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Adv Geo - 7 — 4.4: Equidistance Theorem — Date Nov 10, 2015

Objective

After studying this section, you will be able to

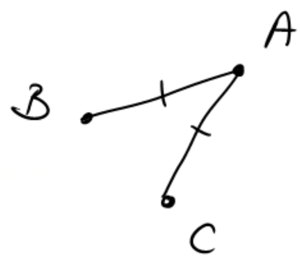
- Recognize the relationship between equidistance and perpendicular bisection

Part One: Introduction

In geometry, the term *distance* has a special meaning.

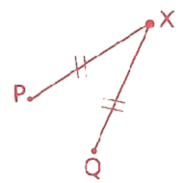
Definition The ***distance*** between two objects is the length of the shortest path joining them.

Postulate ***A line segment is the shortest path between two points.***

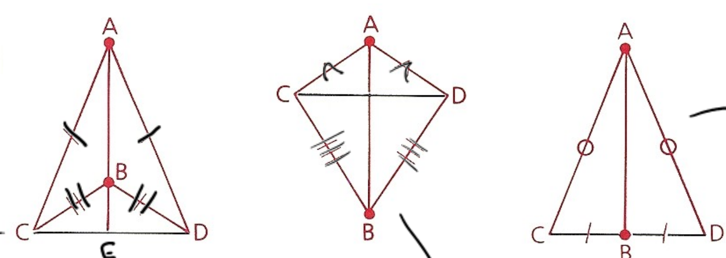


The distance between points R and S is the length of $\overline{RS}$, or RS.

If two points P and Q are the same distance from a third point X, then X is said to be ***equidistant*** from P and Q.



$\overline{PX} \cong \overline{XQ}$
means that
X is equidistant from P and Q.



→ $\overline{AB} \perp$ bis $\overline{CD}$

You should recall many problems with diagrams resembling those above. These diagrams have something in common. In each, both point A and point B are equidistant from the endpoints C and D of $\overline{CD}$. In each case, you could prove that $\overleftrightarrow{AB}$ is the ***perpendicular bisector*** of $\overline{CD}$ just by using the following definition and theorem.

$\overline{AE} \perp$ bis $\overline{CD}$

$\overline{AB} \perp$ bis $\overline{CD}$

Definition The ***perpendicular bisector*** of a segment is the line that bisects and is perpendicular to the segment.

Theorem 24 ***If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.***

Example

$\overline{AP} \cong \overline{PB}$

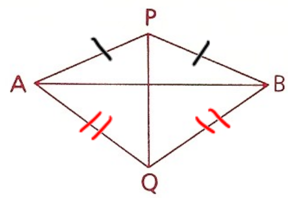
$\overline{AQ} \cong \overline{QB}$

PQ ⊥ bis AB

=dist ⇒ ⊥bis

Given: $\overline{PA} \cong \overline{PB}$,
$\overline{QA} \cong \overline{QB}$

Prove: $\overleftrightarrow{PQ}$ is the ⊥ bisector of $\overline{AB}$.



For a proof of Theorem 24, see sample problem 2 in Section 4.3.

Theorem 25 ***If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.***

Ex:

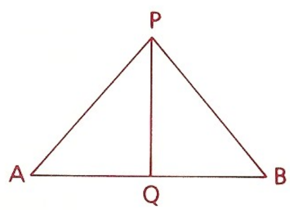
$\overline{PQ}$ ⊥ bis AB

$\overline{PA} \cong \overline{PB}$

⊥bis ⇒ =dist

Given: $\overleftrightarrow{PQ}$ is the ⊥ bisector of $\overline{AB}$.

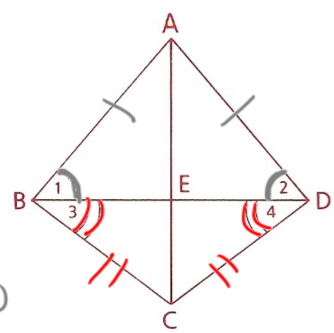
Prove: $\overline{PA} \cong \overline{PB}$



Part Two: Sample Problems

Problem 1 Given: ∠1 ≅ ∠2,
∠3 ≅ ∠4

Prove: $\overleftrightarrow{AE}$ ⊥ bis. $\overline{BD}$



1. ∠1 ≅ ∠2.	1. Given
2. $\overline{AB} \cong \overline{AD}$	2. △ ⇒ △
3. ∠3 ≅ ∠4	3. Given
4. $\overline{BC} \cong \overline{DC}$	4. △ ⇒ △
5. $\overleftrightarrow{AE}$ ⊥ bis $\overline{BD}$	5. =dist ⇒ ⊥ bis

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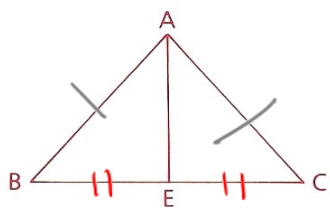
Problem 2 *Prove: The line joining the vertex of an isosceles triangle to the midpoint of the base is perpendicular to the base.*

Given: $\triangle ABC$ *is isosceles, with* $\overline{AB} \cong \overline{AC}$.
E *is the midpoint of* $\overline{BC}$.

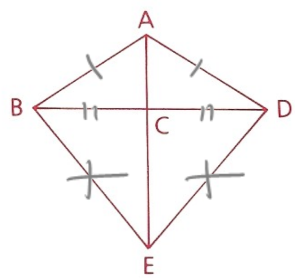
Prove: $\overleftrightarrow{AE} \perp \overline{BC}$



1. $\overline{AB} \cong \overline{AC}$ — 1. Given
2. E mdpt $\overline{BC}$ — 2. Given
3. $\overline{BE} \cong \overline{EC}$ — 3. mdpt ⇒ ≅ segs
4. $\overleftrightarrow{AE} \perp \overline{BC}$ — 4. =dist ⇒ ⊥ bis

Problem 3 *Given:* $\overline{AB} \cong \overline{AD}$,
$\overline{BC} \cong \overline{CD}$

Conclusion: $\overline{BE} \cong \overline{ED}$



1. $\overline{AB} \cong \overline{AD}$ — 1. Given
2. $\overline{BC} \cong \overline{CD}$ — 2. Given
3. $\overleftrightarrow{AC}$ ⊥ bis $\overline{BD}$ — 3. =dist ⇒ ⊥ bis
4. $\overline{BE} \cong \overline{ED}$ — 4. ⊥ bis ⇒ =dist

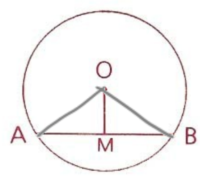
Homework Day 1

Part Three: Problem Sets

Problem Set A

As you work on these proofs, see if the equidistance theorems apply; they can save you a lot of work.

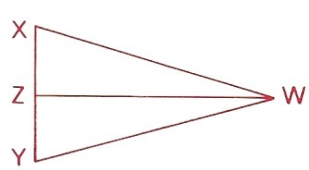
1 Given: ⊙O; M is the midpt. of $\overline{AB}$.
Conclusion: $\overline{OM} \perp \overline{AB}$ (Hint: Draw two auxiliary lines.)



USING NEW THEOREMS:

1. ⊙O — 1. GIVEN
2. DRAW $\overline{OA}$ & $\overline{OB}$ — 2. AUX
3. $\overline{OA} \cong \overline{OB}$ — 3. ⊙ ⇒ ≅ radii
4. M mdpt $\overline{AB}$ — 4. GIVEN
5. $\overline{AM} \cong \overline{MB}$ — 5. mdpt ⇒ ≅ SEGS
6. $\overline{OM} \perp \overline{AB}$ — 6. = dist ⇒ ⊥ bis

2 Given: $\overleftrightarrow{WZ} \perp$ bis. $\overline{XY}$
Prove: △WXY is isosceles. (Hint: This proof can be written in three steps by using Theorem 25.)



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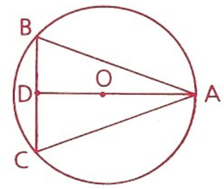
3 Given: ⊙O, $\overline{AB} \cong \overline{AC}$

Conclusion: $\overleftrightarrow{AD}$ ⊥ bis. $\overline{BC}$ (Hint: Show that A and O are each equidistant from B and C.)

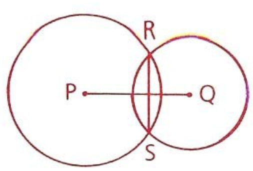


(S in a triangle): more than 1 Δ

(S in a circle): more than 1 ⊙

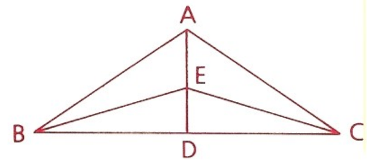
4 Given: (S) P and Q

Prove: $\overleftrightarrow{PQ}$ ⊥ bis. $\overline{RS}$



5 Given: $\overleftrightarrow{AD} \perp$ bis. $\overline{BC}$

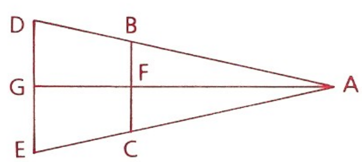
Prove: $\triangle ABE \cong \triangle ACE$



6 Given: $\overleftrightarrow{AG} \perp$ bis. $\overline{BC}$,

$\overleftrightarrow{AG} \perp$ bis. $\overline{DE}$

Conclusion: $\overline{BD} \cong \overline{CE}$



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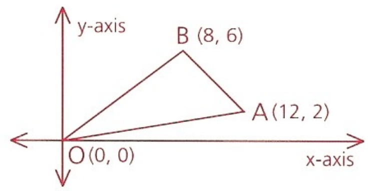
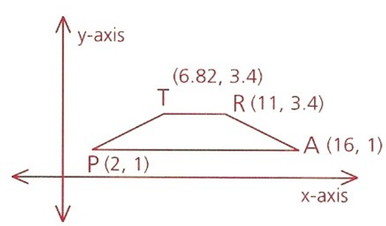
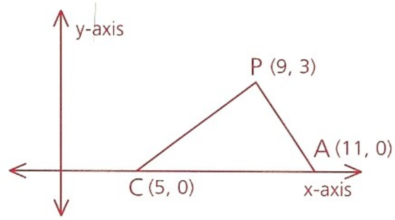
7 How much greater than the x-coordinate of the midpoint of $\overline{OA}$ is the x-coordinate of the midpoint of $\overline{AB}$?



8 In the graph, if a perpendicular is drawn from T to $\overleftrightarrow{PA}$, what will the coordinates of the point where the perpendicular intersects $\overleftrightarrow{PA}$ be?



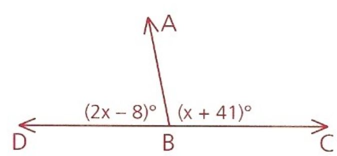
9 If $\triangle CAP$ is slid along the x-axis until C is at (11, 0), what will the new coordinates of P be?



10 A fifth point, E, is located on the diagram so that $m\angle EBC = \sqrt{x} + 83$.

a Is $\overleftrightarrow{AB}$ perpendicular to $\overleftrightarrow{DC}$?

b What do we know about $\overleftrightarrow{AB}$ and $\overleftrightarrow{BE}$?



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Homework Day 2

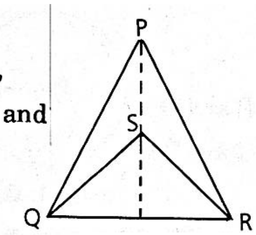
Problem Set B

Remember, the equidistance theorems will help you write a concise proof.

11 Draw isosceles $\triangle PQR$, with P the vertex. Draw the bisectors of the base angles and label their point of intersection S. Prove that $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$. (Hint: Use Theorem 24.)

11 Given: Isos $\triangle PQR$,
$\overline{QS}$ bis $\angle Q$, and
$\overline{RS}$ bis $\angle R$.

Prove: $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$



1 Isos $\triangle PQR$

2 $\overline{PQ} \cong \overline{PR}$

3 $\angle Q \cong \angle R$

4 $\overline{QS}$ bis $\angle Q$.

5 $\overline{RS}$ bis $\angle R$.

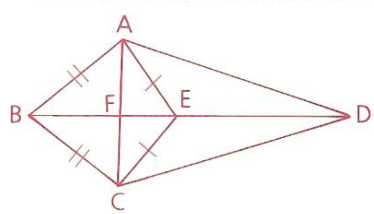
6 $\angle SQR \cong \angle SRQ$

7 $\overline{SQ} \cong \overline{SR}$

8 $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$

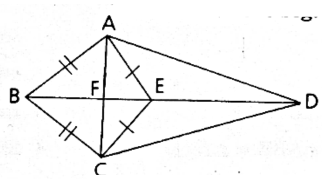
12 Given: $\overline{AB} \cong \overline{BC}$,
$\overline{AE} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{DC}$ (Hint: This can be done in four steps.)



12 Given: $\overline{AB} \cong \overline{BC}$
$\overline{AE} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{DC}$



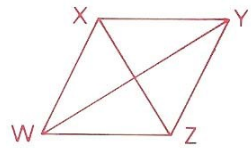
1 $\overline{AB} \cong \overline{BC}$

2 $\overline{AE} \cong \overline{EC}$

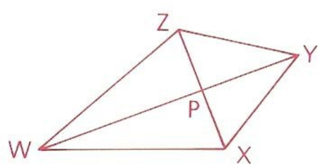
3 $\overline{BE} \perp$ bis $\overline{AC}$.

4 $\overline{AD} \cong \overline{DC}$

13 Given: $\overline{WY}$ and $\overline{XZ}$ $\perp$ bis. each other.
Prove: $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$ (that is, WXYZ is a rhombus)



14 Given: $\overline{WX} \cong \overline{WZ}$, $\overline{XY} \cong \overline{YZ}$ (WXYZ is a *kite*.)
Prove: $\triangle$WPZ is a right $\triangle$.



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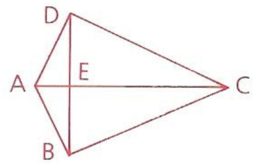
15 Given: $\angle ADC$ and $\angle ABC$ are right $\angle$s.
$\overline{AB} \cong \overline{AD}$

Conclusion: $\overleftrightarrow{AC} \perp$ bis. $\overline{BD}$



16 Prove: The median to the base of an isosceles triangle is also an altitude. (Prove this without using congruent triangles.)

17 Given: F is the midpt. of $\overline{BC}$.
$\overline{DB} \cong \overline{EC}$,
$\overline{DB} \perp \overline{DF}$,
$\overline{EC} \perp \overline{EF}$

Conclusion: $\overleftrightarrow{AF} \perp \overleftrightarrow{BC}$