Name Student

Adv Geo - 7

AMDG

4.3: A Right Angle Theorem

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Objective

After studying this section, you will be able to

- Apply one way of proving that two angles are right angles

Part One: Introduction

Proving that lines are perpendicular depends on proving that they form right angles. For this reason, it is useful to know some ways of proving that angles are right angles. The following theorem will provide you with one such way.

st∠ ⇒ 180°

$2x = 180$

$x = 90$

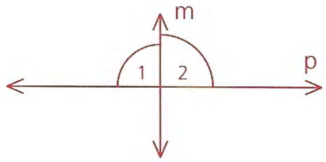
90 ⇒ rt∠

(≅ & supp∠s ⇒ rt∠s)

Theorem 23 ***If two angles are both supplementary and congruent, then they are right angles.***

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 1$ and $\angle 2$ are right angles.



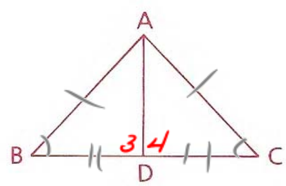
Proof: Since $\angle 1$ and $\angle 2$ form a straight angle (line p), they are supplementary. Therefore, $m\angle 1 + m\angle 2 = 180$. Since $\angle 1 \cong \angle 2$, we can use substitution to get the equation $m\angle 1 + m\angle 1 = 180$, or $m\angle 1 = 90$. Thus, $\angle 1$ is a right angle, and so is $\angle 2$.

In the rest of this book, we shall assume that whenever two angles (such as $\angle 1$ and $\angle 2$ in the diagram for Theorem 23) form a straight angle, the two angles are supplementary. No formal statement of this fact will be necessary.

Sample Problems

Problem 1 *Given:* $\overline{AB} \cong \overline{AC}$, $\overline{BD} \cong \overline{CD}$

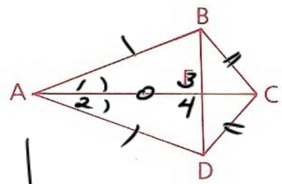
Conclusion: $\overline{AD}$ *is an altitude.*



1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\overline{BD} \cong \overline{CD}$	2. Given
3. $\angle B \cong \angle C$	3. △ ⇒ △
4. $\triangle ABD \cong \triangle ACD$	4. SAS (132)
5. $\angle 3 \cong \angle 4$	5. CPCTC
6. ∠3 supp ∠4	6. st∠ ⇒ supp∠s
7. ∠3 & ∠4 rt∠	7. ≅ & supp∠s ⇒ rt∠s
8. $\overline{AD}$ alt.	8. rt∠s ⇒ alt

Problem 2 *Given:* $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{CD}$

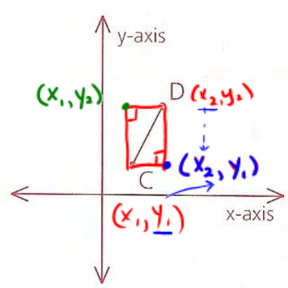
Prove: $\overleftrightarrow{AC}$ *is the* $\perp$ *bisector of* $\overline{BD}$.



Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{BC} \cong \overline{DC}$	2. Given
3. $\overline{AC} \cong \overline{AC}$	3. Ref
4. $\triangle ABC \cong \triangle ADC$	4. SSS (1 2 3)
5. $\angle 1 \cong \angle 2$	5. CPCTC (4)
6. $\overline{AE} \cong \overline{AE}$	6. Ref
7. $\triangle ABE \cong \triangle ADE$	7. SAS (1 5 6)
8. $\angle 3 \cong \angle 4$	8. CPCTC (7)
9. $\angle 3$ supp $\angle 4$.	9. st∠ ⇒ suppl ∠s
10. $\angle 3$ & $\angle 4$ rt∠s	10. ≅ & suppl ∠s ⇒ rt∠s
11. $\overline{AC} \perp \overline{BD}$	11. rt∠ ⇒ ⊥
12. $\overline{BE} \cong \overline{ED}$	12. CPCTC (7)
13. $\overline{AC}$ bis $\overline{BD}$	13. ≅ seg ⇒ bis
14. $\overline{AC} \perp$ bis $\overline{BD}$	14. ⊥ & bis ⇒ ⊥ bis (11,13)

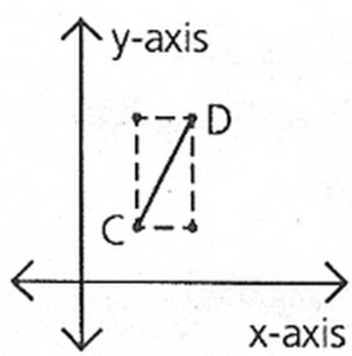
Problem Set B

8 If $\overline{CD}$ is the hypotenuse of a right triangle CAD and A has integral coordinates, find all possible values of the coordinates of A.



8 If $\overline{CD}$ is the hypotenuse, A must be a right angle.

∴ A is at ~~(1, 3) or (2, 1)~~.



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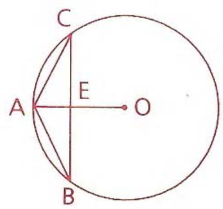
Adv Geo - 4.3: A Right Angle Theorem

9 Given: ⊙O,
$\angle B \cong \angle C$
Conclusion: $\overline{AO} \perp \overline{BC}$

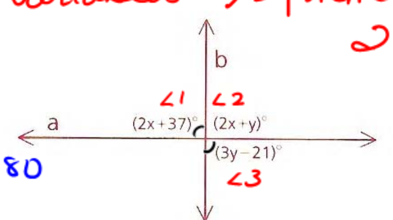


12 Prove that if two circles intersect at two points, A and B, then the line joining the circles' centers is perpendicular to $\overline{AB}$.

13 Prove that the supplement of a right angle is a right angle.

Problem Set C

14 Is b perpendicular to a? Justify your answer.



2 variables ⇒ system ∴ 2 eq.

vert∠s ⇒ ≅∠s

$2x+37 = 3y-21$

$2x-3y = -58$

suppl∠s = 180°

$2x+37+2x+y = 180$

$4x + y = 143$

$\begin{cases} 3(4x+y=143) \\ 2x-3y=-58 \end{cases} \rightarrow \begin{cases} 12x+3y=429 \\ 2x-3y=-58 \end{cases}$

$14x = 371$

$x = 26.5$

$4(26.5)+y = 143$

$106 + y = 143$

$y = 37$

Is a ⊥ b? $2x+y = ?$

$2(26.5)+(37) = ?$

$53 + 37 = 90$ & 90 ⇒ ⊥ ∴ yes a ⊥ b

Name Ms. Kresovic

Adv Geo - 4.3: A Right Angle Theorem

15 The ratio of the complements of two angles is 3:2, and the ratio of their supplements is 9:8. Find the two original angles.

		comp	supp
$\angle A$	x	$90-x$	$180-x$
$\angle B$	y	$90-y$	$180-y$

ORIG $\angle$s
$45°$
$60°$

$$\frac{90-x}{90-y} = \frac{3}{2} \qquad \frac{180-x}{180-y} = \frac{9}{8}$$

$2(90)-2x = 3(90)-3y$ & $8(180-x) = 9(180-y)$

$-2(90) + 3y \quad -2(90) + 3y$

$-2x+3y=90$

$16(90)-8x = 18(90)-9y$

$-16(90) \quad +9y \quad -16(90) \quad +9y$

$-8x+9y = 2(90)$

$$\begin{cases} -3(-2x+3y=90) \\ -8x+9y=2(90) \end{cases}$$

$$\begin{cases} 6x - 9y = -3(90) \\ -8x + 9y = 2(90) \end{cases}$$

$-2x = -1(90)$

$x = 45$

$-2(45)+3y = 90$

$+90 \quad +90$

$3y = 180$

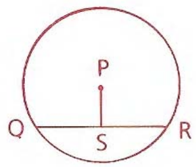
$y = 60.$

16 To the nearest second, what is the first time after 7:00 that the hands of a clock form a right angle?

Homework

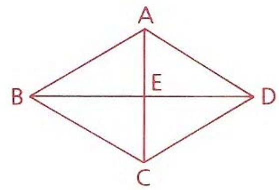
Problem Set A

1 Given: ⊙P;
S is the midpt. of $\overline{QR}$.
Prove: $\overline{PS} \perp \overline{QR}$

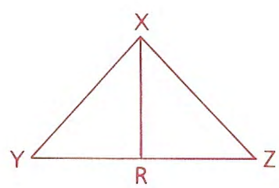


2 Prove: The angle bisector of the vertex angle of an isosceles triangle is perpendicular to the base.

3 Given: $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$
(that is, ABCD is a rhombus)
Conclusion: $\overline{AC} \perp \overline{BD}$
(Hint: Use a detour.)

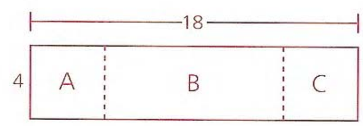


4 Given: $\overrightarrow{XR}$ bisects $\angle YXZ$.
$\angle Y \cong \angle Z$
Conclusion: $\overline{XR}$ is an altitude.

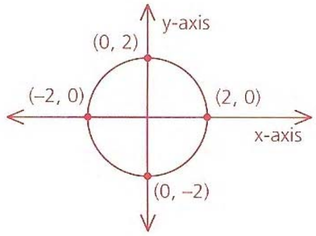


5 A diameter of a circle has endpoints with coordinates (2, 6) and (−4, 10). Find the coordinates of the center of the circle.

6 If squares A and C are folded across the dotted segments onto B, find the area of B that will *not* be covered by either square.



7 Find, to the nearest tenth, the area of the circle.



10 Prove that the median to the base of an isosceles triangle is also an altitude to the base.

11 Given: $\overleftrightarrow{PR}$ bisects $\overline{QS}$.
$\angle RQT \cong \angle RST$
Prove: $\overline{QS} \perp \overline{PR}$