AMDG

Name____________________ Ms. Kresovic

Adv Geo - __________ **2.8: Vertical Angles** Date: ______________

Objectives

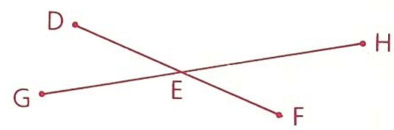
After studying this section, you should be able to

- Recognize opposite rays
- Recognize vertical angles

Opposite Rays

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are ***opposite rays.***

$\overrightarrow{ED}$ and $\overrightarrow{EF}$ are also opposite rays, as are $\overrightarrow{EG}$ and $\overrightarrow{EH}$.



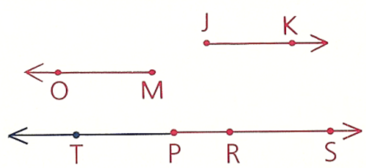
not a term we use in proof.

Definition Two collinear rays that have a common endpoint and extend in different directions are called ***opposite rays***.

Some pairs of rays that are *not* opposite rays are shown below.

$\overrightarrow{JK}$ and $\overrightarrow{MO}$ are not parts of the same line.

$\overrightarrow{PT}$ and $\overrightarrow{RS}$ are not opposite, since they do not have a common endpoint.



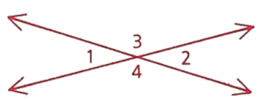
Vertical Angles

Whenever two lines intersect, two pairs of ***vertical angles*** are formed.

Definition Two angles are ***vertical angles*** if the rays forming the sides of one and the rays forming the sides of the other are opposite rays.

$\angle 1$ and $\angle 2$ are vertical angles.

$\angle 3$ and $\angle 4$ are vertical angles.

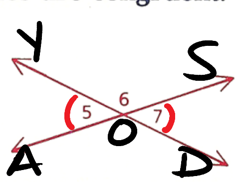


Are $\angle 3$ and $\angle 2$ vertical angles? How do vertical angles compare in size?

Theorem 18 ***Vertical angles are congruent.***

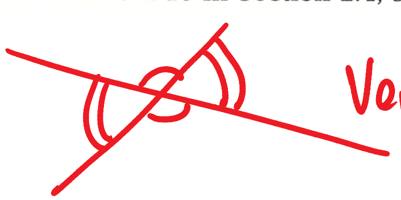
Given: Diagram as shown

Prove: $\angle 5 \cong \angle 7$



Statements	Reasons
1. Diag	1. Given
2. $\angle YOD$ & $\angle SOA$ st$\angle$	2. Diag $\Rightarrow$ st$\angle$
3. $\angle 5$ supp $\angle 6$ $\angle 7$ supp $\angle 6$	3. st$\angle$ $\Rightarrow$ suppl$\angle$s
4. $\angle 5 \cong \angle 7$	4. $\angle$s supp same $\angle$ $\Rightarrow$ $\cong \angle$s

(We proved Theorem 18 in Section 2.4, sample problem 3.)

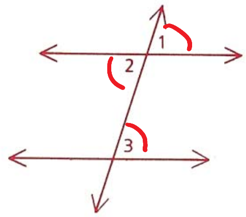


Vert $\angle$s $\Rightarrow$ $\cong \angle$s

Part Two: Sample Problems

Problem 1 *Given:* $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 3$



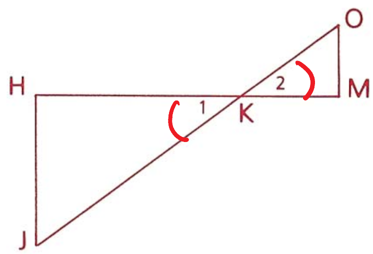
Proof

Statements	Reasons
1 $\angle 2 \cong \angle 3$	1 Given
2 $\angle 1 \cong \angle 2$	2 Vert∠ ⇒ ≅∠s
3 $\angle 1 \cong \angle 3$	3 Trans (1, 2)

Problem 2 *Given:* $\angle O$ is comp. to $\angle 2$.

$\angle J$ is comp. to $\angle 1$.

Conclusion: $\angle O \cong \angle J$



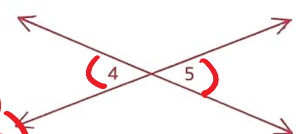
Proof

Statements	Reasons
1 $\angle O$ is comp. to $\angle 2$.	1 Given
2 $\angle J$ is comp. to $\angle 1$.	2 Given
3 $\angle 1 \cong \angle 2$	3 Vert∠s ⇒ ≅∠s
4 $\angle O \cong \angle J$	4 ∠s comp to ≅ ∠s ⇒ ≅∠s (1 2 3)

Problem 3 *Given:* $m\angle 4 = 2x + 5$,

$m\angle 5 = x + 30$

Find: $m\angle 4$



∠4 ≅ ∠5 (Vert)

Solution

$2x + 5 = x + 30$ (subst)

$-x \; -5 \; -x \; -5$

$x = 25$

Therefore, $m\angle 4 = 2(25) + 5$, or 55.

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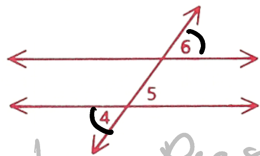
Problem 4

Given: $\angle 4 \cong \angle 6$

Prove: $\angle 5 \cong \angle 6$



Statements	Reasons
1. $\angle 4 \cong \angle 6$	1. Given
2. $\angle 4 \cong \angle 5$	2. Vert $\angle$s $\Rightarrow \cong \angle$s
3. $\angle 5 \cong \angle 6$	3. Trans (1,2)

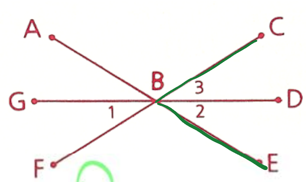
$\angle 6 \cong \angle 4$

$\angle 4 \cong \angle 5$

Problem 5

Given: $\overleftrightarrow{GD}$ bisects $\angle CBE$.

Conclusion: $\angle 1 \cong \angle 2$



Statements	Reasons
1. $\overleftrightarrow{GD}$ bis $\angle CBE$	1. Given
2 $\angle 3 \cong \angle 2$	2. bis $\Rightarrow \cong \angle$s (1)
3. $\angle 1 \cong \angle 3$	3. Vert $\angle$s $\Rightarrow \cong \angle$s
4. $\angle 1 \cong \angle 2$	4. trans

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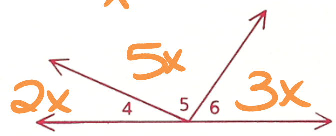
Problem 6

Angles 4, 5, and 6 are in the ratio 2:5:3. → Think "x"
Find the measure of each angle.



$10x = 180$

$x = 18$

m each angle:

$\angle 4: 2(10+8) = 36°$

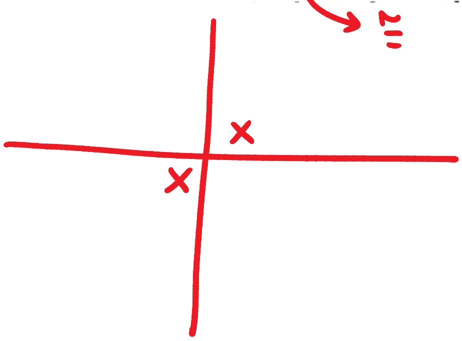
$\angle 5: 5(10+8) = 90°$

$\angle 6 \quad 3(10+8) = 54°$

Problem 7

If a pair of vertical angles are supp., what can we conclude about the angles?

→ ≅



Vert ∠s ⇒ ≅ ∠s

≅ ∠s ⇒ = meas

Supp ∠s ⇒ 180°

$x + x = 180°$

$2x = 180°$

$x = 90°$

90 → rt ∠

rt ∠ ⇒ ⊥

⇒ alt

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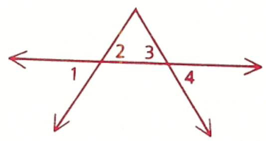
Homework

5 Given: $\angle 1 \cong \angle 4$

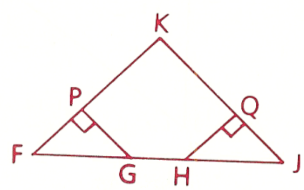
Conclusion: $\angle 2 \cong \angle 3$



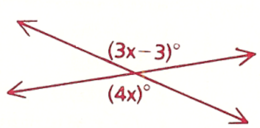
Statements	**Reasons**
1. $\angle 1 \cong \angle 4$	1.
2. $\angle 1 \cong \angle 2$ & $\angle 4 \cong \angle 3$	2.
3. $\angle 2 \cong \angle 3$	3.

6 Given: $\overline{FH} \cong \overline{GJ}$

Prove: $\overline{FG} \cong \overline{HJ}$



7 Is this possible?



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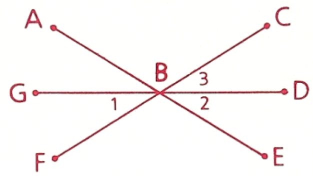
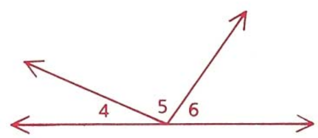
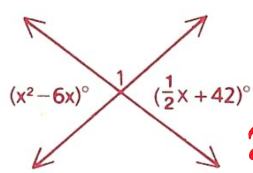
11 Given: $\overleftrightarrow{GD}$ bisects $\angle CBE$.
Conclusion: $\angle 1 \cong \angle 2$



12 Angles 4, 5, and 6 are in the ratio 2:5:3.
Find the measure of each angle.



15 Find m$\angle 1$.



$x^2 - 6x = \frac{1}{2}x + 42$

$2\left(x^2 - 6x - \frac{1}{2}x - 42 = 0\right)$

$2x^2 - 12x - x - 84 = 0$

$2x^2 - 13x - 84 = 0$

$2x^2 + 8x - 21x - 84 = 0$

$2(84) = 168$

$-21 + 8 = -13$