

NAME _____
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Objectives

After studying this section, you will be able to

- Use detours in proofs
- Apply the midpoint formula

Part One: Introduction

Detour Proofs

To solve some problems, it is necessary to prove more than one pair of triangles congruent. We call the proofs we use in such cases

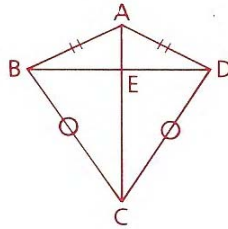
detour proofs.

Analyze carefully the following example.

Example

Given: $\overline{AB} \cong \overline{AD}$,
 $\overline{BC} \cong \overline{CD}$

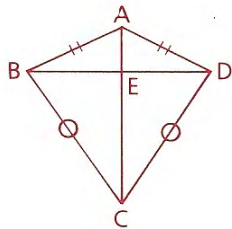
Prove: $\triangle ABE \cong \triangle ADE$



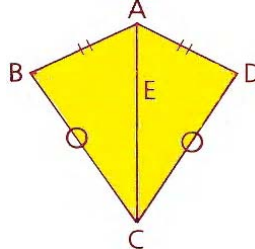
Notice that of the given information only $\overline{AB} \cong \overline{AD}$ seems to be usable. There does not seem to be enough information to prove that $\triangle ABE \cong \triangle ADE$. We must therefore prove something else first, taking a little detour to pick up the congruent parts we need.

Given: $\overline{AB} \cong \overline{AD}$,
 $\overline{BC} \cong \overline{CD}$

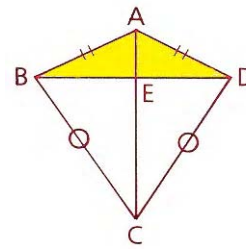
Prove: $\triangle ABE \cong \triangle ADE$



1st set of $\cong \Delta$ s:



2nd set of $\cong \Delta$ s:



Statements	Reasons
1. $\overline{AB} \cong \overline{AD} \ \& \ \overline{BC} \cong \overline{CD}$	1.
2.	2.
3.	3. SSS
4.	4.
5.	5.
6. $\triangle ABE \cong \triangle ADE$	6.

Whenever you are asked to prove that triangles or parts of triangles are congruent and you suspect that a detour may be needed, use the following procedure.

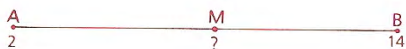
Procedure for Detour Proofs

- 1 Determine which triangles you must prove to be congruent to reach the required conclusion. (In the preceding example, these are $\triangle ABE$ and $\triangle ADE$.)
- 2 Attempt to prove that these triangles are congruent. If you cannot do so for lack of enough given information, take a detour (steps 3–5 below).
- 3 Identify the parts that you must prove to be congruent to establish the congruence of the triangles. (Remember that there are many ways to prove that triangles are congruent. Consider them all.)
- 4 Find a pair of triangles that
 - (a) You can readily prove to be congruent
 - (b) Contain a pair of parts needed for the main proof (parts identified in step 3)
- 5 Prove that the triangles found in step 4 are congruent.
- 6 Use CPCTC and complete the proof planned in step 1.

The Midpoint Formula

In some coordinate-geometry problems, you will need to locate the midpoint of a line segment. A method of doing so is suggested by the following example.

Example On the number line below, the coordinate of A is 2 and the coordinate of B is 14. Find the coordinate of M, the midpoint of \overline{AB} .



There are several ways of solving this problem. One of these is the averaging process (the average of two numbers is equal to half their sum). We will use x_m (read “x sub m”) to represent the coordinate of M.

$$\begin{aligned}x_m &= \frac{2 + 14}{2} \\ &= \frac{16}{2} = 8\end{aligned}$$

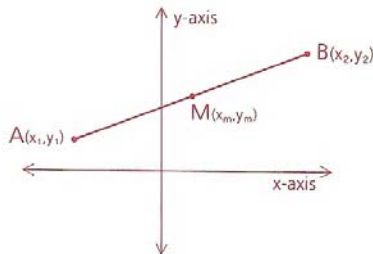
$$\begin{aligned}\text{Check: } AM &= 8 - 2 = 6 \\ MB &= 14 - 8 = 6\end{aligned}$$

Therefore, 8 is the coordinate of M.

We can apply the averaging process to develop a formula, called the **midpoint formula**, that can be used to find the coordinates of the midpoint of any segment in the coordinate plane. The proof of this theorem is left to you.

Theorem 22 If $A = (x_1, y_1)$ and $B = (x_2, y_2)$, then the midpoint $M = (x_m, y_m)$ of \overline{AB} can be found by using the midpoint formula:

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



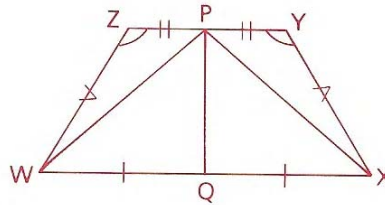
Fun applet: <http://www.mathopenref.com/coordmidpoint.html>

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Part Two: Sample Problems

Problem 1 Given: \overleftrightarrow{PQ} bisects \overline{YZ} .
Q is the midpt. of \overline{WX} .
 $\angle Y \cong \angle Z$, $\overline{WZ} \cong \overline{XY}$
Conclusion: $\angle WQP \cong \angle XQP$



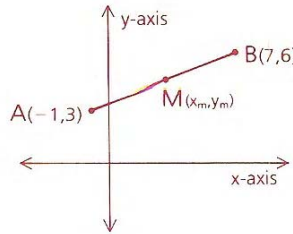
Proof

DETOUR

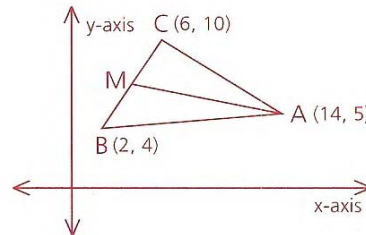
Statements	Reasons
1 \overleftrightarrow{PQ} bisects \overline{YZ} .	1 Given
2	2 If a line bisects a segment, then it divides the segment into two \cong segments.
3 $\angle Z \cong \angle Y$	3 Given
4 $\overline{WZ} \cong \overline{XY}$	4 Given
5 $\triangle ZWP \cong \triangle YXP$	5 SAS (2, 3, 4)
6 $\overline{WP} \cong \overline{PX}$	6 CPCTC
7 Q is the midpt. of \overline{WX} .	7 Given
8 $\overline{WQ} \cong \overline{QX}$	8
9 $\overline{PQ} \cong \overline{PQ}$	9
10 $\triangle WQP \cong \triangle XQP$	10
11 $\angle WQP \cong \angle XQP$	11

Problem 2 Find the coordinates of M, the midpoint of \overline{AB} .

Solution



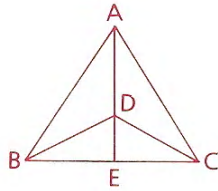
Problem 3 In $\triangle ABC$, find the coordinates of the point at which the median from A intersects \overline{BC} .



Solution Since a median is drawn to a midpoint, use the midpoint formula to find the midpoint M of \overline{BC} .

Problem Set C

- 16 Given: $\overline{AB} \cong \overline{AC}$;
 \overrightarrow{BD} bisects $\angle ABE$.
 \overrightarrow{CD} bisects $\angle ACE$.
 Conclusion: \overline{AE} bisects \overline{BC} .



<i>Statements</i>	<i>Reasons</i>
1 $\overline{AB} \cong \overline{AC}$	1
2 \overrightarrow{BD} bis $\angle ABE$.	2
3 \overrightarrow{CD} bis $\angle ACE$.	3
4 $\angle ABC \cong \angle ACB$	4
5 $\angle DBE \cong \angle DCE$	5
6 $\overline{BD} \cong \overline{CD}$	6
7 $\overline{AD} \cong \overline{AD}$	7
8 $\triangle ADB \cong \triangle ADC$	8
9 $\angle BAD \cong \angle CAD$	9
10 $\overline{AE} \cong \overline{AE}$	10
11 $\triangle AEB \cong \triangle AEC$	11
12 $\overline{BE} \cong \overline{EC}$	12
13 \overline{AE} bis \overline{BC} .	13

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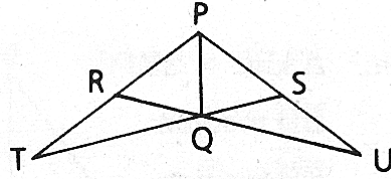
AMDG
Lines in the Plane – Chapter 4
4.1: Detours & Midpoints

Date _____

Ms. Kresovic

17 Given: $\overline{PT} \cong \overline{PU}$
 $\overline{PR} \cong \overline{PS}$

Prove: \overrightarrow{PQ} bis $\angle RPS$.



1 $\overline{PT} \cong \overline{PU}, \overline{PR} \cong \overline{PS}$ 1

2 $\angle TPS \cong \angle UPR$ 2

3 $\triangle TPS \cong \triangle UPR$ 3

4 $\angle PRU \cong \angle PST$ 4

5 $\angle T \cong \angle U$ 5

6 $\angle TRQ$ supp $\angle PRU$ 6

7 $\angle USQ$ supp $\angle PST$ 7

8 $\angle TRQ \cong \angle USQ$ 8

9 $\overline{TR} \cong \overline{US}$ 9

10 $\triangle TRQ \cong \triangle USQ$ 10

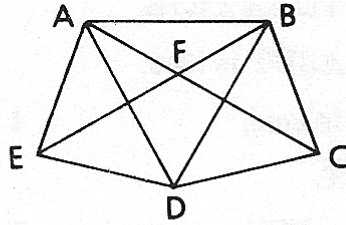
11 $\overline{RQ} \cong \overline{SQ}$ 11

12 $\triangle PRQ \cong \triangle PSQ$ 12

13 $\angle RPQ \cong \angle SPQ$ 13

14 \overrightarrow{PQ} bis $\angle RPS$. 14

- 18 Given: $\overline{AD} \cong \overline{DB}$
 $\overline{AE} \cong \overline{BC}$
 $\overline{CD} \cong \overline{ED}$



Prove: $\triangle AFB$ is isos.

- | | | |
|----|--|----|
| 1 | $\overline{AD} \cong \overline{DB}, \overline{AE} \cong \overline{BC}$ | 1 |
| 2 | $\overline{CD} \cong \overline{ED}$ | 2 |
| 3 | $\triangle ADE \cong \triangle BDC$ | 3 |
| 4 | $\angle ADE \cong \angle BDC$ | 4 |
| 5 | $\angle ADB \cong \angle ADB$ | 5 |
| 6 | $\angle EDB \cong \angle CDA$ | 6 |
| 7 | $\triangle EDB \cong \triangle CDA$ | 7 |
| 8 | $\overline{EB} \cong \overline{CA}$ | 8 |
| 9 | $\overline{AB} \cong \overline{AB}$ | 9 |
| 10 | $\triangle ABE \cong \triangle BAC$ | 10 |
| 11 | $\angle ABF \cong \angle BAF$ | 11 |
| 12 | $\overline{BF} \cong \overline{FA}$ | 12 |
| 13 | $\triangle ABF$ is isos. | 13 |

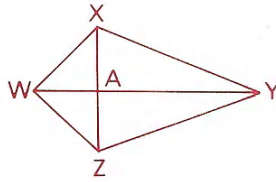
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Homework

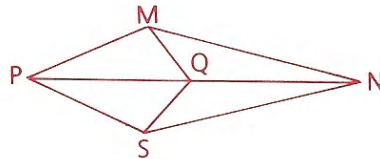
Problem Set A

- 1 Given: $\overline{WX} \cong \overline{WZ}$, $\overline{XY} \cong \overline{ZY}$
Prove: $\triangle XAY \cong \triangle ZAY$



Statements	Reasons
1 $\overline{WX} \cong \overline{WZ}$	1 Given
2 $\overline{XY} \cong \overline{ZY}$	2 Given
3 _____	3 Reflexive Property
4 $\triangle WXY \cong \triangle WZY$	4 _____
5 $\angle XYW \cong \angle ZYW$	5 _____
6 _____	6 Reflexive Property
7 $\triangle XAY \cong \triangle ZAY$	7 _____

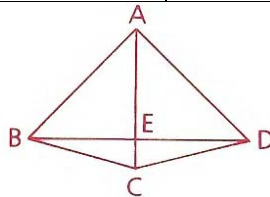
- 2 Given: $\overline{MN} \cong \overline{NS}$,
 $\overline{MP} \cong \overline{PS}$
Prove: $\angle MQP \cong \angle SQP$



(With the Givens on the same line, 7 steps.)

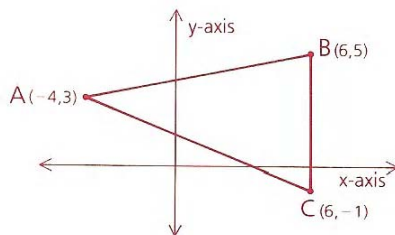
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

- 3 Given: A is equidistant from B and D
(that is, $AB = AD$).
 \overrightarrow{AC} bisects $\angle BAD$.
Prove: \overline{AC} bisects \overline{BD} .



Statements	Reasons
1. A is =dist from B and D; $\overline{AB} \cong \overline{AD}$.	1.
2. \overrightarrow{AC} bis $\angle BAD$.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

- 4 Find the coordinates of the midpoint of each side of $\triangle ABC$.



Make sure you communicate to the reader what you are doing. Use subtitles or subheadings, like this:

Mdpt AB

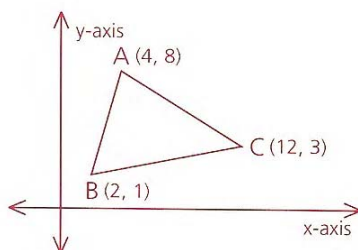
Mdpt BC

Mdpt CA

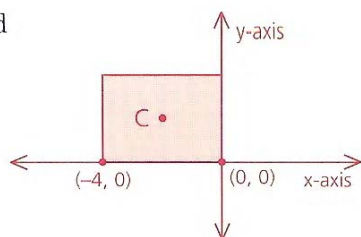
- 5 Find the coordinates of the point where the median from A intersects \overline{BC} .

Median \Rightarrow _____

Find _____



- 8 If the shaded square has center at C and an area of A_{\square} , find C and A_{\square} .



Find the area of the square

Find C

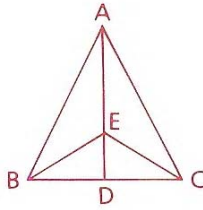
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Problem Set B

9 Given: $\triangle ABC$ is isosceles, with base \overline{BC} .
 $\overline{AD} \perp \overline{BC}$

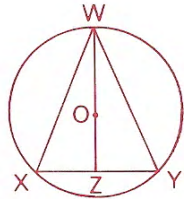
Prove: $\triangle BEC$ is isosceles.



11 steps, each given on a separate line.
Hint: Number the angles

Statements	Reasons
1. $\overline{AD} \perp \overline{BC}$	1.
2.	2.
3. $\triangle ABC$ is isos, base \overline{BC} .	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.

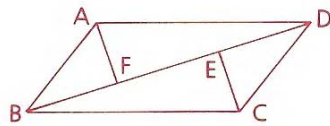
10 Given: $\odot O$, $\overline{WX} \cong \overline{WY}$
Prove: \overleftrightarrow{WZ} bisects \overline{XY} .



Statements	Reasons
1. $\odot O$	1.
2. Draw \overline{OX} and \overline{OY}	2.
3.	3.
4. $\overline{WX} \cong \overline{WY}$	4.
5.	5.
6. $\triangle WXO \cong \triangle WYO$	6.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.
12.	12.
13.	13.
14.	14.

11 Given: $\overline{AD} \cong \overline{BC}$, $\overline{AF} \cong \overline{EC}$,
 $\overleftrightarrow{BD} \perp \overleftrightarrow{AF}$, $\overleftrightarrow{BD} \perp \overleftrightarrow{EC}$

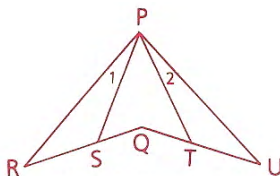
Conclusion: $\overline{AB} \cong \overline{DC}$



Statements	Reasons
1. $\overleftrightarrow{BD} \perp \overleftrightarrow{AF}$ and \overleftrightarrow{EC}	1.
2.	2.
3. $\overline{AD} \cong \overline{BC}$, $\overline{AF} \cong \overline{EC}$	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.

12 Given: $\overline{PR} \cong \overline{PU}$,
 $\overline{QR} \cong \overline{QU}$,
 $\overline{RS} \cong \overline{UT}$

Conclusion: $\angle 1 \cong \angle 2$

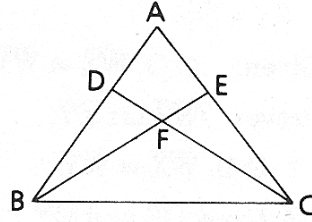


Statements	Reasons
1. $\overline{PR} \cong \overline{PU}$, $\overline{QR} \cong \overline{QU}$, & $\overline{RS} \cong \overline{UT}$	1.
2. Draw \overline{PQ}	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

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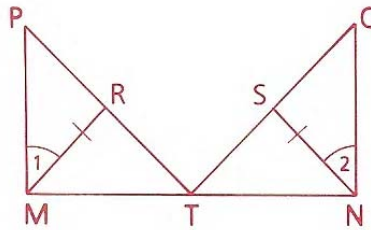
- 13 Given: $\overline{AB} \cong \overline{AC}$
 $\overline{AD} \cong \overline{AE}$



Prove: $\triangle FBC$ is isos.

- | | | |
|---|--|---|
| 1 | $\overline{AB} \cong \overline{AC}, \overline{AD} \cong \overline{AE}$ | 1 |
| 2 | $\angle A \cong \angle A$ | 2 |
| 3 | $\triangle BAE \cong \triangle CAD$ | 3 |
| 4 | $\angle ABE \cong \angle ACD$ | 4 |
| 5 | $\angle ABC \cong \angle ACB$ | 5 |
| 6 | $\angle FCB \cong \angle FBC$ | 6 |
| 7 | $\overline{BF} \cong \overline{FC}$ | 7 |
| 8 | $\triangle FBC$ is isos. | 8 |

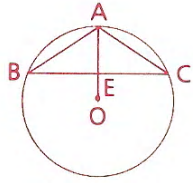
- 14 Given: T is the midpt. of \overline{MN} .
 $\angle PMT$ and $\angle QNT$ are right \angle s.
 $\overline{MR} \cong \overline{SN}, \angle 1 \cong \angle 2$



Conclusion: $\angle P \cong \angle Q$

Statements	Reasons
1. T mdpt of \overline{MN}	1.
2. $\overline{MT} \cong \overline{TN}$	2.
3. $\angle PMT, \angle QNT$ are rt \angles.	3.
4. $\angle PMT \cong \angle QNT$	4.
5. $\angle 1 \cong \angle 2$	5.
6. $\angle RMT \cong \angle SNT$	6.
7. $\overline{MR} \cong \overline{SN}$	7.
8.	8.
9.	9.
10.	10.
11.	11.
12.	12.
13.	13.
14.	14.

- 15 Given: $\odot O$, $\angle B \cong \angle C$
Prove: \overline{AO} bisects \overline{BC} .



11 steps, each given on a separate line