Objectives

After studying this section, you will be able to

- Apply the principle of CPCTC
- Recognize some basic properties of circles

CPCTC

Suppose that in the figure $\triangle ABC = \triangle DEF$. Is it therefore true that $\angle B = \angle E$? If you refer to Section 3.1, you will find that we have already answered yes to this question in the definition of congruent triangles.

In the portions of the book that follow, we shall often draw such a conclusion after knowing that some triangles are congruent. We shall use CPCTC as the reason. CPCTC is short for "Corresponding Parts of Congruent Triangles are Congruent." By corresponding parts, we mean only the matching angles and sides of the respective triangles.

Introduction to Circles

Point $O$ is the center of the circle shown at the right. By definition, every point of the circle is the same distance from the center. The center, however, is not an element of the circle; the circle consists only of the "rim." A circle is named by its center; this circle is called circle $O$ (or $\odot O$).

Points $A$, $B$, and $C$ lie on circle $P (\odot P)$.
- $PA$ is called a radius.
- $PA$, $PB$, and $PC$ are called radii.

From previous math courses you may remember formulas for the area and the circumference of a circle:

\[
A = \pi r^2 \\
C = 2\pi r
\]

By pressing the $\pi$ key on a scientific calculator, you can find that $\pi \approx 3.141592654$.

Theorem 19  All radii of a circle are congruent.
Problem 1
Given: \( \odot P \)
Conclusion: \( \overline{AB} \cong \overline{CD} \)

Proof

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \odot P )</td>
<td>1 \textit{Given}</td>
</tr>
<tr>
<td>2 ( \overline{PA} \cong \overline{PB} \cong \overline{PC} \cong \overline{PD} )</td>
<td>2 ( \odot \cong \odot )</td>
</tr>
<tr>
<td>3 ( \angle CPD \cong \angle APB )</td>
<td>3 \textit{Vertical}</td>
</tr>
<tr>
<td>4 ( \triangle CPD \cong \triangle APB )</td>
<td>4 \textit{SAS} (( \angle 3 ) ( \angle 2 ) ( \angle 5 ))</td>
</tr>
<tr>
<td>5 ( \overline{AB} \cong \overline{CD} )</td>
<td>5 \textit{CPCTC} (( \angle 4 ))</td>
</tr>
</tbody>
</table>

Problem 2
Given: \( \odot O \);
\( \angle T \) is comp. to \( \angle MOT \).
\( \angle S \) is comp. to \( \angle POS \).
Prove: \( \overline{MO} \cong \overline{PO} \)

Proof

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<tr>
<td>1 ( \odot O )</td>
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</tr>
<tr>
<td>2 ( \overline{OT} \cong \overline{OS} )</td>
<td>2</td>
</tr>
<tr>
<td>3 ( \angle T ) is comp. to ( \angle MOT ).</td>
<td>3</td>
</tr>
<tr>
<td>4 ( \angle S ) is comp. to ( \angle POS ).</td>
<td>4</td>
</tr>
<tr>
<td>5 ( \angle MOT \cong \angle POS )</td>
<td>5</td>
</tr>
<tr>
<td>6 ( \angle T \cong \angle S )</td>
<td>6</td>
</tr>
<tr>
<td>7 ( \triangle MOT \cong \triangle POS ) (Watch the correspondence.)</td>
<td>7</td>
</tr>
<tr>
<td>8 ( \overline{MO} \cong \overline{PO} )</td>
<td>8</td>
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Problem 3
Find the exact area & circumference of a circle whose radius is 12.5 cm.
**Problem 4**

Explain why the area of the shaded region is $100 - 25\pi$. 

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**Problem 5**

Given: $\angle A = \angle E$,  
$AB = BE$,  
$FB \perp AE$,  
$\angle 2 = \angle 3$  
Prove: $CB = DB$
Problem 8

21 Given: $AE \cong FC$,  
FB $\cong DE$,  
$\angle CFB \cong \angle AED$  
Prove: $\angle 1 \cong \angle 2$

Problem 9

Given: H is the midpt. of $GJ$.  
M is the midpt. of $OK$.  
$GO \cong JK$,  
$GJ \cong OK$,  
$\angle G \cong \angle K$,  
OK $= 27$,  
m$\angle GOH = x + 24$, m$\angle GHO = 2y - 7$,  
m$\angle MK = 3y - 23$, m$\angle MJK = 4x - 105$  
Find: m$\angle GOH$, m$\angle GHO$, and GH
Homework

Problem Set A

1. Given: \( \overline{AB} \cong \overline{DE} \), 
   \( \overline{BC} \cong \overline{EF} \), 
   \( \overline{AC} \cong \overline{DF} \)
   Prove: \( \angle A \cong \angle D \)

2. Given: \( \angle HGJ \cong \angle KJG \), 
   \( \angle KGJ \cong \angle HJG \)
   Conclusion: \( \overline{HG} \cong \overline{KJ} \)

3. Given: \( \circ O \), 
   \( \overline{RO} \perp \overline{MP} \)
   Prove: \( \overline{MR} \cong \overline{PR} \)

4. Given: \( T \) and \( R \) trisect \( \overline{SW} \), 
   \( \overline{XS} \cong \overline{XW} \), 
   \( \angle S \cong \angle W \)
   Prove: \( \overline{XT} \cong \overline{XR} \)
5) Given: \( \angle B \cong \angle Y; \)
   \( C \) is the midpt. of \( BY. \)
   Conclusion: \( AB \cong YZ \)

8) Given: \( \odot O, \)
    \( CD \cong DE \)
   Prove: \( \angle COD \cong \angle DOE \)

7) Find, to the nearest tenth, the area & circumference of a circle whose radius is 12.5 cm.

8) \( \triangle ABC \cong \triangle DEF, \)
   \( \angle A = 90^\circ, \angle B = 50^\circ, \angle C = 40^\circ, \)
   \( m \angle E = 12x + 30, \ m \angle F = \frac{y}{2} - 10, \)
   \( m \angle D = \sqrt{z} \)
   Solve for \( x, y, \) and \( z. \)
9. Given: $\overrightarrow{FH}$ bisects $\angle GFJ$ and $\angle GHJ$.
   Conclusion: $FG = FJ$

10. Given: $\angle M = \angle R$,
    $\angle RPS \equiv \angle MOK$,
    $MP = RO$
    Conclusion: $KM \equiv RS$

14. Given: $\angle 5 \equiv \angle 6$,
    $\angle JHG \equiv \angle O$,
    $GH \equiv MO$
    Conclusion: $\angle J \equiv \angle P$

15. Given: $\angle RST \equiv \angle RVT$,
    $\angle RVS \equiv \angle TSV$
    Conclusion: $RS \equiv VT$
16 Given: \( \angle Z \cong \angle B, \quad ZY \cong WX \)
Prove: \( \angle W \cong \angle Y \)

17 Given: \( \angle AEC \cong \angle DEB, \quad DE \cong CE, \quad \angle ABE \cong \angle DCE \)
Prove: \( AB \cong CD \)

19 a Find the coordinates of point P.
   b Find the area of the circle.