NAME Stu Dent
Adv Geo period 5

AMDG
Lines in the Plane – Chapter 4
4.1: Detours & Midpoints

Ms. Kresovic
Date F 30 OCT 15

Objectives

After studying this section, you will be able to

- Use detours in proofs
- Apply the midpoint formula ✓

Part One: Introduction

Detour Proofs

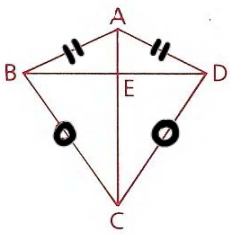
To solve some problems, it is necessary to prove more than one pair of triangles congruent. We call the proofs we use in such cases ***detour proofs***.

Analyze carefully the following example.

Example

Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{CD}$

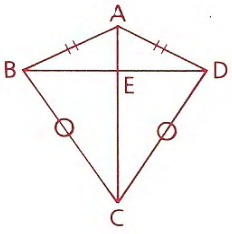
Prove: $\triangle ABE \cong \triangle ADE$



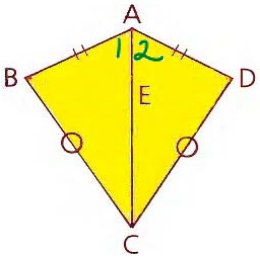
Notice that of the given information only $\overline{AB} \cong \overline{AD}$ seems to be usable. There does not seem to be enough information to prove that $\triangle ABE \cong \triangle ADE$. We must therefore prove something else first, taking a little detour to pick up the congruent parts we need.

Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{CD}$

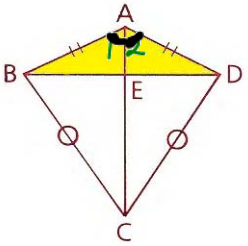
Prove: $\triangle ABE \cong \triangle ADE$



1st set of ≅ Δs:



2nd set of ≅ Δs:



Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$ & $\overline{BC} \cong \overline{CD}$	1. GIVEN
2. $\overline{AC} \cong \overline{AC}$	2. Ref
3. $\triangle CBA \cong \triangle CDA$	3. SSS (1 1 2)
4. $\angle 1 \cong \angle 2$	4. CPCTC
5. $\overline{AE} \cong \overline{AE}$	5. Ref
6. $\triangle ABE \cong \triangle ADE$	6. SAS (1 4 5)

Whenever you are asked to prove that triangles or parts of triangles are congruent and you suspect that a detour may be needed, use the following procedure.

Procedure for Detour Proofs

1 Determine which triangles you must prove to be congruent to reach the required conclusion. (In the preceding example, these are △ABE and △ADE.)

2 Attempt to prove that these triangles are congruent. If you cannot do so for lack of enough given information, take a detour (steps 3–5 below).

3 Identify the parts that you must prove to be congruent to establish the congruence of the triangles. (Remember that there are many ways to prove that triangles are congruent. Consider them all.)

4 Find a pair of triangles that
   (a) You can readily prove to be congruent
   (b) Contain a pair of parts needed for the main proof (parts identified in step 3)

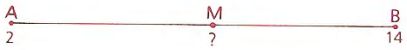
5 Prove that the triangles found in step 4 are congruent.

6 Use CPCTC and complete the proof planned in step 1.

The Midpoint Formula

In some coordinate-geometry problems, you will need to locate the midpoint of a line segment. A method of doing so is suggested by the following example.

Example *On the number line below, the coordinate of* A *is* 2 *and the coordinate of* B *is* 14. *Find the coordinate of* M, *the midpoint of* $\overline{AB}$.



There are several ways of solving this problem. One of these is the *averaging* process (the average of two numbers is equal to half their sum). We will use x_m (read "x sub m") to represent the coordinate of M.

$$x_m = \frac{2 + 14}{2}$$
$$= \frac{16}{2} = 8$$

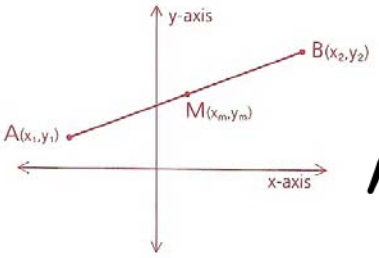
Check: AM = 8 − 2 = 6
MB = 14 − 8 = 6

Therefore, 8 is the coordinate of M.

We can apply the averaging process to develop a formula, called the ***midpoint formula***, that can be used to find the coordinates of the midpoint of any segment in the coordinate plane. The proof of this theorem is left to you.

Theorem 22 ***If*** $A = (x_1, y_1)$ ***and*** $B = (x_2, y_2)$, ***then the midpoint*** $M = (x_m, y_m)$ ***of*** $\overline{AB}$ ***can be found by using the midpoint formula:***

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



$(\bar{x}, \bar{y})$

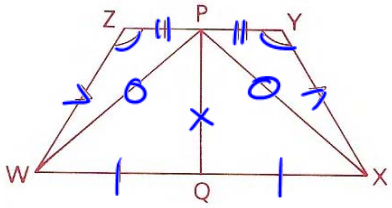
C (11, 9)
D (5, 31)

M (8, 20)

Fun applet: http://www.mathopenref.com/coordmidpoint.html

Part Two: Sample Problems

Problem 1 Given: $\overleftrightarrow{PQ}$ *bisects* $\overline{YZ}$.
Q *is the midpt. of* $\overline{WX}$.
$\angle Y \cong \angle Z$, $\overline{WZ} \cong \overline{XY}$
Conclusion: $\angle WQP \cong \angle XQP$



Proof

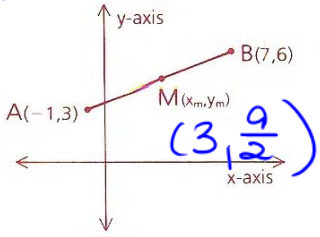
DETOUR

Statements	Reasons
1 $\overleftrightarrow{PQ}$ bisects $\overline{YZ}$.	1 Given
2 $\overline{ZP} \cong \overline{PY}$	2 If a line bisects a segment, then it divides the segment into two ≅ segments.
3 $\angle Z \cong \angle Y$	3 Given
4 $\overline{WZ} \cong \overline{XY}$	4 Given
5 $\triangle ZWP \cong \triangle YXP$	5 SAS (2, 3, 4)
6 $\overline{WP} \cong \overline{PX}$	6 CPCTC
7 Q is the midpt. of $\overline{WX}$.	7 Given
8 $\overline{WQ} \cong \overline{QX}$	8 mdpt ⇒ ≅ segs
9 $\overline{PQ} \cong \overline{PQ}$	9 ref
10 $\triangle WQP \cong \triangle XQP$	10 SSS (6 8 9)
11 $\angle WQP \cong \angle XQP$	11 CPCTC

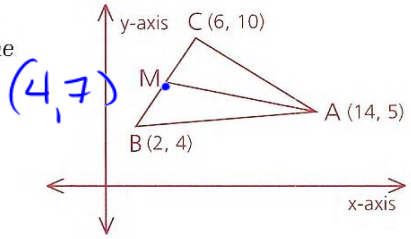
SAS
ASA
SSS
HL

Problem 2 *Find the coordinates of* M, *the midpoint of* $\overline{AB}$.

Solution



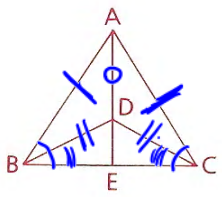
Problem 3 *In* $\triangle ABC$, *find the coordinates of the point at which the median from* A *intersects* $\overline{BC}$.



Solution Since a median is drawn to a midpoint, use the midpoint formula to find the midpoint M of $\overline{BC}$.

Problem Set C

16 Given: $\overline{AB} \cong \overline{AC}$;
$\overrightarrow{BD}$ bisects $\angle ABE$.
$\overrightarrow{CD}$ bisects $\angle ACE$.
Conclusion: $\overline{AE}$ bisects $\overline{BC}$.

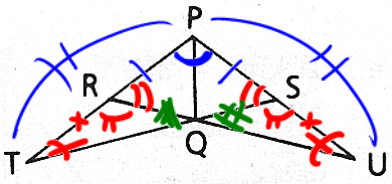


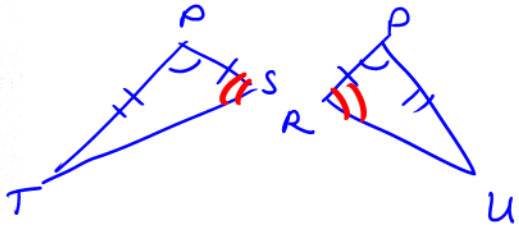
Statements	*Reasons*
1 $\overline{AB} \cong \overline{AC}$	1 Given
2 $\overrightarrow{BD}$ bis $\angle ABE$.	2 Given
3 $\overrightarrow{CD}$ bis $\angle ACE$.	3 Given
4 $\angle ABC \cong \angle ACB$	4 $\angle\angle \Rightarrow \triangle\triangle$ (1)
5 $\angle DBE \cong \angle DCE$	5 $\div$ (2 3 4)
6 $\overline{BD} \cong \overline{CD}$	6 $\triangle\triangle$ $\angle\angle$ (5)
7 $\overline{AD} \cong \overline{AD}$	7 Ref
8 $\triangle ADB \cong \triangle ADC$	8 SSS (1, 6, 7)
9 $\angle BAD \cong \angle CAD$	9 CPCTC (8)
10 $\overline{AE} \cong \overline{AE}$	10 Ref
11 $\triangle AEB \cong \triangle AEC$	11 SAS
12 $\overline{BE} \cong \overline{EC}$	12 CPCTC (11)
13 $\overline{AE}$ bis $\overline{BC}$.	13 $2 \cong$ segs $\Rightarrow$ bis

17 Given: $\overline{PT} \cong \overline{PU}$

$\overline{PR} \cong \overline{PS}$

Prove: $\overrightarrow{PQ}$ bis $\angle RPS$.



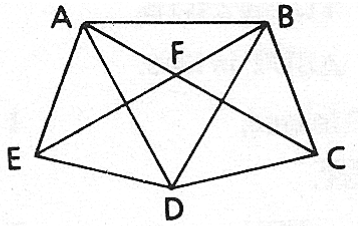


1	$\overline{PT} \cong \overline{PU}, \overline{PR} \cong \overline{PS}$	1	Given
2	$\angle TPS \cong \angle UPR$	2	Ref
3	$\triangle TPS \cong \triangle UPR$	3	SAS (1 2 1)
4	$\angle PRU \cong \angle PST$	4	CPCTC (3)
5	$\angle T \cong \angle U$	5	CPCTC (3)
6	$\angle TRQ$ supp $\angle PRU$	6	StL ⇒ suppLs
7	$\angle USQ$ supp $\angle PST$	7	StL ⇒ suppLs
8	$\angle TRQ \cong \angle USQ$	8	Ls suppto ≅ Ls ⇒ ≅ Ls (5 6 7)
9	$\overline{TR} \cong \overline{US}$	9	Subtraction (1 8 1)
10	$\triangle TRQ \cong \triangle USQ$	10	ASA (5 9 8)
11	$\overline{RQ} \cong \overline{SQ}$	11	CPCTC
12	$\triangle PRQ \cong \triangle PSQ$	12	SAS (1 4 11)
13	$\angle RPQ \cong \angle SPQ$	13	CPCTC (12)
14	$\overrightarrow{PQ}$ bis $\angle RPS$.	14	2 ≅ Ls ⇒ bis

18 Given: $\overline{AD} \cong \overline{DB}$

$\overline{AE} \cong \overline{BC}$

$\overline{CD} \cong \overline{ED}$



Prove: $\triangle AFB$ is isos.

1	$\overline{AD} \cong \overline{DB}, \overline{AE} \cong \overline{BC}$	1
2	$\overline{CD} \cong \overline{ED}$	2
3	$\triangle ADE \cong \triangle BDC$	3
4	$\angle ADE \cong \angle BDC$	4
5	$\angle ADB \cong \angle ADB$	5
6	$\angle EDB \cong \angle CDA$	6
7	$\triangle EDB \cong \triangle CDA$	7
8	$\overline{EB} \cong \overline{CA}$	8
9	$\overline{AB} \cong \overline{AB}$	9
10	$\triangle ABE \cong \triangle BAC$	10
11	$\angle ABF \cong \angle BAF$	11
12	$\overline{BF} \cong \overline{FA}$	12
13	$\triangle ABF$ is isos.	13

NAME **Lines in the Plane – Chapter 4** Ms. Kresovic

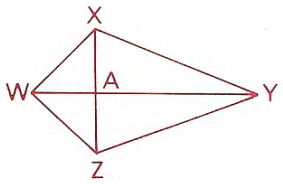
Adv Geo period ___ 4.1: Detours & Midpoints Date

Homework

Problem Set A

1 Given: $\overline{WX} \cong \overline{WZ}$, $\overline{XY} \cong \overline{ZY}$

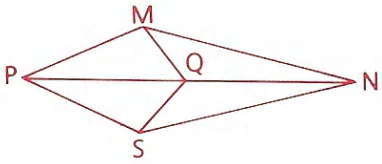
Prove: $\triangle XAY \cong \triangle ZAY$



Statements	Reasons
1 $\overline{WX} \cong \overline{WZ}$	1 Given
2 $\overline{XY} \cong \overline{ZY}$	2 Given
3 _____	3 Reflexive Property
4 $\triangle WXY \cong \triangle WZY$	4 _____
5 $\angle XYW \cong \angle ZYW$	5 _____
6 _____	6 Reflexive Property
7 $\triangle XAY \cong \triangle ZAY$	7 _____

2 Given: $\overline{MN} \cong \overline{NS}$, $\overline{MP} \cong \overline{PS}$

Prove: $\angle MQP \cong \angle SQP$



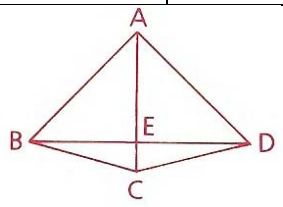
(With the Givens on the same line, 7 steps.)

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

3 Given: A is *equidistant from* B and D (that is, AB = AD).

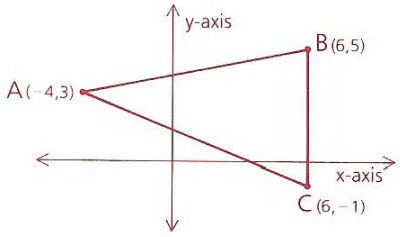
$\overrightarrow{AC}$ bisects $\angle BAD$.

Prove: $\overline{AC}$ bisects $\overline{BD}$.



Statements	Reasons
1. **A is =dist from B and D; $\overline{AB} \cong \overline{AD}$.**	1.
2. **$\overrightarrow{AC}$ bis $\angle BAD$.**	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

4 Find the coordinates of the midpoint of each side of $\triangle ABC$.



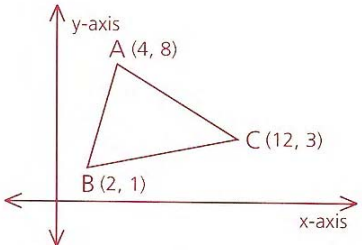
Make sure you communicate to the reader what you are doing. Use subtitles or subheadings, like this:

Mdpt AB | Mdpt BC | Mdpt CA

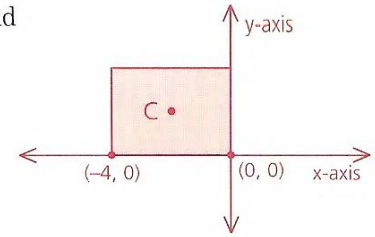
5 Find the coordinates of the point where the median from A intersects $\overline{BC}$.

Median ⇒ _____

Find _____



8 If the shaded square has center at C and an area of $A_{\square}$, find C and $A_{\square}$.

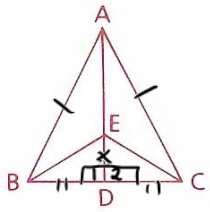


Find the area of the square | Find C

Problem Set B

9 Given: $\triangle ABC$ is isosceles, with base $\overline{BC}$.
$\overline{AD} \perp \overline{BC}$

Prove: $\triangle BEC$ is isosceles.



11 steps, each given on a separate line.
Hint: Number the angles

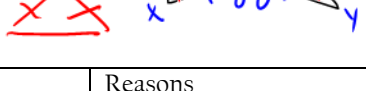
Statements	Reasons
1. $\overline{AD} \perp \overline{BC}$	1. Given
2. $\angle 1$ & $\angle 2$ Nt Ls	2. $\perp \Rightarrow$ Nt Ls
3. $\triangle ABC$ is isos, base $\overline{BC}$.	3. Given
H 4. $\overline{AB} \cong \overline{AC}$	4. Isos $\triangle \Rightarrow 2 \cong$ sds
L 5. $\overline{AD} \cong \overline{AD}$	5. Ref
6. $\triangle ABD \cong \triangle ACD$	6. HL (2 45)
7. $\overline{BD} \cong \overline{CD}$	7. CPCTC
8. $\angle 1 \cong \angle 2$	8. Nt $\angle$s $\Rightarrow \cong \angle$s
9. $\overline{ED} \cong \overline{ED}$	9. Ref
10. $\triangle DEB \cong \triangle DEC$	10. SAS
11. $\overline{BE} \cong \overline{CE}$	11. CPCTC

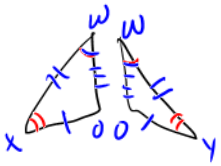
12 $\triangle BEC$ Isos

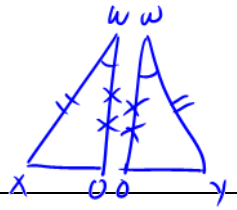
12. 2 $\cong$ sds $\Rightarrow$ Isos $\triangle$

10 Given: $\odot O$, $\overline{WX} \cong \overline{WY}$

Prove: $\overleftrightarrow{WZ}$ bisects $\overline{XY}$.





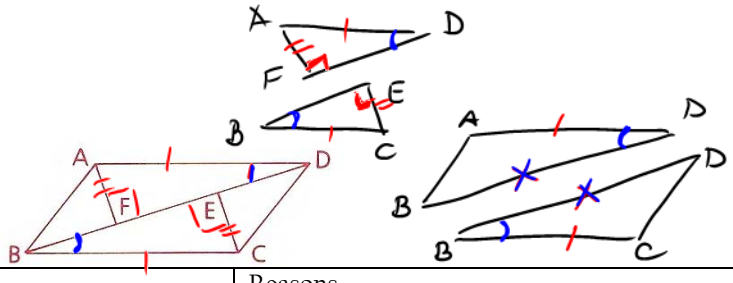


Statements	Reasons
1. $\odot O$	1. GIVEN
2. Draw $\overline{OX}$ and $\overline{OY}$	2. AUX
3. $\overline{OX} \cong \overline{OY}$	3. $\odot \Rightarrow \cong$ rad
4. $\overline{WX} \cong \overline{WY}$	4. Given
5. $\overline{OW} \cong \overline{OW}$	5. Ref
6. $\triangle WXO \cong \triangle WYO$	6. SSS
7. $\angle XWO \cong \angle YWO$, $\angle WOX \cong \angle WOY$	7. CPCTC
8. $\angle OXZ \cong$ 4OYZ	8. $\triangle \Rightarrow \triangle$s
9. $\angle WXZ \cong \angle WYZ$	9. Add
10. $\triangle WXZ \cong \triangle WYZ$	10. ASA
11. $\overline{XZ} \cong \overline{YZ}$	11. CPCTC
12. $\overline{WZ}$ bis $\overline{XY}$	12. $\cong$ seg $\Rightarrow$ bis
13.	13.
14.	14.

ALT:
7. $\angle XWO \cong \angle YWO$
8. $\overline{WZ} \cong \overline{WZ}$
9. $\triangle WXO \cong \triangle WYO$
10. $\overline{XZ} \cong \overline{YZ}$
11. $\overline{WZ}$ bis $\overline{XY}$

11 Given: $\overline{AD} \cong \overline{BC}$, $\overline{AF} \cong \overline{EC}$,
$\overleftrightarrow{BD} \perp \overleftrightarrow{AF}$, $\overleftrightarrow{BD} \perp \overleftrightarrow{EC}$

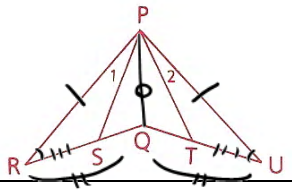
Conclusion: $\overline{AB} \cong \overline{DC}$



Statements	Reasons
1. $\overleftrightarrow{BD} \perp \overleftrightarrow{AF}$ and $\overleftrightarrow{EC}$	1. GIVEN
2. ∠AFD & ∠CEB rt∠s	2. ⊥ ⇒ rt∠s (1)
3. $\overline{AD} \cong \overline{BC}$, $\overline{AF} \cong \overline{EC}$	3. GIVEN
4. △AFD ≅ △CEB	4. HL (2 3 3)
5. ∠ADB ≅ ∠CBD	5. CPCTC
6. $\overline{BD} \cong \overline{DB}$	6. Ref
7. △ADB ≅ △CBD	7. SAS (3 5 6)
8. $\overline{AB} \cong \overline{DC}$	8. CPCTC
9.	9.
10.	10.
11.	11.

12 Given: $\overline{PR} \cong \overline{PU}$,
$\overline{QR} \cong \overline{QU}$,
$\overline{RS} \cong \overline{UT}$

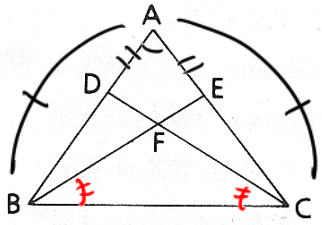
Conclusion: $\angle 1 \cong \angle 2$



Statements	Reasons
1. $\overline{PR} \cong \overline{PU}$, $\overline{QR} \cong \overline{QU}$, & $\overline{RS} \cong \overline{UT}$	1. GIVEN
2. Draw $\overline{PQ}$	2. Aux
3. $\overline{PQ} \cong \overline{PQ}$	3. Ref
4. △PRQ ≅ △PUQ	4. SSS (1 1 3)
5. ∠PRQ ≅ ∠PUQ	5. CPCTC
6. △PRS ≅ △PUT	6. SAS
7. ∠1 ≅ ∠2	7. CPCTC

13 Given: $\overline{AB} \cong \overline{AC}$
$\overline{AD} \cong \overline{AE}$

Prove: $\triangle FBC$ is isos.



1 $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AE}$ — 1 GIVEN

2 $\angle A \cong \angle A$ — 2 Ref

3 $\triangle BAE \cong \triangle CAD$ — 3 SAS

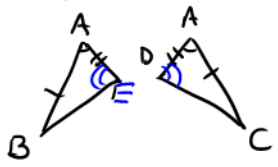
4 $\angle ABE \cong \angle ACD$ — 4 CPCTC

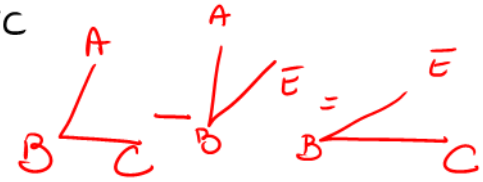
5 $\angle ABC \cong \angle ACB$ — 5 $\measuredangle\measuredangle \Rightarrow \triangle\triangle$

6 $\angle FCB \cong \angle FBC$ — 6 Subtract (5 - 4)

7 $\overline{BF} \cong \overline{FC}$ — 7 $\triangle\triangle \Rightarrow \measuredangle\measuredangle$

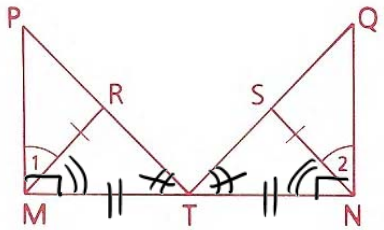
8 $\triangle FBC$ is isos. — 8 2 $\cong$ sds $\Rightarrow$ Isos $\triangle$





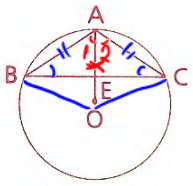
14 Given: T is the midpt. of $\overline{MN}$.
$\angle PMT$ and $\angle QNT$ are right $\angle$s.
$\overline{MR} \cong \overline{SN}$, $\angle 1 \cong \angle 2$

Conclusion: $\angle P \cong \angle Q$



Statements	Reasons
1. **T mdpt of $\overline{MN}$**	1. Given
2. $\overline{MT} \cong \overline{TN}$	2. mdpt $\Rightarrow \cong$ segs
3. $\angle PMT, \angle QNT$ are rt $\angle$s.	3. Given
4. $\angle PMT \cong \angle QNT$	4. rt $\angle$s $\Rightarrow \cong \angle$s
5. $\angle 1 \cong \angle 2$	5. Given
6. $\angle RMT \cong \angle SNT$	6. Subtract
7. $\overline{MR} \cong \overline{SN}$	7. Given
8. $\triangle RMT \cong \triangle SNT$	8. SAS
9. $\angle RTM \cong \angle STN$	9. CPCTC
10. $\triangle PMT \cong \triangle QNT$	10. ASA (9, 2, 4)
11. $\angle P \cong \angle Q$	11. CPCTC
12.	12.
13.	13.
14.	14.

15 Given: $\odot O$, $\angle B \cong \angle C$
Prove: $\overline{AO}$ bisects $\overline{BC}$.



11 steps, each given on a separate line

1. $\angle ABE \cong \angle ACE$	1. Given
2. $\overline{AB} \cong \overline{AC}$	2. $\triangle \Rightarrow$ isos $\triangle$
3. $\odot O$ Draw BO & CO	3. Given Aux
4. $\overline{OB} \cong \overline{OC}$	4. $\odot \Rightarrow \cong$ radii
5. $\overline{AO} \cong \overline{AO}$	5. Ref
6. $\triangle ABO \cong \triangle ACO$	6. SSS (2 4 5)
7. $\angle 1 \cong \angle 2$	7. CPCTC (6)
8. $\overline{AE} \cong \overline{AE}$	8. Ref
9. $\triangle ABE \cong \triangle ACE$	9. SAS (2 7 8)
10. $\overline{BE} \cong \overline{CE}$	10. CPCTC
11. $\overline{AO}$ bis $\overline{BC}$	11. $\cong$ seg $\Rightarrow$ bisection