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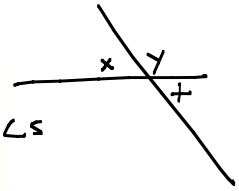
Note Tit

2.4 Congruent Supplements and Complements

Objective

After studying this section, you will be able to

- Prove angles congruent by means of four new theorems



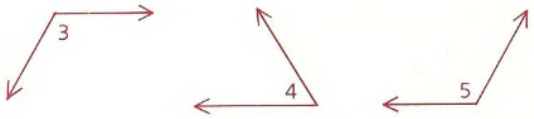
Theorem 4 ***If angles are supplementary to the same angle, then they are congruent.***

∠s supp to same ∠ are ≅∠s

Proof for Thm construct thm to prove itself

Given: ∠3 is supp. to ∠4.
∠5 is supp. to ∠4.

Prove: ∠3 ≅ ∠5

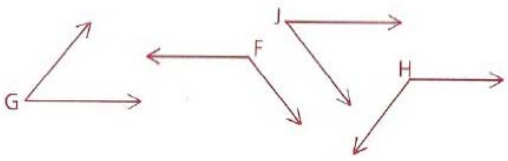


Proof: ∠3 is supp. to ∠4, so m∠3 + m∠4 = 180.
Therefore, m∠3 = 180 − m∠4.
∠5 is supp. to ∠4, so m∠5 + m∠4 = 180.
Therefore, m∠5 = 180 − m∠4.
Since ∠3 and ∠5 have the same measure, ∠3 ≅ ∠5.

Theorem 5 ***If angles are supplementary to congruent angles, then they are congruent.***

Given: ∠F is supp. to ∠G.
∠H is supp. to ∠J.
∠G ≅ ∠J

Conclusion: ∠F ≅ ∠H



Theorem 6 ***If angles are complementary to the same angle, then they are congruent.***

Theorem 7 ***If angles are complementary to congruent angles, then they are congruent.***

Before starting the assignment, memorize Theorems 4–7. The key to the use of these theorems is to look for the double use of the word complementary or supplementary in a problem.

2.4

Theorem (T)

If angles are supplementary to the same angle, then they are congruent.

Given: ∠3 is supp. to ∠4.
∠5 is supp. to ∠4.
Prove: ∠3 ≅ ∠5

Statements	Reasons
1. ∠3 is supp. to ∠4	1. Given
2. m∠3 + m∠4 = 180	2. suppLs ⇒ 180°
3. m∠3 = 180 − m∠4	3. subtract
4. ∠5 is supp. to ∠4	4. Given
5. m∠5 + m∠4 = 180	5. suppLs ⇒ 180°
6. m∠5 = 180 − m∠4	6. subtract
7. ∠3 ≅ ∠5	7. same meas ⇒ ≅ Ls

T

If angles are supplementary to congruent angles, then they are congruent.

Given: ∠F is supp. to ∠G.
∠H is supp. to ∠J.
∠G ≅ ∠J
Conclusion: ∠F ≅ ∠H

Statements	Reasons
1. ∠F is supp. to ∠G	1. Given
2. ∠F + ∠G = 180	2. suppLs ⇒ 180°
3. ∠F = 180 − ∠G	3. subtract
4. ∠H is supp. to ∠J	4. Given
5. ∠H + ∠J = 180	5. suppLs ⇒ 180°
6. ∠H = 180 − ∠J	6. subtract
7. ∠G ≅ ∠J	7. Given
8. ∠F ≅ ∠H	8. same meas ⇒ ≅ Ls

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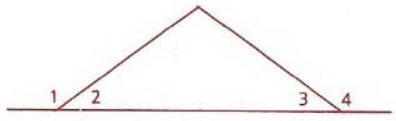
2.4 Congruent Supplements and Complements

Examples

Problem 1

Given: $\angle 1$ is supp. to $\angle 2$.
$\angle 3$ is supp. to $\angle 4$.
$\angle 1 \cong \angle 4$

Conclusion: $\angle 2 \cong \angle 3$



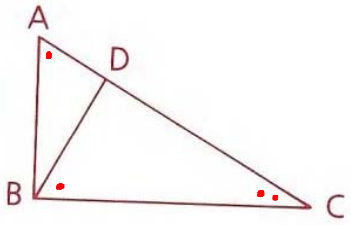
Proof

Statements	Reasons
1 $\angle 1$ is supp. to $\angle 2$.	1 Given
2 $\angle 3$ is supp. to $\angle 4$.	2 Given
3 $\angle 1 \cong \angle 4$	3 Given
4 $\angle 2 \cong \angle 3$	4 $\angle$s supp to $\cong$ $\angle$s $\Rightarrow \cong \angle$

Problem 2

Given: $\angle A$ is comp. to $\angle C$.
$\angle DBC$ is comp. to $\angle C$.

Conclusion: ?



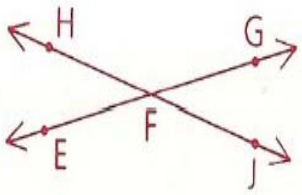
Proof

Statements	Reasons
1 $\angle A$ is comp. to $\angle C$.	1 Given
2 $\angle DBC$ is comp. to $\angle C$.	2 Given
3 $\angle A \cong \angle DBC$	3 $\angle$s comp same $\angle \Rightarrow \cong \angle$s

Problem 3

Given: Diagram as shown

Prove: $\angle HFE \cong \angle GFJ$



Proof

Statements	Reasons
1 Diagram as shown.	1 Given
2 $\angle EFG$ is a straight $\angle$.	2 Diag $\Rightarrow$ st$\angle$
3 $\angle HFE$ is supp. to $\angle HFG$.	3 st$\angle \Rightarrow$ supp$\angle$s
4 $\angle HFJ$ is a straight $\angle$.	4 Diag $\Rightarrow$ st$\angle$
5 $\angle GFJ$ is supp. to $\angle HFG$.	5 st$\angle \Rightarrow$ supp$\angle$s
6 $\angle HFE \cong \angle GFJ$	6 $\angle$s supp same $\angle \Rightarrow \cong \angle$s

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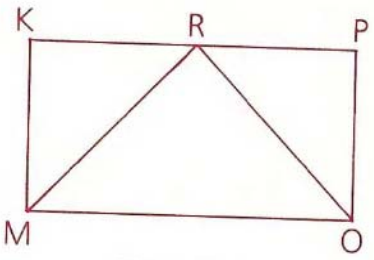
2.4 Congruent Supplements and Complements

Problem 4

Given: $\overline{KM} \perp \overline{MO}$, $\overline{PO} \perp \overline{MO}$, $\angle KMR \cong \angle POR$

Prove: $\angle ROM \cong \angle RMO$



Proof

Statements	Reasons
1 $\overline{KM} \perp \overline{MO}$	1 Given
2 $\angle KMO$ is a right $\angle$.	2 $\perp \Rightarrow$ rt $\angle$
3 $\angle RMO$ is comp. to $\angle KMR$.	3 rt $\angle \Rightarrow$ comp $\angle$s
~~4 In a similar manner, $\angle ROM$ is comp. to $\angle POR$.~~	~~4~~
5 $\angle KMR \cong \angle POR$	5 Given
6 $\angle ROM \cong \angle RMO$	6 $\angle$s comp to $\cong$ $\angle$s are $\cong$

Don't do this

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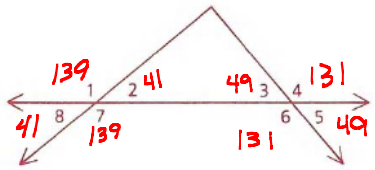
2.4 Congruent Supplements and Complements

Homework 1–19 all

Before starting the assignment, memorize Theorems 4–7. The key to the use of these theorems is to look for the double use of the word *complementary* or *supplementary* in a problem.

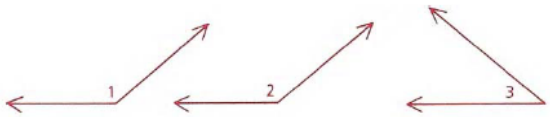
1 Given: $\angle 2$ is comp. to $\angle 3$.
$\angle 4 = 131°$



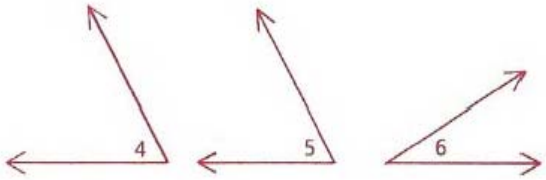
Find the measure of each of the following angles.

a $\angle 3$
b $\angle 6$
c $\angle 5$
d $\angle 2$
e $\angle 1$
f $\angle 8$
g $\angle 7$

2 Given: $\angle 1$ is supp. to $\angle 3$.
$\angle 2$ is supp. to $\angle 3$.
Prove: $\angle 1 \cong \angle 2$



3 Given: $\angle 4$ is comp. to $\angle 6$.
$\angle 5$ is comp. to $\angle 6$.
Prove: $\angle 4 \cong \angle 5$

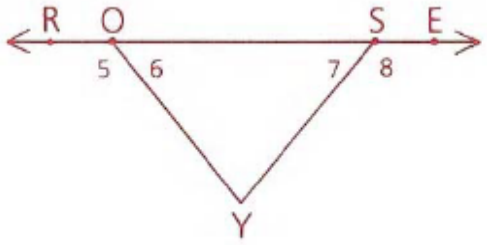


4 One of two supplementary angles is four times the other. Find the larger angle.

5 One of two complementary angles is 20° larger than the other. Find the measure of each.

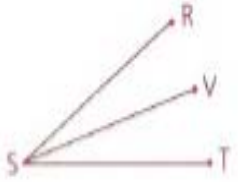
8 Given: Diagram as shown,
$\angle 6 \cong \angle 7$

Prove: $\angle 5 \cong \angle 8$



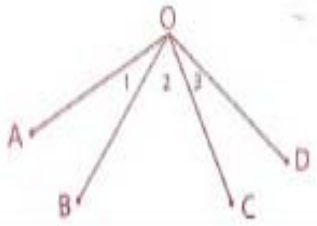
9 Given: $\overrightarrow{SV}$ bisects $\angle RST$.

Conclusion: $\angle RSV \cong \angle TSV$



10 Given: $\overleftrightarrow{OA} \perp \overleftrightarrow{OC}$, $\overleftrightarrow{OB} \perp \overleftrightarrow{OD}$

Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\overleftrightarrow{OA} \perp \overleftrightarrow{OC}$	1. Given
2. $\angle AOC$ rt $\angle$	2. $\perp \Rightarrow$ rt $\angle$
3. $\angle 1$ comp $\angle 2$	3. rt $\angle \Rightarrow$ comp $\angle$s
4. $\overleftrightarrow{OB} \perp \overleftrightarrow{OD}$	4. Given
5. $\angle BOD$ rt $\angle$	5. $\perp \Rightarrow$ rt $\angle$
6. $\angle 3$ comp $\angle 2$	6. rt $\angle \Rightarrow$ comp $\angle$s
7. $\angle 1 \cong \angle 3$	7. $\angle$s comp same $\angle \Rightarrow \cong \angle$

12 The measure of the supp. of an ∠ exceeds 3 times the measure of the comp. of the ∠ by 10. Find the measure of the comp.

$180 - x = 3(90 - x) + 10$

$2(90) - x = 3(90) - 3x + 10$

$-2(90) \quad +3x \quad 2(90) \quad +3x$

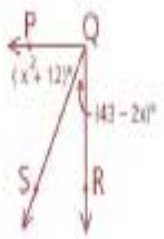
$2x = 90 + 10$

$2x = 100$

$x = 50$

$\text{Comp} = 40^\circ$

15 Given: $\overline{PQ} \perp \overline{QR}$
Find: $m\angle PQS$



17 If three times the supp. of an ∠ is subtracted from seven times the comp. of the ∠, the answer is the same as that obtained by trisecting a right ∠. Find the supplement.