplementary to $\angle 2$	3. If two angles form a straight angle, then they are supplementary angles.
4. $m\angle 1 + m\angle 2 = 180°$	4. If two angles are supplementary, then they sum to 180°.
5. $m\angle 1 + m\angle 1 = 180°$	5. Substitution
6. $m\angle 1 = 90°$	6. Division
7. $\angle 1$ & $\angle 2$ are right angles	7. If an angles measure is 90°, then it is a right angle.

Paragraph proof:
Since $\angle 1$ and $\angle 2$ form a straight angle (line p), they are supplementary. Therefore, $m\angle 1 + m\angle 2 = 180$. Since $\angle 1 \cong \angle 2$, we can use substitution to get the equation $m\angle 1 + m\angle 1 = 180$, or $m\angle 1 = 90$. Thus, $\angle 1$ is a right angle, and so is $\angle 2$.

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4.4

Definition The ***distance*** between two objects is the length of the shortest path joining them.

Postulate ***A line segment is the shortest path between two points.***

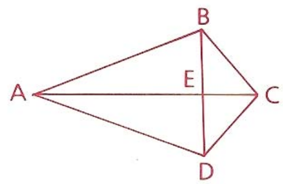
The distance between points R and S is the length of $\overline{RS}$, or RS.

Definition The ***perpendicular bisector*** of a segment is the line that bisects and is perpendicular to the segment.

Theorem 24 ***If two points are each equidistant from the end-points of a segment, then the two points determine the perpendicular bisector of that segment.***

Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{CD}$

Prove: $\overleftrightarrow{AC}$ *is the* $\perp$ *bisector of* $\overline{BD}$.



Proof

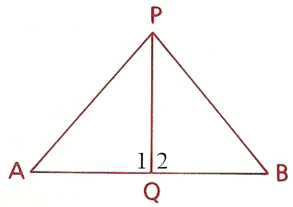
DETOUR

1 $\overline{AB} \cong \overline{AD}$	1 Given
2 $\overline{BC} \cong \overline{CD}$	2 Given
3 $\overline{AC} \cong \overline{AC}$	3 Reflexive Property
4 $\triangle ABC \cong \triangle ADC$	4 SSS (1, 2, 3)
5 $\angle BAC \cong \angle DAC$	5 CPCTC
6 $\overline{AE} \cong \overline{AE}$	6 Reflexive Property
7 $\triangle ABE \cong \triangle ADE$	7 SAS (1, 5, 6)
8 $\overline{BE} \cong \overline{ED}$	8 CPCTC
9 $\overleftrightarrow{AC}$ bisects $\overline{BD}$.	9 If a line divides a segment into two $\cong$ segments, it bisects the segment.
10 $\angle AEB \cong \angle AED$	10 CPCTC (step 7)
11 $\angle AED$ and $\angle AEB$ are right $\angle$s.	11 If two $\angle$s are both supp. and $\cong$, then they are right $\angle$s.
12 $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$	12 If two lines intersect to form right $\angle$s, they are $\perp$.
13 $\overleftrightarrow{AC}$ is the $\perp$ bisector of $\overline{BD}$.	13 Combination of steps 9 and 12

Theorem 25 ***If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.***

Given: $\overleftrightarrow{PQ}$ is the ⊥ bisector of $\overline{AB}$.

Prove: $\overline{PA} \cong \overline{PB}$



Statements	*Reasons*
1. $\overline{PQ}$ ⊥ bis $\overline{AB}$	1. Given
2. $\overline{AQ} \cong \overline{QB}$	2. If a segment bisects a segment, then it divides the segment into two congruent segments. (1)
3. ∠1 & ∠2 are right angles	3. If two segments are perpendicular, then they form right angles. (1)
4. ∠1 ≅ ∠2	4. All right angles are congruent. (2)
5. $\overline{PQ} \cong \overline{PQ}$	5. Any segment is congruent to itself; the Reflexive Property.
6. ΔPAQ ≅ ΔPBQ	6. SAS (2, 4, 5)
7. $\overline{PA} \cong \overline{PB}$	7. Corresponding parts of congruent triangles are congruent; CPCTC. (6)

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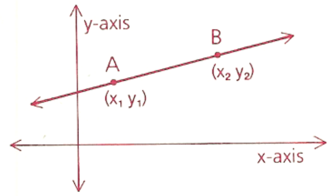
4.6

Definition The ***slope*** m of a nonvertical line, segment, or ray containing (x_1, y_1) and (x_2, y_2) is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } m = \frac{y_1 - y_2}{x_1 - x_2} \text{ or } m = \frac{\Delta y}{\Delta x}$$



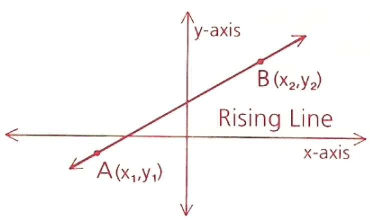
Note In more-advanced mathematics classes, it is common to use Δy (read "delta y") instead of $y_2 - y_1$ and Δx ("delta x") instead of $x_2 - x_1$. The symbol Δ is used to indicate change, so that Δy, for example, means "the change in y-coordinates between two points."

Do not confuse no slope with a slope of zero. On a horizontal line, $y_2 = y_1$, but $x_2 \neq x_1$. Therefore, the numerator is zero, while the denominator is not. Hence, a horizontal line has zero slope.

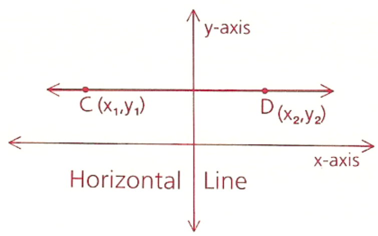
Visual Interpretation of Slope

The numerical value of a slope gives us a clue to the direction a line is taking. The following diagrams illustrate this notion.

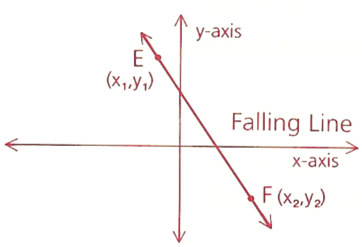
Positive Slope



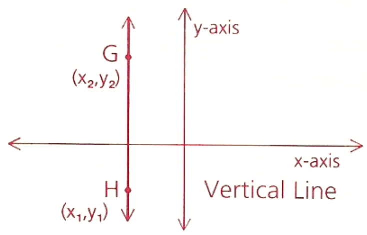
Zero Slope



Negative Slope



No Slope



In summary,

- Rising line ⟺ positive slope
- Horizontal line ⟺ zero slope
- Falling line ⟺ negative slope
- Vertical line ⟺ no slope

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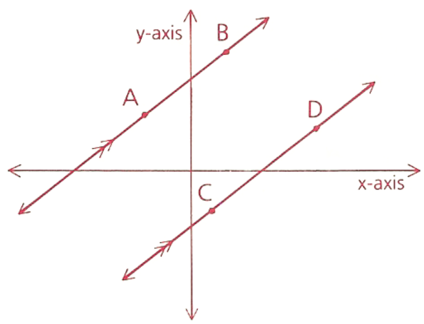
Slopes of Parallel and Perpendicular Lines

The proofs of the following four theorems require a knowledge of the properties of similar triangles and will be omitted here.

Theorem 26 ***If two nonvertical lines are parallel, then their slopes are equal.***

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: Slope $\overleftrightarrow{AB}$ = slope $\overleftrightarrow{CD}$



Theorem 27 ***If the slopes of two nonvertical lines are equal, then the lines are parallel.***

mAB = mCD

Theorem 28 ***If two lines are perpendicular and neither is vertical, each line's slope is the opposite reciprocal of the other's.***

(mAB)(mCD) = −1

Theorem 29 ***If a line's slope is the opposite reciprocal of another line's slope, the two lines are perpendicular.***

You will not hand in this ASN for points. However you are required to know & apply the axioms.