AMDG

Name Stu Dent   Ms. Kresovic

Adv Geo - 8   4.4: Equidistance Theorem   Date NOV 10-2015

Objective

After studying this section, you will be able to

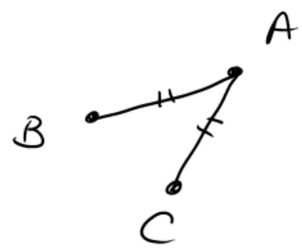
- Recognize the relationship between equidistance and perpendicular bisection

Part One: Introduction

In geometry, the term *distance* has a special meaning.

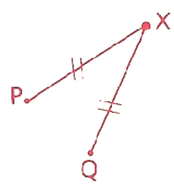
Definition The ***distance*** between two objects is the length of the shortest path joining them.

Postulate ***A line segment is the shortest path between two points.***

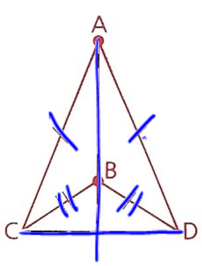


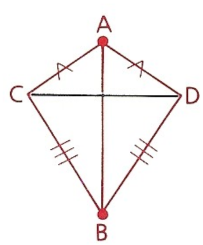
The distance between points R and S is the length of $\overline{RS}$, or RS.

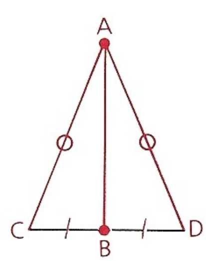
If two points P and Q are the same distance from a third point X, then X is said to be ***equidistant*** from P and Q.



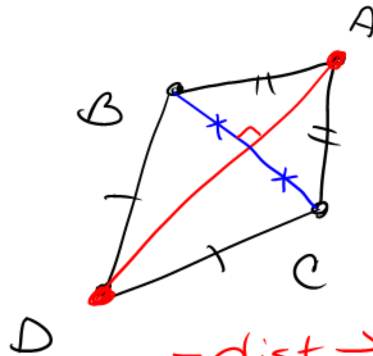
$\overline{PX} \cong \overline{XQ}$
means that
X is equidistant from P and Q.







You should recall many problems with diagrams resembling those above. These diagrams have something in common. In each, both point A and point B are equidistant from the endpoints C and D of $\overline{CD}$. In each case, you could prove that $\overleftrightarrow{AB}$ is the ***perpendicular bisector*** of $\overline{CD}$ just by using the following definition and theorem.

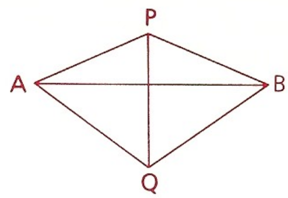


=dist ⇒ ⊥ bis

Definition The ***perpendicular bisector*** of a segment is the line that bisects and is perpendicular to the segment.

Theorem 24 ***If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.***

Given: $\overline{PA} \cong \overline{PB}$,
$\overline{QA} \cong \overline{QB}$
Prove: $\overleftrightarrow{PQ}$ is the $\perp$ bisector of $\overline{AB}$.

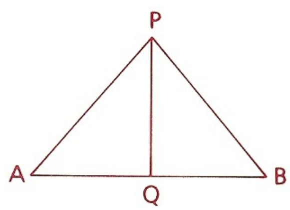


For a proof of Theorem 24, see sample problem 2 in Section 4.3.

Theorem 25 ***If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.***

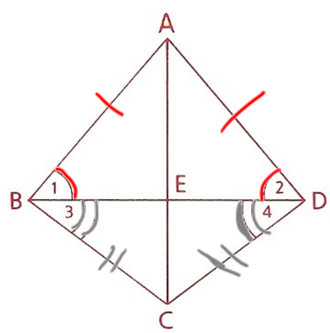
⊥ bis ⇒ =dist

Given: $\overleftrightarrow{PQ}$ is the $\perp$ bisector of $\overline{AB}$.
Prove: $\overline{PA} \cong \overline{PB}$



Part Two: Sample Problems

Problem 1 Given: $\angle 1 \cong \angle 2$,
$\angle 3 \cong \angle 4$
Prove: $\overleftrightarrow{AE} \perp$ *bis.* $\overline{BD}$



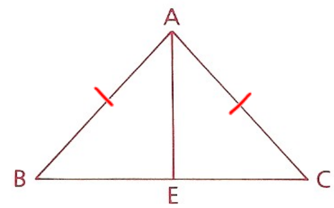
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overline{BA} \cong \overline{DA}$	2. $\triangle \Rightarrow \angle\angle$
3. $\angle 3 \cong \angle 4$	3. Given
4. $\overline{BC} \cong \overline{DC}$	4. $\triangle \Rightarrow \angle\angle$
5. $\overleftrightarrow{AE} \perp$ bis $\overline{BD}$	5. =dist ⇒ ⊥ bis

Name Ms. Kresovic

Adv Geo - 4.4: Equidistance Theorem

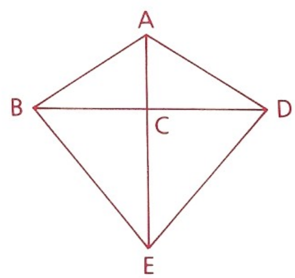
Problem 2 *Prove: The line joining the vertex of an isosceles triangle to the midpoint of the base is perpendicular to the base.*

Given: $\triangle ABC$ *is isosceles, with* $\overline{AB} \cong \overline{AC}$.
E *is the midpoint of* $\overline{BC}$.
Prove: $\overleftrightarrow{AE} \perp \overline{BC}$



1. $\overline{AB} \cong \overline{AC}$	1. Given
2. E mdpt BC	2. Given
3. $\overline{BE} \cong \overline{EC}$	3. mdpt ⇒ ≅ seg
4. $\overleftrightarrow{AE} \perp \overline{BC}$	4. = dist ⇒ ⊥ bis

Problem 3 *Given:* $\overline{AB} \cong \overline{AD}$,
$\overline{BC} \cong \overline{CD}$
Conclusion: $\overline{BE} \cong \overline{ED}$



Homework Day 1

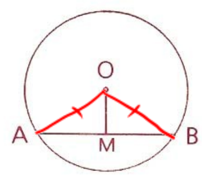
Part Three: Problem Sets

Problem Set A

As you work on these proofs, see if the equidistance theorems apply; they can save you a lot of work.

1 Given: ⊙O; M is the midpt. of $\overline{AB}$.

Conclusion: $\overline{OM} \perp \overline{AB}$ (Hint: Draw two auxiliary lines.)

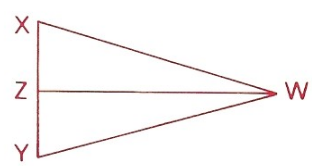


USING NEW THEOREM

1. ⊙O	1. Given
2. Draw $\overline{OA}$ & $\overline{OB}$	2. aux
3. $\overline{OA} \cong \overline{OB}$	3. ⊙ ⇒ ≅ radii
4. M mdpt $\overline{AB}$	4. Given
5. $\overline{AM} \cong \overline{MB}$	5. mdpt ⇒ ≅ segs
6. OM ⊥ $\overline{AB}$	6. = dist ⇒ ⊥ bis

2 Given: $\overleftrightarrow{WZ}$ ⊥ bis. $\overline{XY}$

Prove: △WXY is isosceles. (Hint: This proof can be written in three steps by using Theorem 25.)



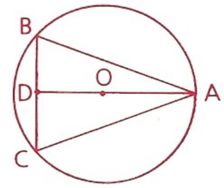
1. $\overleftrightarrow{WZ}$ ⊥ bis $\overline{XY}$	1. Given
2. $\overline{WX} \cong \overline{WY}$	2. ⊥ bis ⇒ = dist
3. △WXY isos, base $\overline{XY}$	3. 2 ≅ segs ⇒ isos △

Name Ms. Kresovic

Adv Geo - 4.4: Equidistance Theorem

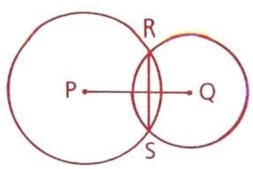
3 Given: $\odot O$, $\overline{AB} \cong \overline{AC}$

Conclusion: $\overleftrightarrow{AD} \perp$ bis. $\overline{BC}$ (Hint: Show that A and O are each equidistant from B and C.)



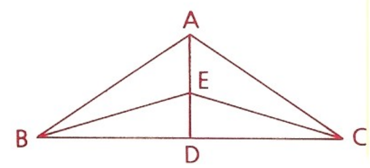
4 Given: Ⓢ P and Q

Prove: $\overleftrightarrow{PQ} \perp$ bis. $\overline{RS}$



5 Given: $\overleftrightarrow{AD} \perp$ bis. $\overline{BC}$

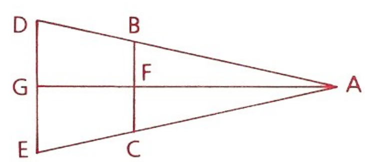
Prove: $\triangle ABE \cong \triangle ACE$



6 Given: $\overleftrightarrow{AG} \perp$ bis. $\overline{BC}$,

$\overleftrightarrow{AG} \perp$ bis. $\overline{DE}$

Conclusion: $\overline{BD} \cong \overline{CE}$



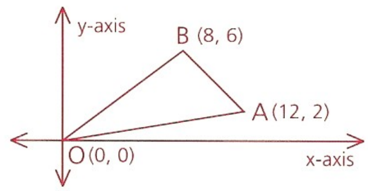
AMDG

Name
Adv Geo -

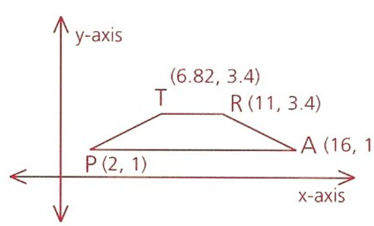
4.4: Equidistance Theorem

Ms. Kresovic
Nov 13

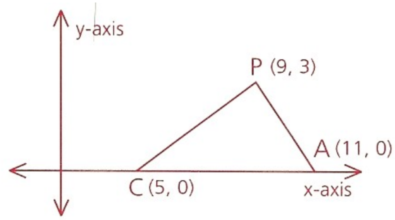
7 How much greater than the x-coordinate of the midpoint of $\overline{OA}$ is the x-coordinate of the midpoint of $\overline{AB}$?



8 In the graph, if a perpendicular is drawn from T to $\overleftrightarrow{PA}$, what will the coordinates of the point where the perpendicular intersects $\overleftrightarrow{PA}$ be?



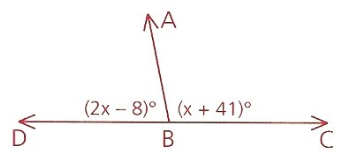
9 If $\triangle CAP$ is slid along the x-axis until C is at (11, 0), what will the new coordinates of P be?



10 A fifth point, E, is located on the diagram so that $m\angle EBC = \sqrt{x} + 83$.

a Is $\overleftrightarrow{AB}$ perpendicular to $\overleftrightarrow{DC}$?

b What do we know about $\overleftrightarrow{AB}$ and $\overleftrightarrow{BE}$?



AMDG

Name | Ms. Kresovic

Adv Geo - | 4.4: Equidistance Theorem | Nov 13

Homework Day 2

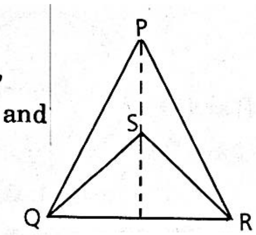
Problem Set B

Remember, the equidistance theorems will help you write a concise proof.

11 Draw isosceles $\triangle PQR$, with P the vertex. Draw the bisectors of the base angles and label their point of intersection S. Prove that $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$. (Hint: Use Theorem 24.)

11 Given: Isos $\triangle PQR$, $\overline{QS}$ bis $\angle Q$, and $\overline{RS}$ bis $\angle R$.

Prove: $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$



1 Isos $\triangle PQR$

2 $\overline{PQ} \cong \overline{PR}$

3 $\angle Q \cong \angle R$

4 $\overline{QS}$ bis $\angle Q$.

5 $\overline{RS}$ bis $\angle R$.

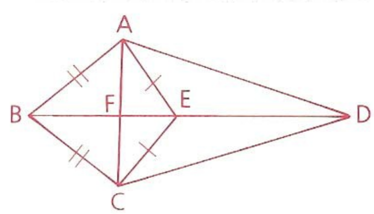
6 $\angle SQR \cong \angle SRQ$

7 $\overline{SQ} \cong \overline{SR}$

8 $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$

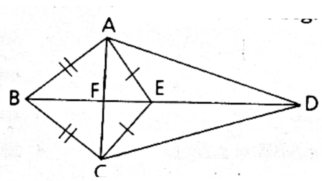
12 Given: $\overline{AB} \cong \overline{BC}$, $\overline{AE} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{DC}$ (Hint: This can be done in four steps.)



12 Given: $\overline{AB} \cong \overline{BC}$, $\overline{AE} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{DC}$



1 $\overline{AB} \cong \overline{BC}$

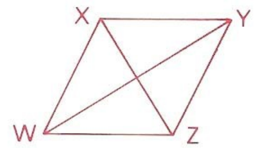
2 $\overline{AE} \cong \overline{EC}$

3 $\overline{BE} \perp$ bis $\overline{AC}$.

4 $\overline{AD} \cong \overline{DC}$

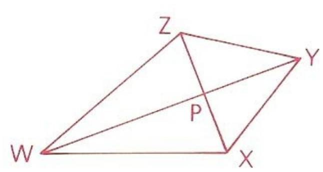
13 Given: $\overline{WY}$ and $\overline{XZ}$ $\perp$ bis. each other.

Prove: $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$ (that is, WXYZ is a rhombus)



14 Given: $\overline{WX} \cong \overline{WZ}$, $\overline{XY} \cong \overline{YZ}$ (WXYZ is a *kite*.)

Prove: $\triangle WPZ$ is a right $\triangle$.

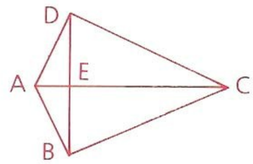


AMDG

Name Ms. Kresovic

Adv Geo - 4.4: Equidistance Theorem Nov 13

15 Given: $\angle ADC$ and $\angle ABC$ are right $\angle$s.
$\overline{AB} \cong \overline{AD}$
Conclusion: $\overleftrightarrow{AC} \perp$ bis. $\overline{BD}$



16 Prove: The median to the base of an isosceles triangle is also an altitude. (Prove this without using congruent triangles.)

17 Given: F is the midpt. of $\overline{BC}$.
$\overline{DB} \cong \overline{EC}$,
$\overline{DB} \perp \overline{DF}$,
$\overline{EC} \perp \overline{EF}$

Conclusion: $\overleftrightarrow{AF} \perp \overleftrightarrow{BC}$