

Name

Adv Geo -

2.4 Congruent Supplements and Complements

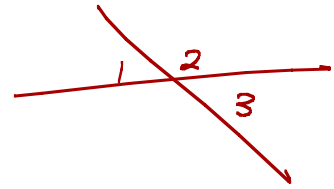
Note Tit

Objective

After studying this section, you will be able to

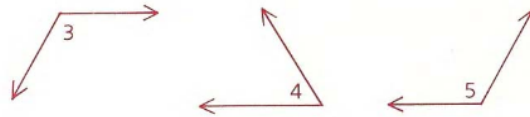
- Prove angles congruent by means of four new theorems

Theorem 4 *If angles are supplementary to the same angle, then they are congruent.*



Given: $\angle 3$ is supp. to $\angle 4$.
 $\angle 5$ is supp. to $\angle 4$.

Prove: $\angle 3 \cong \angle 5$



Proof: $\angle 3$ is supp. to $\angle 4$, so $m\angle 3 + m\angle 4 = 180$.
 Therefore, $m\angle 3 = 180 - m\angle 4$.
 $\angle 5$ is supp. to $\angle 4$, so $m\angle 5 + m\angle 4 = 180$.
 Therefore, $m\angle 5 = 180 - m\angle 4$.
 Since $\angle 3$ and $\angle 5$ have the same measure, $\angle 3 \cong \angle 5$.

If angles are supplementary to the same angle, then they are congruent.

Given: $\angle 3$ is supp. to $\angle 4$.
 $\angle 5$ is supp. to $\angle 4$.

Prove: $\angle 3 \cong \angle 5$

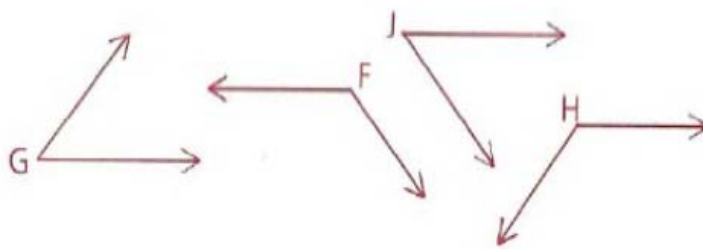


Statements	Reasons
1. $\angle 3$ is supp. to $\angle 4$	1. Given
2. $m\angle 3 + m\angle 4 = 180$	2. $\text{suppl } \angle s \Rightarrow 180^\circ$ (1)
3. $m\angle 3 = 180 - m\angle 4$	3. subtract (2)
4. $\angle 5$ is supp. to $\angle 4$	4. Given
5. $m\angle 5 + m\angle 4 = 180$	5. $\text{suppl } \angle s \Rightarrow 180^\circ$ (4)
6. $m\angle 5 = 180 - m\angle 4$	6. subtract (5)
7. $\angle 3 \cong \angle 5$	7. $\text{same meas } \Rightarrow \cong \angle s$ (3, 6)

Theorem 5 *If angles are supplementary to congruent angles, then they are congruent.*

Given: $\angle F$ is supp. to $\angle G$.
 $\angle H$ is supp. to $\angle J$.
 $\angle G \cong \angle J$

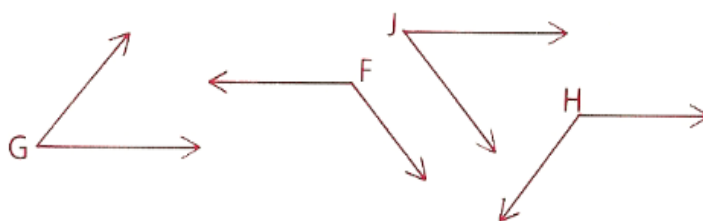
Conclusion: $\angle F \cong \angle H$



If angles are supplementary to congruent angles, then they are congruent.

Given: $\angle F$ is supp. to $\angle G$.
 $\angle H$ is supp. to $\angle J$.
 $\angle G \cong \angle J$

Conclusion: $\angle F \cong \angle H$



Statements	Reasons
1. $\angle F$ is supp. to $\angle G$	1. <i>Given</i>
2. $\angle F + \angle G = 180^\circ$	2. <i>suppl \angles $\Rightarrow 180^\circ$</i>
3. $\angle F = 180^\circ - \underline{\angle G}$	3. <i>subtract.</i>
4. $\angle H$ is supp. to $\angle J$	4. <i>Given</i>
5. $\angle H + \angle J = 180^\circ$	5. <i>suppl \angles $\Rightarrow 180^\circ$</i>
6. $\angle H = 180^\circ - \underline{\angle J}$	6. <i>subtract</i>
7. $\underline{\angle G \cong \angle J}$	7. <i>Given</i>
8. $\angle F \cong \angle H$	8. <i>same meas $\Rightarrow \cong \angle$s</i>

Theorem 6 *If angles are complementary to the same angle, then they are congruent.*

Theorem 7 *If angles are complementary to congruent angles, then they are congruent.*

Before starting the assignment, memorize Theorems 4–7. The key to the use of these theorems is to look for the double use of the word *complementary* or *supplementary* in a problem.

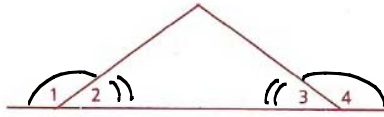
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Examples

Problem 1

Given: $\angle 1$ is supp. to $\angle 2$.
 $\angle 3$ is supp. to $\angle 4$.
 $\angle 1 \cong \angle 4$



Conclusion: $\angle 2 \cong \angle 3$

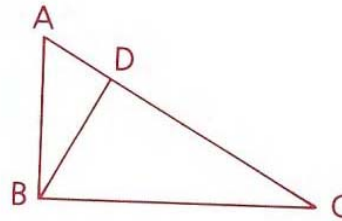
Proof

Statements	Reasons
1 $\angle 1$ is supp. to $\angle 2$.	1 Given
2 $\angle 3$ is supp. to $\angle 4$.	2 Given
3 $\angle 1 \cong \angle 4$	3 Given
4 $\angle 2 \cong \angle 3$	4 $\angle s$ supp to $\cong \angle s \Rightarrow \cong \angle s$

Problem 2

Given: $\angle A$ is comp. to $\angle C$.
 $\angle DBC$ is comp. to $\angle C$.

Conclusion: ?



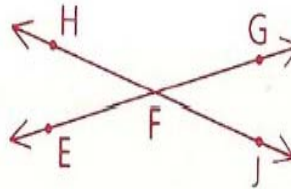
Proof

Statements	Reasons
1 $\angle A$ is comp. to $\angle C$.	1 Given
2 $\angle DBC$ is comp. to $\angle C$.	2 Given
3 $\angle A \cong \angle DBC$	3 $\angle s$ comp to same $\angle \Rightarrow \cong \angle s$

Problem 3

Given: Diagram as shown

Prove: $\angle HFE \cong \angle GFJ$



Proof

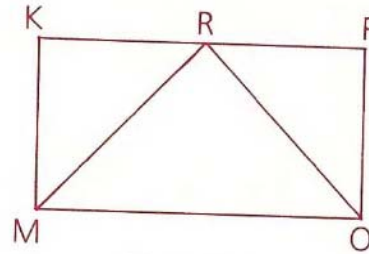
Statements	Reasons
1 Diagram as shown.	1 Given
2 $\angle EFG$ is a straight \angle .	2 $Diag \Rightarrow st\angle$
3 $\angle HFE$ is supp. to $\angle HFG$.	3 $st\angle \Rightarrow supp\angle s$
4 $\angle HFJ$ is a straight \angle .	4 $Diag \Rightarrow st\angle$
5 $\angle GFJ$ is supp. to $\angle HFG$.	5 $st\angle \Rightarrow supp\angle s$
6 $\angle HFE \cong \angle GFJ$	6 $\angle s$ supp to same $\angle \Rightarrow \cong \angle s$

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Problem 4

Given: $\overline{KM} \perp \overline{MO}$,
 $\overline{PO} \perp \overline{MO}$,
 $\angle KMR \cong \angle POR$
 Prove: $\angle ROM \cong \angle RMO$



Proof

Statements	Reasons
1 $\overline{KM} \perp \overline{MO}, \overline{PO} \perp \overline{MO}$	1 Given
2 $\angle KMO$ is a right \angle . $\angle POM$ rt \angle	2 $\perp \Rightarrow$ rt \angle
3 $\angle RMO$ is comp. to <u>$\angle KMR$</u> .	3 rt $\angle \Rightarrow$ comp \angle s
4 In a similar manner, $\angle ROM$ is comp. to <u>$\angle POR$</u> .	4 rt $\angle \Rightarrow$ comp \angle s
5 <u>$\angle KMR \cong \angle POR$</u>	5 Given
6 $\angle ROM \cong \angle RMO$	6 \angle s comp to <u>$\cong \angle$s</u> \Rightarrow $\cong \angle$ s

Don't do this

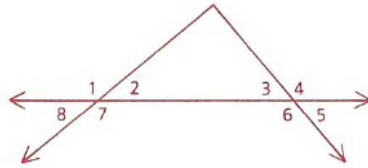
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Homework 1-19 all

Before starting the assignment, memorize Theorems 4-7. The key to the use of these theorems is to look for the double use of the word *complementary* or *supplementary* in a problem.

- 1 Given: $\angle 2$ is comp. to $\angle 3$.
 $\angle 4 = 131^\circ$

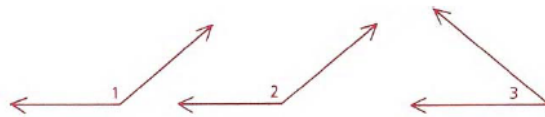


Find the measure of each of the following angles.

- a $\angle 3$ c $\angle 5$ e $\angle 1$ g $\angle 7$
b $\angle 6$ d $\angle 2$ f $\angle 8$

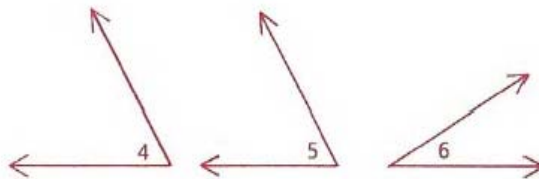
- 2 Given: $\angle 1$ is supp. to $\angle 3$.
 $\angle 2$ is supp. to $\angle 3$.

Prove: $\angle 1 \cong \angle 2$



- 3 Given: $\angle 4$ is comp. to $\angle 6$.
 $\angle 5$ is comp. to $\angle 6$.

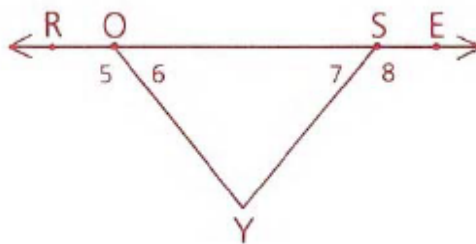
Prove: $\angle 4 \cong \angle 5$



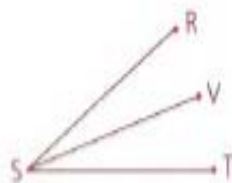
- 4 One of two supplementary angles is four times the other. Find the larger angle.

- 5 One of two complementary angles is 20° larger than the other. Find the measure of each.

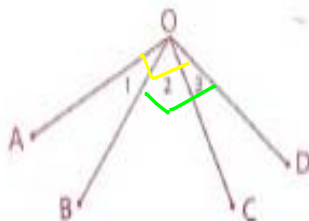
8 Given: Diagram as shown,
 $\angle 6 \cong \angle 7$
 Prove: $\angle 5 \cong \angle 8$



9 Given: \vec{SV} bisects $\angle RST$,
 Conclusion: $\angle RSV \cong \angle TSV$



10 Given: $\vec{OA} \perp \vec{OC}$,
 $\vec{OB} \perp \vec{OD}$
 Prove: $\angle 1 \cong \angle 3$



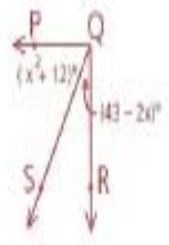
Statements	Reasons
1. $\vec{OA} \perp \vec{OC}$	1. GIVEN
2. $\angle AOC$ $\text{rt} \angle$	2. $\perp \Rightarrow \text{rt} \angle$
3. $\angle 1$ comp <u>$\angle 2$</u>	3. $\text{rt} \angle \Rightarrow \text{comp} \angle$ s
4. $\vec{OB} \perp \vec{OD}$	4. Given
5. $\angle BOD$ $\text{rt} \angle$	5. $\perp \Rightarrow \text{rt} \angle$
6. $\angle 3$ comp <u>$\angle 2$</u>	6. $\text{rt} \angle \Rightarrow \text{comp} \angle$ s
7. $\angle 1 \cong \angle 3$	7. \angle s comp <u>same</u> $\angle \Rightarrow \cong \angle$

- 12 The measure of the supp. of an \angle exceeds 3 times the measure of the comp. of the \angle by 10. Find the measure of the comp.

$$(180 - x) = 3(90 - x) + 10$$
$$\begin{array}{r} 180 - x \\ + 3x \\ \hline 180 + 2x = 280 \end{array} \qquad \begin{array}{r} 270 - 3x + 10 \\ + 3x \\ \hline 280 \end{array}$$

$$180 + 2x = 280$$

- 15 Given: $\overline{PQ} \perp \overline{QR}$
Find: $m\angle PQS$



- 17 If three times the supp. of an \angle is subtracted from seven times the comp. of the \angle , the answer is the same as that obtained by trisecting a right \angle . Find the supplement.