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Adv Geo - ___________ **2.7: Transitive & Substitution Properties** Date ________________

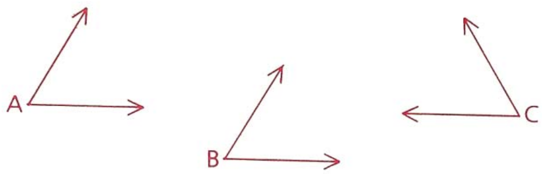
## Objectives

*After studying this section, you will be able to*

- Apply the transitive properties of angles and segments
- Apply the Substitution Property

## Transitive Properties

Suppose that $\angle A \cong \angle B$ and $\angle A \cong \angle C$. Is $\angle B \cong \angle C$?




The transitive property of algebra can be used to prove this general rule.

**Theorem 16** ***If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other. (Transitive Property)***

Theorem 16 can be used twice to prove the next theorem.

**Theorem 17** ***If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other. (Transitive Property)***

## Substitution Property

In your algebra studies and in some of the problems you have worked this year, you have solved for a variable such as x and then ***substituted*** the value you found for that variable.

**Example** *If* $\angle A \cong \angle B$, *find* $m\angle A$.

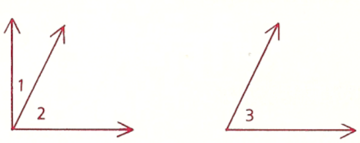

$2x - 4 = x + 10$

$x = 14$

We can now substitute 14 for x in $m\angle A = x + 10$ to find that $m\angle A = 14 + 10 = 24$.

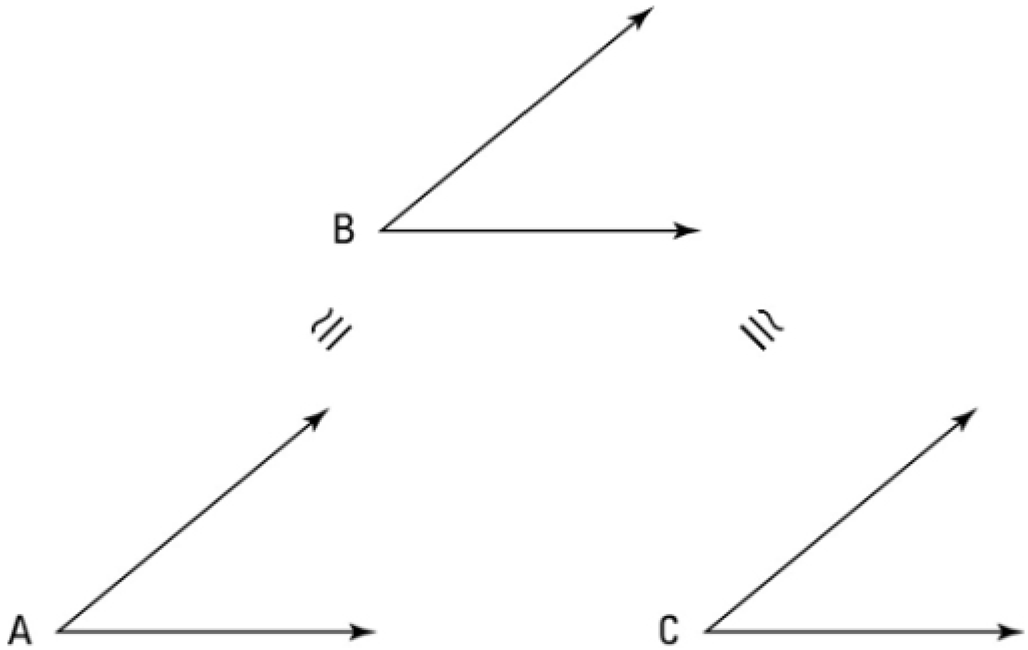
The Substitution Property can also be applied when no variables are involved.

If $\angle 1$ is comp. to $\angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1$ is comp. to $\angle 3$ by ***substitution.***



You're probably already familiar with the Transitive Property and the Substitution Property from algebra. If $a = b$ and $b = c$, then $a = c$, right? That's transitivity. And if $a = b$ and $b < c$, then $a < c$. That's substitution. Easy enough. Below, you see these theorems in greater detail:

$a = b$
$\underline{\quad b = c}$
$a \quad = c$




- **Transitive Property (for three segments or angles):** If two segments (or angles) are each congruent to a third segment (or angle), then they're congruent to each other.

  For example, if $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$ ($\angle A$ and $\angle C$ are each congruent to $\angle B$, so they're congruent to each other).

  The Transitive Property for three things is illustrated in the above figure.

$\angle A \cong \angle B$
$\underline{\quad \angle B \cong \angle C}$
$\angle A \quad \cong \angle C$

**Notice that all steps involve ONLY ≅ !

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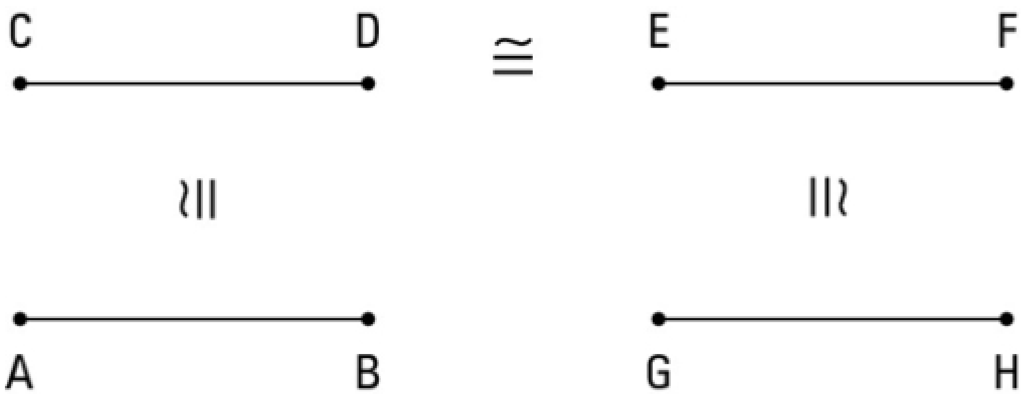
- **Transitive Property (for four segments or angles):** If two segments (or angles) are congruent to congruent segments (or angles), then they're congruent to each other.

  For example, if $\overline{AB} \cong \overline{CD}$, $\overline{CD} \cong \overline{EF}$, and $\overline{EF} \cong \overline{GH}$, then $\overline{AB} \cong \overline{GH}$. ($\overline{AB}$ and $\overline{GH}$ are congruent to the congruent segments $\overline{CD}$ and $\overline{EF}$, so they're congruent to each other.)

  The Transitive Property for four things is illustrated in the below figure.




- **Substitution Property:** If two geometric objects (segments, angles, triangles, or whatever) are congruent and you have a statement involving one of them, you can pull the switcheroo and replace the one with the other. (Note that you will not be able to find the term "switcheroo" in your geometry glossary.)

  For example, if $\angle X \cong \angle Y$ and $\angle Y$ is supplementary to $\angle Z$, then $\angle X$ is supplementary to $\angle Z$.

  A figure isn't especially helpful for this property, so one isn't included here.

To avoid getting the Transitive and Substitution Properties mixed up, just follow these guidelines:

- Use the *Transitive Property* as the reason in a proof when the statement on the same line involves congruent things.
- Use the *Substitution Property* when the statement does not involve a congruence.

**Substitution or Transitive?**

In algebra:

If $a = b$ and $b = c$, then $a = c$, right? That's transitivity.

And if $a = b$ and $b < c$, then $a < c$. That's substitution.

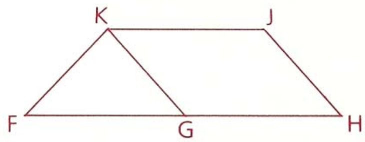
In Geometric Proof:

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$ then $\angle A \cong \angle C$, right? That's transitivity. (Everything's ≅)

And if $\angle A \cong \angle B$ and $\angle A = 50°$ then $\angle B = 50°$. That's substitution. (Not **<u>every</u>** thing's ≅)

# Part Two: Sample Problems

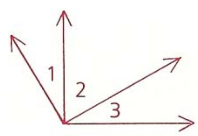
**Problem 1** *Given:* $\overline{FG} \cong \overline{KJ}$,
$\overline{GH} \cong \overline{KJ}$
*Prove:* $\overleftrightarrow{KG}$ *bisects* $\overline{FH}$.




**Proof**

| Statements | Reasons |
|---|---|
| 1 $\overline{FG} \cong \overline{KJ}$ | 1 Given |
| 2 $\overline{GH} \cong \overline{KJ}$ | 2 Given |
| 3 $\overline{FG} \cong \overline{GH}$ | 3 |
| 4 $\overleftrightarrow{KG}$ bisects $\overline{FH}$. | 4 |

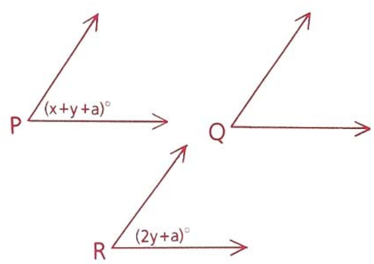
**Problem 2** *Given:* $\angle 1 + \angle 2 = 90°$,
$\angle 1 \cong \angle 3$
*Prove:* $\angle 3 + \angle 2 = 90°$




**Proof**

| Statements | Reasons |
|---|---|
| 1 $\angle 1 + \angle 2 = 90°$ | 1 Given |
| 2 $\angle 1 \cong \angle 3$ | 2 Given |
| 3 $\angle 3 + \angle 2 = 90°$ | 3 |

**Problem 3** *If* $\angle P \cong \angle R$ *and* $\angle Q \cong \angle R$, *express* $m\angle Q$ *in terms of x and a.*




**Solution**

$\angle P = \angle R$

$x + y + a = 2y + a$

$\angle Q \cong \angle R$

$2y = x + y$

$y = x$

---

$m\angle P = x + y + a = x + x + a = 2x + a$

By the transitive property, $\angle P \cong \angle R$
$\angle R \cong \angle Q$

---

If angles are congruent, then they have the same measure.
Therefore,
$m\angle Q$

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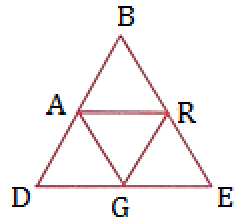
**Problem 4**

Given: BA + AG = DE

AG= GR

Prove: BA + GR = DE




**Problem 5**

Given: $\overline{EU} \cong \overline{ID}$,

$\overline{CL} \cong \overline{ID}$

Prove: $\overline{EC} \cong \overline{UL}$


| Statements | Reasons |
|---|---|
| 1. $\overline{EU} \cong \overline{ID}$, | 1. Given |
| 2. $\overline{CL} \cong \overline{ID}$ | 2. Given |
| 3. $\overline{EU} \cong \overline{CL}$ | 3. |
| 4. $\overline{UC} \cong \overline{UC}$ | 4. Reflexive |
| 5. $\overline{EC} \cong \overline{UL}$ | 5. |

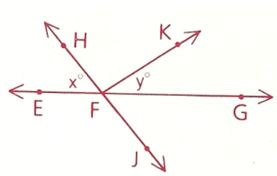
**Problem 6**

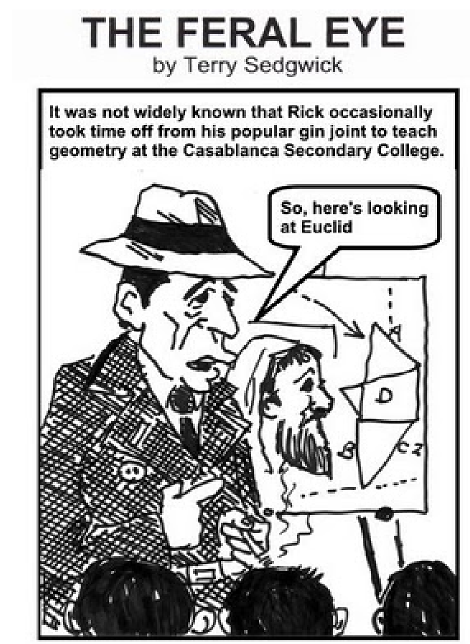
Find the measures of each of the following angles in terms of $x$ and $y$.

**a** ∠HFK

**b** ∠EFK

**c** ∠HFG



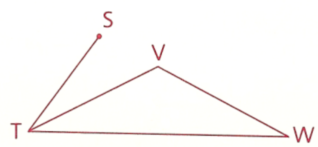



**Problem 7**

The complement of an angle is 24° greater than twice the angle.
Find the measure of the complement.

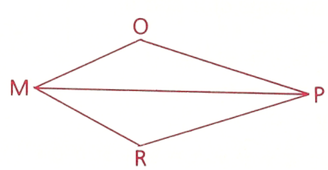
**Problem 8**

$\angle W \cong \angle STV$;
$\overrightarrow{TV}$ bisects $\angle STW$.
$\angle W = (2x - 5)°$,
$\angle VTW = (x + 15)°$
Find: $m\angle STW$




**Problem 9**

Given: $\angle OMP \cong \angle RPM$;
$\overrightarrow{MP}$ bisects $\angle OMR$.
$\overrightarrow{PM}$ bisects $\angle OPR$.
Prove: $\angle OMR \cong \angle OPR$



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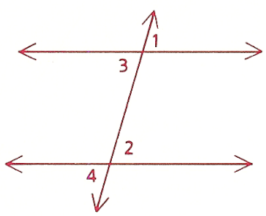
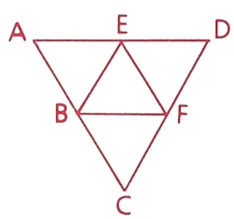
**Homework**

**2** Given: $\angle 1 \cong \angle 2$,
$\angle 2 \cong \angle 3$
Conclusion: $\angle 1 \cong \angle 3$

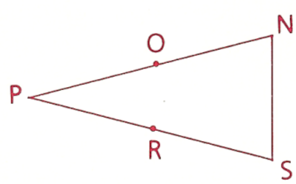

**3** Given: $\angle 1 \cong \angle 3$,
$\angle 2 \cong \angle 3$,
$\angle 2 \cong \angle 4$
Prove: $\angle 1 \cong \angle 4$



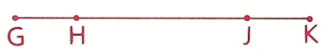

**4** Given: BC + BE = AD,
BE = EF
Prove: BC + EF = AD




**5** Given: O is the midpt. of $\overline{NP}$.
R is the midpt. of $\overline{SP}$.
$\overline{NP} \cong \overline{SP}$
Conclusion: $\overline{SR} \cong \overline{NO}$




**6** Given: $\overline{GJ} \cong \overline{HK}$
Conclusion: $\overline{GH} \cong \overline{JK}$



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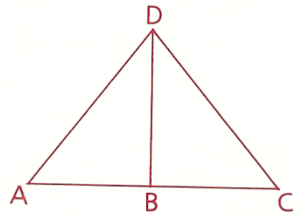
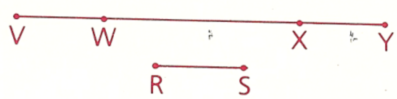
**12** Given: ∠A is comp. to ∠ADB.
∠C is comp. to ∠CDB.
$\overrightarrow{DB}$ bisects ∠ADC.
Conclusion: ∠A ≅ ∠C




You MUST complete 10 & 15. We will start the next class with a peer assessment of this proof. If it is not completed, you will not be able to participate and score a 0 for participation.

**10** Given: $\overline{VW} \cong \overline{RS}$,
$\overline{XY} \cong \overline{RS}$
Prove: $\overline{VX} \cong \overline{WY}$




**15** Given: ∠A is a right ∠.
∠B is a right ∠.
∠B ≅ ∠D
Prove: ∠A ≅ ∠D