AMDG

Name
Adv Geo – 8

4: Lines in the Plane
4.4: Equidistance Theorems

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Objective

After studying this section, you will be able to

- Recognize the relationship between equidistance and perpendicular bisection

Part One: Introduction

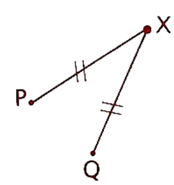
In geometry, the term *distance* has a special meaning.

Definition The ***distance*** between two objects is the length of the shortest path joining them.

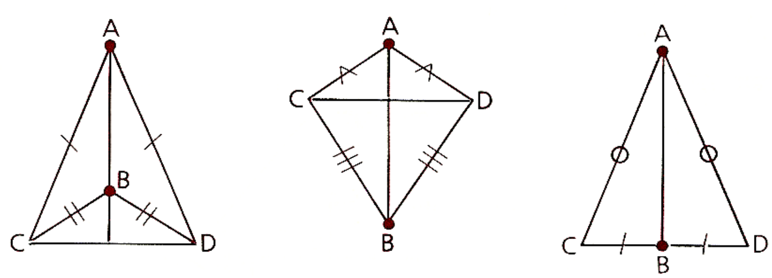
Postulate ***A line segment is the shortest path between two points.***

The distance between points R and S is the length of $\overline{RS}$, or RS.

If two points P and Q are the same distance from a third point X, then X is said to be ***equidistant*** from P and Q.

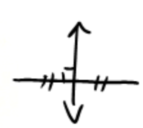


$\overline{PX} \cong \overline{XQ}$
means that
X is equidistant from P and Q.



You should recall many problems with diagrams resembling those above. These diagrams have something in common. In each, both point A and point B are equidistant from the endpoints C and D of $\overline{CD}$. In each case, you could prove that $\overleftrightarrow{AB}$ is the ***perpendicular bisector*** of $\overline{CD}$ just by using the following definition and theorem.

Definition The ***perpendicular bisector*** of a segment is the line that bisects and is perpendicular to the segment.



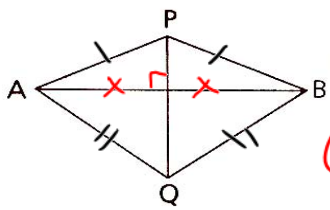
Theorem 24 ***If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.***

GSP

2 PTS = DIST
(PA = PB & QA = QB)
Then PQ ⊥ BIS AB
(= DIST ⇒ ⊥ BIS)

Given: $\overline{PA} \cong \overline{PB}$,
$\overline{QA} \cong \overline{QB}$

Prove: $\overleftrightarrow{PQ}$ is the $\perp$ bisector of $\overline{AB}$.



For a proof of Theorem 24, see sample problem 2 in Section 4.3.

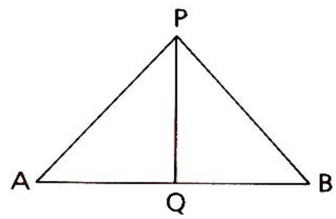
converse

Theorem 25 ***If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.***

(⊥ BIS ⇒ = DIST)

Given: $\overleftrightarrow{PQ}$ is the $\perp$ bisector of $\overline{AB}$.

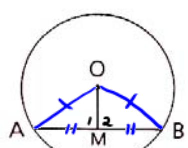
Prove: $\overline{PA} \cong \overline{PB}$



You can easily prove this theorem by using the definition of *perpendicular bisector* and some congruent triangles.

Homework

1 Given: $\odot$O; M is the midpt. of $\overline{AB}$.
Conclusion: $\overline{OM} \perp \overline{AB}$ (Hint: Draw two auxiliary lines.)

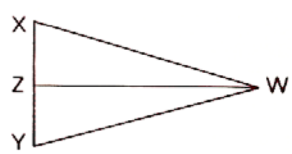


NEW WAY (∵ OF NEW THE THEOREM)!

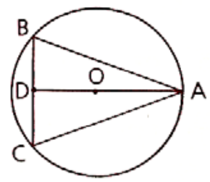
OLD WAY:

1. $\odot$O	1. Given
2. DRAW $\overline{OA}$ & $\overline{OB}$	2. AUX
3. $\overline{OA} \cong \overline{OB}$	3. $\odot \Rightarrow \cong$ radii
4. M mdpt $\overline{AB}$	4. Given
5. $\overline{AM} \cong \overline{MB}$	5. mdpt $\Rightarrow \cong$ segs
6. $\overline{OM} \cong \overline{OM}$	6. Ref
7. $\triangle OAM \cong \triangle OBM$	7. SSS (3 5 6)
8. $\angle 1 \cong \angle 2$	8. CPCTC
9. $\angle 1$ supp $\angle 2$	9. st$\angle \Rightarrow$ supp
10. $\angle 1$ & $\angle 2$ rt$\angle$s	10. $\cong$ & supp $\angle$s $\Rightarrow$ rt$\angle$
11. $\overline{OM} \perp \overline{AB}$	11. rt$\angle \Rightarrow \perp$
12. OM ⊥ bis AB	

2 Given: $\overleftrightarrow{WZ} \perp$ bis. $\overline{XY}$
Prove: $\triangle$WXY is isosceles. (Hint: This proof can be written in three steps by using Theorem 25.)



3 Given: $\odot$O, $\overline{AB} \cong \overline{AC}$
Conclusion: $\overleftrightarrow{AD} \perp$ bis. $\overline{BC}$ (Hint: Show that A and O are each equidistant from B and C.)



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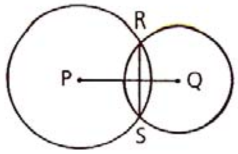
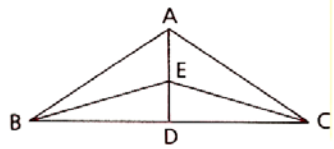
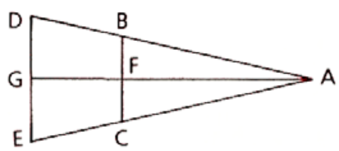
4 Given: Ⓢ P and Q
Prove: $\overleftrightarrow{PQ} \perp$ bis. $\overline{RS}$



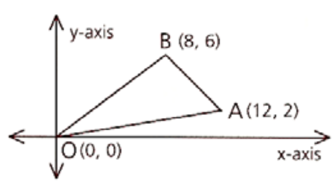
5 Given: $\overleftrightarrow{AD} \perp$ bis. $\overline{BC}$
Prove: $\triangle ABE \cong \triangle ACE$



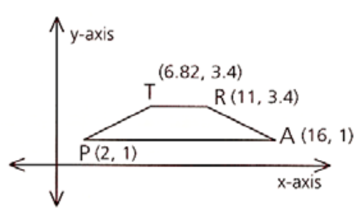
6 Given: $\overleftrightarrow{AG} \perp$ bis. $\overline{BC}$,
$\overleftrightarrow{AG} \perp$ bis. $\overline{DE}$
Conclusion: $\overline{BD} \cong \overline{CE}$



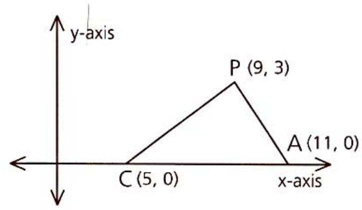
7 How much greater than the x-coordinate of the midpoint of $\overline{OA}$ is the x-coordinate of the midpoint of $\overline{AB}$?



8 In the graph, if a perpendicular is drawn from T to $\overleftrightarrow{PA}$, what will the coordinates of the point where the perpendicular intersects $\overleftrightarrow{PA}$ be?



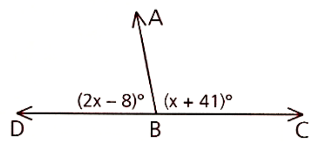
9 If $\triangle CAP$ is slid along the x-axis until C is at (11, 0), what will the new coordinates of P be?



10 A fifth point, E, is located on the diagram so that $m\angle EBC = \sqrt{x} + 83$.

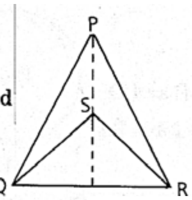
a Is $\overleftrightarrow{AB}$ perpendicular to $\overleftrightarrow{DC}$?

b What do we know about $\overleftrightarrow{AB}$ and $\overleftrightarrow{BE}$?



11 Given: Isos $\triangle PQR$,
$\overline{QS}$ bis $\angle Q$, and
$\overline{RS}$ bis $\angle R$.

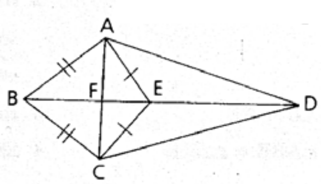
Prove: $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$



1 Isos $\triangle PQR$	1
2 $\overline{PQ} \cong \overline{PR}$	2
3 $\angle Q \cong \angle R$	3
4 $\overline{QS}$ bis $\angle Q$.	4
5 $\overline{RS}$ bis $\angle R$.	5
6 $\angle SQR \cong \angle SRQ$	6
7 $\overline{SQ} \cong \overline{SR}$	7
8 $\overleftrightarrow{PS} \perp \overleftrightarrow{QR}$	8

12 Given: $\overline{AB} \cong \overline{BC}$
$\overline{AE} \cong \overline{EC}$

Prove: $\overline{AD} \cong \overline{DC}$



1 $\overline{AB} \cong \overline{BC}$	1
2 $\overline{AE} \cong \overline{EC}$	2
3 $\overline{BE} \perp$ bis $\overline{AC}$.	3
4 $\overline{AD} \cong \overline{DC}$	4