Name ______________________________ Ms. Kresovic

Adv Geo - **2.6 Multiplication & Division Properties** Date _______________

Objective

After studying this section, you will be able to

- Apply the multiplication and division properties of segments and angles

Theorem 14 ***If segments (or angles) are congruent, their like multiples are congruent. (Multiplication Property)***

Theorem 15 ***If segments (or angles) are congruent, their like divisions are congruent. (Division Property)***

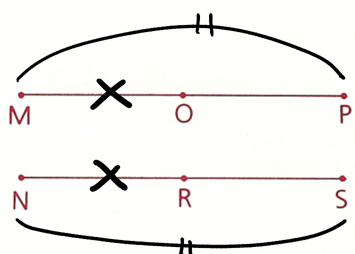
Using the Multiplication and Division Properties in Proofs

1. Look for a double use of the word *midpoint* or *trisect* or *bisects* in the given information.
2. The Multiplication Property is used when the segments or angles in the conclusion are *greater than* those in the given information.
3. The Division Property is used when the segments or angles in the conclusion are *smaller than* those in the given information.

Add → big
Subtract → small

Part Two: Sample Problems

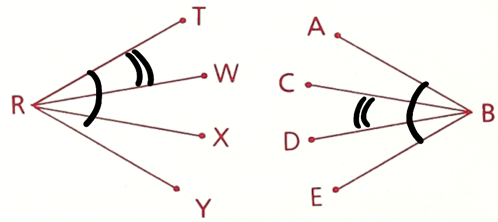
Problem 1 *Given:* $\overline{MP} \cong \overline{NS}$;
O is the midpoint of $\overline{MP}$.
R is the midpoint of $\overline{NS}$.
Prove: $\overline{MO} \cong \overline{NR}$



Proof

Statements	Reasons
1 $\overline{MP} \cong \overline{NS}$	1 Given
2 O is the midpoint of $\overline{MP}$.	2 Given
3 R is the midpoint of $\overline{NS}$.	3 Given
4 $\overline{MO} \cong \overline{NR}$	4 ÷ (1,2,3)

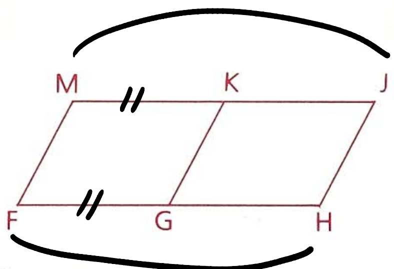
Problem 2 Given: $\angle TRY \cong \angle ABE$;
$\overrightarrow{RW}$ and $\overrightarrow{RX}$ trisect $\angle TRY$.
$\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.
Conclusion: $\angle TRW \cong \angle CBD$



Proof

Statements	Reasons
1 $\angle TRY \cong \angle ABE$	1 Given
2 $\overrightarrow{RW}$ and $\overrightarrow{RX}$ trisect $\angle TRY$.	2 Given
3 $\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.	3 Given
4 $\angle TRW \cong \angle CBD$	4 $\div$

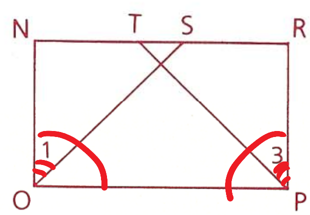
Problem 3 Given: $\overline{MK} \cong \overline{FG}$;
$\overline{KG}$ bisects $\overline{MJ}$ and $\overline{FH}$.
Prove: $\overline{MJ} \cong \overline{FH}$



Proof

Statements	Reasons
1 $\overline{MK} \cong \overline{FG}$	1 Given
2 $\overline{KG}$ bisects $\overline{MJ}$ and $\overline{FH}$.	2 Given
3 $\overline{MJ} \cong \overline{FH}$	3 Mult

Problem 4 Given: $\angle NOP \cong \angle RPO$;
$\overrightarrow{PT}$ bisects $\angle RPO$.
$\overrightarrow{OS}$ bisects $\angle NOP$.
$\angle NSO$ is comp. to $\angle 1$.
$\angle RTP$ is comp. to $\angle 3$.
Prove: $\angle NSO \cong \angle RTP$



Proof

Statements	Reasons
1 $\angle NOP \cong \angle RPO$	1 Given
2 $\overrightarrow{PT}$ bisects $\angle RPO$.	2 Given
3 $\overrightarrow{OS}$ bisects $\angle NOP$.	3 Given
4 $\angle 1 \cong \angle 3$	4 $\div$
5 $\angle NSO$ is comp. to $\angle 1$.	5 Given
6 $\angle RTP$ is comp. to $\angle 3$.	6 Given
7 $\angle NSO \cong \angle RTP$	7 $\angle$s comp $\cong \angle$s (4 5 6) $\Rightarrow \cong \angle$s

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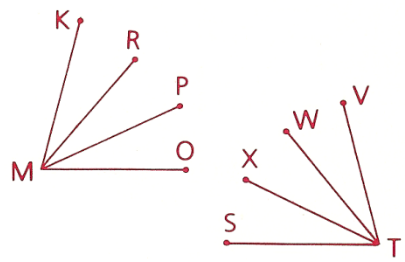
Homework

Before starting the proofs in this problem set, reread the chart on page 90.

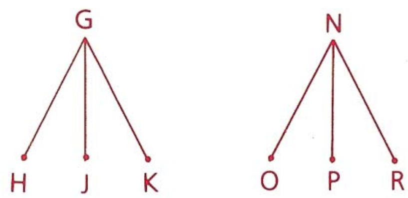
1 Given: $\angle KMR \cong \angle VTW$;
$\overrightarrow{MR}$ and $\overrightarrow{MP}$ trisect $\angle KMO$.
$\overrightarrow{TX}$ and $\overrightarrow{TW}$ trisect $\angle STV$.

Prove: $\angle KMO \cong \angle STV$

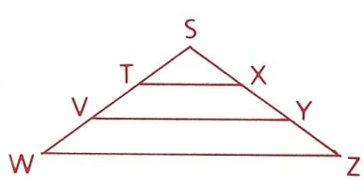


2 Use the given information to find the value of x.

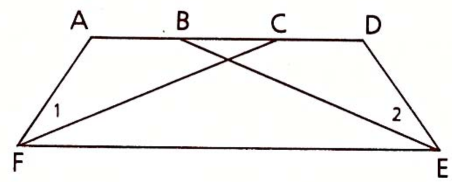
a $\angle HGJ \cong \angle ONP$;
$\overrightarrow{GJ}$ and $\overrightarrow{NP}$ are $\angle$ bisectors.
$\angle HGK = 50°$,
$\angle ONR = (2x + 10)°$



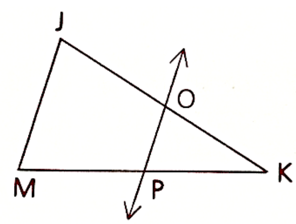
b $\overline{SW} \cong \overline{SZ}$;
$\overleftrightarrow{TX}$ and $\overleftrightarrow{VY}$ trisect $\overline{SW}$ and $\overline{SZ}$.
$ST = 12$,
$YZ = x - 4$



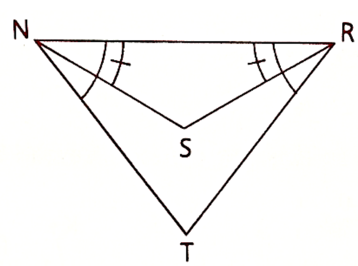
4 Given: $\angle AFE \cong \angle DEF$;
$\overrightarrow{FC}$ bisects $\angle AFE$.
$\overrightarrow{EB}$ bisects $\angle DEF$.
Conclusion: $\angle 1 \cong \angle 2$



5 Given: $\overline{JK} \cong \overline{MK}$;
$\overleftrightarrow{OP}$ bisects $\overline{JK}$ and $\overline{MK}$.
Prove: $\overline{JO} \cong \overline{PK}$



6 Given: $\angle TNR \cong \angle TRN$,
$\angle NRS \cong \angle RNS$
Conclusion: $\underline{\quad ? \quad}$



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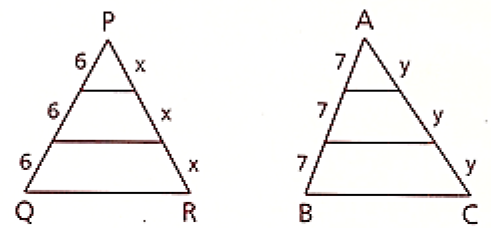
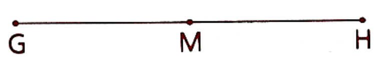
7 **a** If $\overline{PQ} \cong \overline{PR}$ in $\triangle PQR$, what can we conclude?



b If AC = AB + 3 in $\triangle ABC$, what can we conclude?

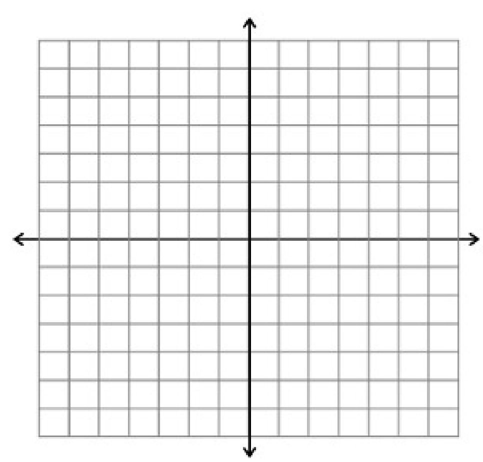
8 Given: M is the midpoint of $\overline{GH}$.
Conclusion: $\overline{GM} \cong \overline{MH}$



9 Given: $(x_1, y_1) = (5, 1)$,
$(x_2, y_2) = (9, 3)$

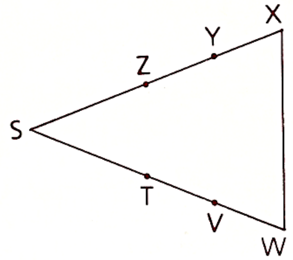
(a) Find: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(b) What is this point called?



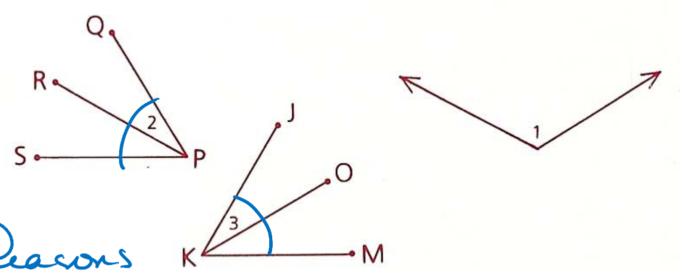
Problem Set B

11 Given: $\overline{SZ} \cong \overline{ST}$,
$\overline{XY} \cong \overline{VW}$;
Y is the midpt. of $\overline{ZX}$.
V is the midpt. of $\overline{TW}$.
Prove: $\overline{SX} \cong \overline{SW}$



Statements	Reasons
1. $\overline{XY} \cong \overline{WV}$ Y is the midpoint of $\overline{XZ}$ V is the midpoint of $\overline{WT}$	1. Given
2.	2. If segments are congruent, then their like multiples are congruent (Multiplication Property)
3.	3. Given
4.	4. If congruent segments are added to congruent segments, then their sums are congruent.

12 Given: $\overrightarrow{PR}$ bisects $\angle QPS$.
$\overrightarrow{KO}$ bisects $\angle JKM$.
$\angle 1$ is supp. to $\angle JKM$.
$\angle 1$ is supp. to $\angle QPS$.
Conclusion: $\angle 2 \cong \angle 3$



Statements	Reasons
1. $\angle JKM$ supp $\angle 1$ $\angle QPS$ supp $\angle 1$	1. Given
2. $\angle JKM \cong \angle QPS$	2. $\angle$s supp to same $\Rightarrow \cong \angle$s
3. $\overrightarrow{KO}$ bis $\angle JKM$ $\overrightarrow{PR}$ bis $\angle QPS$	3. Given
4. $\angle 2 \cong \angle 3$	4. Division

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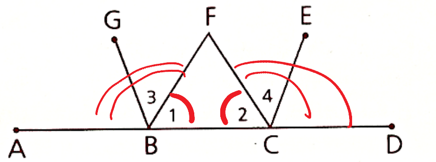
13 Given: $\angle 1 \cong \angle 2$;
$\overrightarrow{BG}$ bisects $\angle ABF$.
$\overrightarrow{CE}$ bisects $\angle FCD$.
Prove: $\angle 3 \cong \angle 4$



revisit 2.4

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1$ supp $\angle ABF$; $\angle 2$ supp $\angle FCD$	2. st $\angle$ ⇒ supps
3. $\angle ABF \cong \angle FCD$	3. $\angle$s supp to $\cong$ $\angle$s ⇒ $\cong$ $\angle$s
4. $\overrightarrow{BG}$ bis $\angle ABF$; $\overrightarrow{CE}$ bis $\angle FCD$	4. Given
5. $\angle 3 \cong \angle 4$	5. Divide

SUPPLEMENTS & COMPLEMENTS & ANGLES

OH MY!

14 If four times the supplement of an angle is added to eight times the angle's complement, the sum is equivalent to three straight angles. Find the measure of the angle that is supplementary to the complement.

$$4(180-x) + 8(90-x) = 3(180)$$

$$4(180) - 4x + 4(180) - 8x = 3(180)$$

$$-12x + 8(180) = 3(180)$$

$$+12x \quad -3(180) \quad -3(180) \quad +12x$$

$$5(180) = 12x$$

$$5 \cdot 6 \cdot 2 \cdot 15 \qquad 6 \cdot 2\, x$$

$75 = x$ → ANGLE

$15 = 90 - 75$ ← COMP

Supp of Comp: $180 - 15 = \boxed{165^\circ}$

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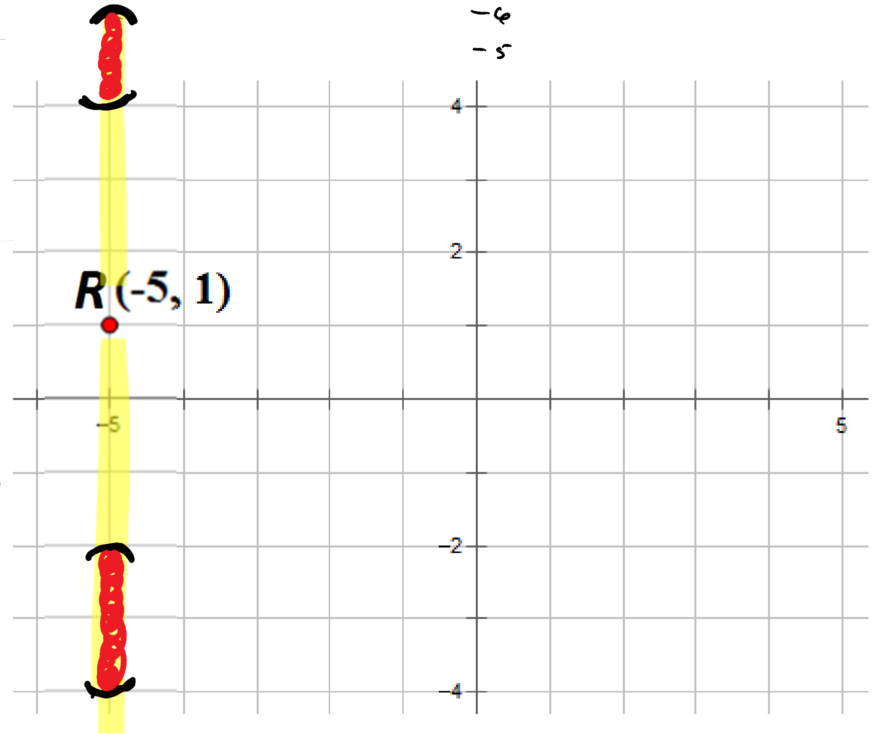
Problem Set C

15 Point T is located on the graph so that $\overleftrightarrow{RT}$ is perpendicular to the x-axis and $3 < RT < 5$. Find the restrictions on the coordinates of T.

$T:(x,y) \mid x = -5$

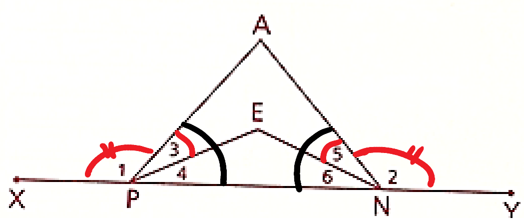
&

$(4<y<6) \cup (-4<y<-2)$



16 Given: Diagram as shown
$\angle 1 \cong \angle 2$,
$\overrightarrow{PE}$ bis. $\angle APN$,
$\overrightarrow{NE}$ bis. $\angle ANP$

Prove: $\angle XPE \cong \angle ENY$



Statements	Reasons
1. Diagram as shown	1. Given
2. $\angle XPY$ is a straight $\angle$	2. Diag ⇒ st∠ (1
3. $\angle 1$ is supplementary to $\angle APN$	3. st∠ ⇒ supp∠s (2)
4. $\angle 2$ is supplementary to $\angle ANP$	4. st∠ ⇒ supp∠s (2)
5. $\angle 1 \cong \angle 2$	5. Given
6. $\angle APN \cong \angle ANP$	6. ∠s supp to ≅ ∠s ⇒ ≅ ∠s (345)
7. $\overrightarrow{PE}$ bisects $\angle APN$	7. Given
8. $\overrightarrow{NE}$ bisects $\angle ANP$	8. Given
9. $\angle 3 \cong \angle 5$	9. ÷ (6 7 8)
10. $\angle XPE \cong \angle YNE$	10. Add (5 & 9)