

Name *Student*
 Adv Geo - *5*

AMDG

4.4: Equidistance Theorem

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Objective

After studying this section, you will be able to

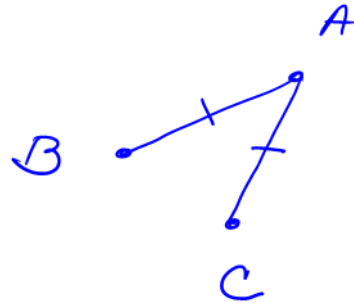
- Recognize the relationship between equidistance and perpendicular bisection

Part One: Introduction

In geometry, the term distance has a special meaning.

Definition The *distance* between two objects is the length of the shortest path joining them.

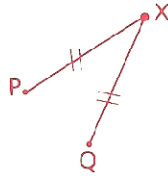
Postulate A line segment is the shortest path between two points.



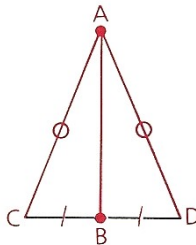
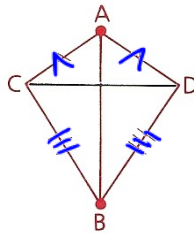
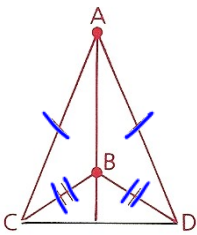
The distance between points R and S is the length of \overline{RS} , or RS.



If two points P and Q are the same distance from a third point X, then X is said to be *equidistant* from P and Q.



$\overline{PX} \cong \overline{XQ}$
 means that
 X is equidistant from P and Q.



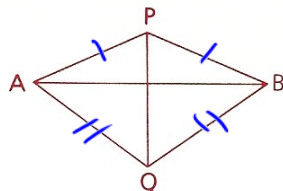
You should recall many problems with diagrams resembling those above. These diagrams have something in common. In each, both point A and point B are equidistant from the endpoints C and D of \overleftrightarrow{CD} . In each case, you could prove that \overleftrightarrow{AB} is the **perpendicular bisector** of \overline{CD} just by using the following definition and theorem.

Definition The **perpendicular bisector** of a segment is the line that bisects and is perpendicular to the segment.

Theorem 24 *If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.*

Given: $\overline{PA} \cong \overline{PB}$,
 $\overline{QA} \cong \overline{QB}$

Prove: \overleftrightarrow{PQ} is the \perp bisector of \overline{AB} .

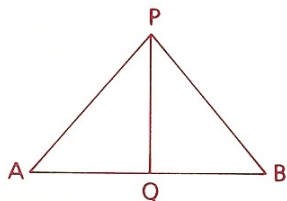


For a proof of Theorem 24, see sample problem 2 in Section 4.3.

Theorem 25 *If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.*

Given: \overleftrightarrow{PQ} is the \perp bisector of \overline{AB} .

Prove: $\overline{PA} \cong \overline{PB}$



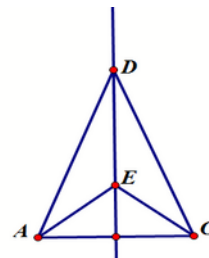
Example

$$\overline{PA} \cong \overline{PB}$$

$$\overline{QA} \cong \overline{QB}$$

$$\overline{PQ} \perp \text{bis } \overline{AB} \quad (= \text{dist} \Rightarrow \perp \text{bis})$$

$m \overline{DC} = 5.80$ cm
 $m \overline{DA} = 5.80$ cm
 $m \overline{EC} = 2.70$ cm
 $m \overline{AE} = 2.70$ cm

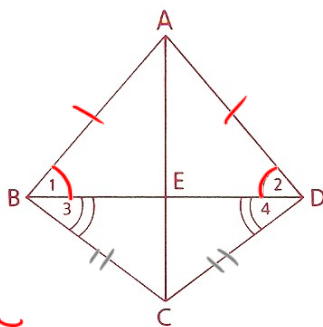


Part Two: Sample Problems

Problem 1

Given: $\angle 1 \cong \angle 2$,
 $\angle 3 \cong \angle 4$

Prove: $\overleftrightarrow{AE} \perp \text{bis. } \overline{BD}$



1. $\angle 1 \cong \angle 2$

2. $\overline{AB} \cong \overline{AD}$

3. $\angle 3 \cong \angle 4$

4. $\overline{BC} \cong \overline{DC}$

5. $\overleftrightarrow{AE} \perp \text{bis } \overline{BD}$

1. Given

2. $\triangle \Rightarrow \triangle$

3. Given

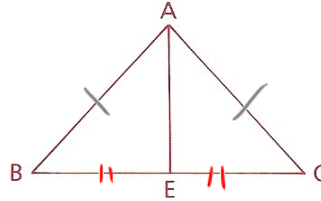
4. $\triangle \Rightarrow \triangle$

5. $= \text{dist} \Rightarrow \perp \text{bis}$

4.4: Equidistance Theorem

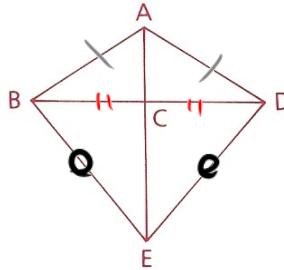
Problem 2 Prove: The line joining the vertex of an isosceles triangle to the midpoint of the base is perpendicular to the base.

Given: $\triangle ABC$ is isosceles, with $\overline{AB} \cong \overline{AC}$.
 E is the midpoint of \overline{BC} .
 Prove: $\overleftrightarrow{AE} \perp \overline{BC}$



- | | |
|--|-----------------------------------|
| 1. $\triangle ABC$ isos, $\overline{AB} \cong \overline{AC}$ | 1. Given |
| 2. E mdpt BC | 2. Given |
| 3. $\overline{BE} \cong \overline{EC}$ | 3. mdpt $\Rightarrow \cong$ segs |
| 4. $\overleftrightarrow{AE} \perp \overline{BC}$ | 4. = dist $\Rightarrow \perp$ bis |

Problem 3 Given: $\overline{AB} \cong \overline{AD}$,
 $\overline{BC} \cong \overline{CD}$
 Conclusion: $\overline{BE} \cong \overline{ED}$



- | | |
|--|-------------------------------------|
| 1. $\overline{AB} \cong \overline{AD}$ | 1. Given |
| 2. $\overline{BC} \cong \overline{CD}$ | 2. Given |
| 3. $\overleftrightarrow{AC} \perp$ bis \overline{BD} | 3. = dist $\Rightarrow \perp$ bis |
| 4. $\overline{BE} \cong \overline{ED}$ | 4. \perp bis \Rightarrow = dist |

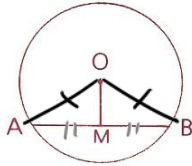
Homework Day 1

Part Three: Problem Sets

Problem Set A

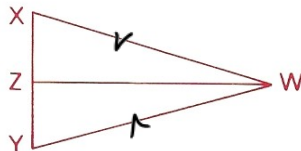
As you work on these proofs, see if the equidistance theorems apply; they can save you a lot of work.

- 1 Given: $\odot O$; M is the midpt. of \overline{AB} .
 Conclusion: $\overline{OM} \perp \overline{AB}$ (Hint: Draw two auxiliary lines.)



- | | |
|---|------------------------------------|
| 1. $\odot O$ | 1. GIVEN |
| 2. Draw \overline{OA} & \overline{OB} | 2. Aux |
| 3. $\overline{OA} \cong \overline{OB}$ | 3. $\odot \Rightarrow \cong$ radii |
| 4. M midpt \overline{AB} | 4. Given |
| 5. $\overline{AM} \cong \overline{MB}$ | 5. midpt $\Rightarrow \cong$ segs |
| 6. $\overline{OM} \perp \overline{AB}$ | 6. = dist $\Rightarrow \perp$ bis |

- 2 Given: $\overleftrightarrow{WZ} \perp$ bis. \overline{XY}
 Prove: $\triangle WXY$ is isosceles. (Hint: This proof can be written in three steps by using Theorem 25.)



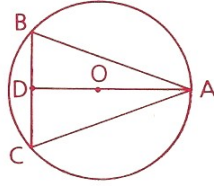
- | | |
|--|--|
| 1. $\overleftrightarrow{WZ} \perp$ bis \overline{XY} | 1. Given |
| 2. $\overline{WX} \cong \overline{WY}$ | 2. \perp bis \Rightarrow = dist |
| 3. $\triangle WXY$ isos | 3. \cong segs \Rightarrow isos \triangle |

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4.4: Equidistance Theorem

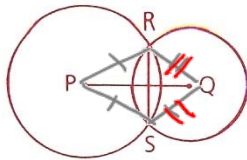
3 Given: $\odot O$, $\overline{AB} \cong \overline{AC}$

Conclusion: $\overleftrightarrow{AD} \perp \text{bis. } \overline{BC}$ (Hint: Show that A and O are each equidistant from B and C.)



4 Given: $\odot P$ and $\odot Q$

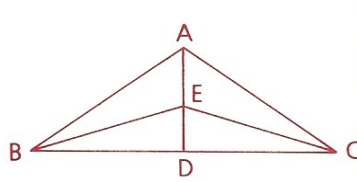
Prove: $\overleftrightarrow{PQ} \perp \text{bis. } \overline{RS}$



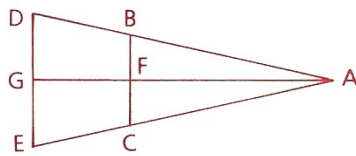
1. $\odot P$ & $\odot Q$
2. Draw \overline{PR} , \overline{PS} , \overline{QR} , & \overline{QS}
3. $\overline{PR} \cong \overline{PS}$ & $\overline{QR} \cong \overline{QS}$
4. $\overleftrightarrow{PQ} \perp \text{bis. } \overline{RS}$

1. Given
2. Aux
3. $\odot \Rightarrow \cong \text{Radii}$
4. $\cong \text{dist} \Rightarrow \perp \text{bis}$

- 5 Given: $\overleftrightarrow{AD} \perp \text{bis. } \overline{BC}$
Prove: $\triangle ABE \cong \triangle ACE$

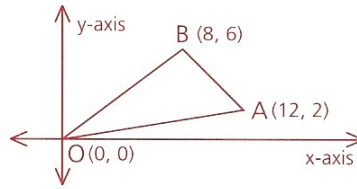


- 6 Given: $\overleftrightarrow{AG} \perp \text{bis. } \overline{BC}$,
 $\overleftrightarrow{AG} \perp \text{bis. } \overline{DE}$
Conclusion: $\overline{BD} \cong \overline{CE}$

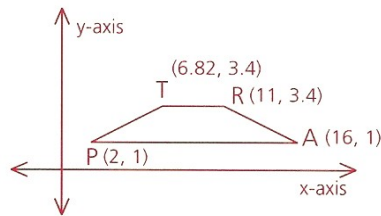


4.4: Equidistance Theorem

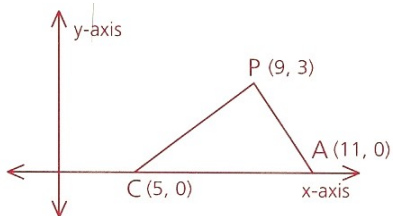
- 7 How much greater than the x-coordinate of the midpoint of \overline{OA} is the x-coordinate of the midpoint of \overline{AB} ?



- 8 In the graph, if a perpendicular is drawn from T to \overleftrightarrow{PA} , what will the coordinates of the point where the perpendicular intersects \overleftrightarrow{PA} be?

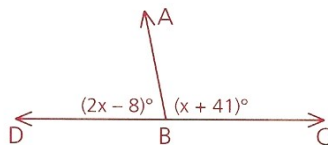


- 9 If $\triangle CAP$ is slid along the x-axis until C is at (11, 0), what will the new coordinates of P be?



- 10 A fifth point, E, is located on the diagram so that $m\angle EBC = \sqrt{x} + 83$.

- a Is \overleftrightarrow{AB} perpendicular to \overleftrightarrow{DC} ?
 b What do we know about \overleftrightarrow{AB} and \overleftrightarrow{BE} ?



Homework Day 2

Problem Set B

Remember, the equidistance theorems will help you write a concise proof.

- 11 Draw isosceles $\triangle PQR$, with P the vertex. Draw the bisectors of the base angles and label their point of intersection S. Prove that $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$. (Hint: Use Theorem 24.)

- 11 Given: Isos $\triangle PQR$,
 \overline{QS} bis $\angle Q$, and
 \overline{RS} bis $\angle R$.

Prove: $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$

1 Isos $\triangle PQR$

2 $\overline{PQ} \cong \overline{PR}$

3 $\angle Q \cong \angle R$

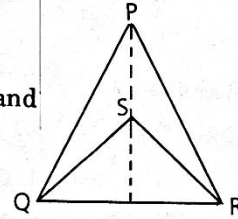
4 \overline{QS} bis $\angle Q$.

5 \overline{RS} bis $\angle R$.

6 $\angle SQR \cong \angle SRQ$

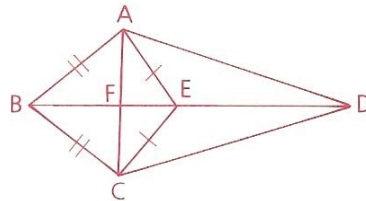
7 $\overline{SQ} \cong \overline{SR}$

8 $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$



- 12 Given: $\overline{AB} \cong \overline{BC}$,
 $\overline{AE} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{DC}$ (Hint: This can be done in four steps.)



- 12 Given: $\overline{AB} \cong \overline{BC}$
 $\overline{AE} \cong \overline{EC}$

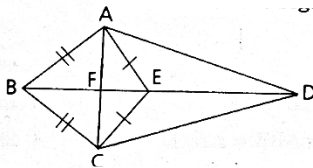
Prove: $\overline{AD} \cong \overline{DC}$

1 $\overline{AB} \cong \overline{BC}$

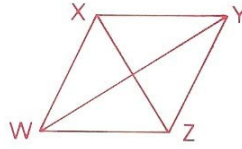
2 $\overline{AE} \cong \overline{EC}$

3 $\overline{BE} \perp \text{bis } \overline{AC}$.

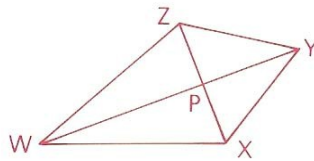
4 $\overline{AD} \cong \overline{DC}$



- 13 Given: \overline{WY} and $\overline{XZ} \perp$ bis. each other.
Prove: $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$ (that is,
 $WXYZ$ is a rhombus)



- 14 Given: $\overline{WX} \cong \overline{WZ}$, $\overline{XY} \cong \overline{YZ}$
(WXYZ is a kite.)
Prove: $\triangle WPZ$ is a right \triangle .



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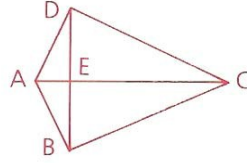
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Nov 13

4.4: Equidistance Theorem

15 Given: $\angle ADC$ and $\angle ABC$ are right \angle s.

$$\overline{AB} \cong \overline{AD}$$

Conclusion: $\overleftrightarrow{AC} \perp \text{bis. } \overline{BD}$



16 Prove: The median to the base of an isosceles triangle is also an altitude. (Prove this without using congruent triangles.)

17 Given: F is the midpt. of \overline{BC} .

$$\overline{DB} \cong \overline{EC},$$

$$\overline{DB} \perp \overline{DF},$$

$$\overline{EC} \perp \overline{EF}$$

Conclusion: $\overleftrightarrow{AF} \perp \overleftrightarrow{BC}$

