AMDG

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1.8: Statements of Logic

Objective

After studying this section, you will be able to

- Recognize conditional statements
- Recognize the negation of a statement
- Recognize the converse, the inverse, and the contrapositive of a statement
- Use the chain rule to draw conclusions

Review of Conditional Statements

In this section, we will review and extend the discussion of conditional statements in Section 1.7. Recall that a conditional statement is a sentence that is in the form "If . . ., then . . ." Many declarative sentences can be rewritten in conditional form.

Declarative Sentence:	*Conditional Form:*
Two straight angles are congruent.	If two angles are straight angles, then they are congruent.

Remember that

- The clause following the word *if* is called the hypothesis
- The clause following the word *then* is called the conclusion
- The conditional statement "If p, then q" can be written in symbols as $p \Rightarrow q$

Negation

The ***negation*** of any statement p is the statement "not p." (Thus, the negation of "It is raining" is "It is not raining.") The symbol for "not p" is $\sim p$. Notice also that the negation of "It is not raining" is "It is raining"—in general, not (not p) = p, or $\sim\sim p = p$.

Converse, Inverse, and Contrapositive

Every conditional statement "If p, then q" has three other statements associated with it. (You have already been introduced to the first of these—the converse).

1 A converse (If q, then p.)
2 An ***inverse*** (If $\sim p$, then $\sim q$.)
3 A ***contrapositive*** (If $\sim q$, then $\sim p$.)

Example *Find the converse, the inverse, and the contrapositive of the statement "If you live in Atlanta, then you live in Georgia."*

The statement is in the form "If p, then q," with p being "You live in Atlanta" and q being "You live in Georgia."

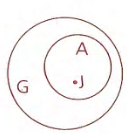
Converse: "If you live in Georgia, then you live in Atlanta." (If q, then p.)

Inverse: "If you don't live in Atlanta, then you don't live in Georgia." (If $\sim p$, then $\sim q$.)

Contrapositive: "If you don't live in Georgia, then you don't live in Atlanta." (If $\sim q$, then $\sim p$.)

You may have noticed that some of the statements in the preceding example are not necessarily true, although the original statement is true. A useful tool for determining whether or not a conditional statement is true or false is a ***Venn diagram***. Assume that the following statement is true: "If Jenny lives in Atlanta, then Jenny must live in Georgia."

All the people who live in Georgia are represented by points on the large circle and in its interior (G).



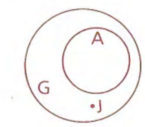
All the people who live in Atlanta are represented by points on the small circle and in its interior (A).

Notice that every person in set A, including Jenny (J), is also in set G.

The Venn diagram for this conditional statement may be used to test whether its converse, inverse, and contrapositive are true or false.

Converse: "If Jenny lives in Georgia, then she must live in Atlanta."

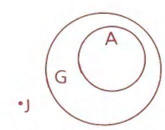
This statement is not necessarily true, as shown by the diagram. Notice that point J may lie in G but not in A. This means that Jenny could live in Georgia and yet not live in Atlanta.



In general, the converse of a conditional statement is not necessarily true. Try a similar argument with the same Venn diagram to convince yourself that the inverse of a conditional statement is also not necessarily true.

Contrapositive: "If Jenny does not live in Georgia, then she does not live in Atlanta."

This time point J lies outside of G, so it *cannot* lie in A. Any point that is not in G is also not in A. Therefore, the contrapositive is true.



This analysis suggests the following important theorem:

Theorem 3 ***If a conditional statement is true, then the contrapositive of the statement is also true. (If p, then q $\Leftrightarrow$ If $\sim q$, then $\sim p$.)***

In other words, a statement and its contrapositive are *logically equivalent*.

Chains of Reasoning

Each proof that you do involves a series of steps in a logical sequence. In many cases, the sequence will take the following form.

If $p \Rightarrow q$ and $q \Rightarrow r$, then $p \Rightarrow r$.

This is called the ***chain rule***, and a series of conditional statements so connected is known as a ***chain of reasoning***.

Example *If we accept the two statements "If you study hard, then you will earn a good grade" ($p \Rightarrow q$) and "If you earn a good grade, then your family will be happy" ($q \Rightarrow r$), what can we conclude?*

We can conclude that $p \Rightarrow r$—that is, if you study hard, then your family will be happy.

Problem Set A

1 Write each sentence in conditional ("If . . ., then . . .") form.

a Eighteen-year-olds may vote in federal elections.

b Opposite angles of a parallelogram are congruent.

2 Write the converse, the inverse, and the contrapositive of each statement. Determine the truth of each of the new statements.

a If each side of a triangle has a length of 10, then the triangle's perimeter is 30.

CONVERSE

INVERSE

CONTRAPOSITIVE

b If an angle is acute, then it has a measure greater than 0 and less than 90.

CONVERSE

INVERSE

CONTRAPOSITIVE

3 If a conditional statement and its converse are both true, the statement is said to be *biconditional*. Which of these statements is biconditional?

a If two angles are congruent, then they have the same measure.

b If two angles are straight angles, then they are congruent.

This is the biconditional arrow:

$\Leftrightarrow$

Triangle $\Leftrightarrow$ polygon with 3 sides

4 Draw a Venn diagram for the true conditional statement "If a person lives in Chicago, then the person lives in Illinois." Assuming that each of the following "Given . . ." statements is true, determine the truth of the conclusion.

a Given: Penny lives in Chicago.
Conclusion: Penny lives in Illinois.

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b Given: Benny lives in Illinois.
Conclusion: Benny lives in Chicago.

c Given: Kenny does not live in Chicago.
Conclusion: Kenny must live in Illinois.

d Given: Denny does not live in Illinois.
Conclusion: Denny lives in Chicago.

What conclusion can you draw, using all of the following statements?

$\sim q \rightarrow s$ $t \rightarrow \sim r$ $q \rightarrow t$ $u \rightarrow \sim s$

5 Write a concluding statement for each of the following chains of reasoning.

a
$a \Rightarrow b$
$d \Rightarrow \sim c$
$\sim c \Rightarrow a$
$b \Rightarrow f$

b
$p \Rightarrow \sim q$
$r \Rightarrow q$
$s \Rightarrow r$

c If weasels walk wisely, then cougars call their cubs.
If goats go to graze, then horses head for home.
If cougars call their cubs, then goats go to graze.
If bobcats begin to browse, then weasels walk wisely.

7 Rewrite the following sentence in conditional form and find its converse, inverse, and contrapositive: "A square is a quadrilateral with four congruent sides."

CONVERSE

INVERSE

CONTRAPOSITIVE

8 Write the converse, the inverse, and the contrapositive of each statement.

a If a ray bisects an angle, it divides the angle into two congruent angles.

CONVERSE

INVERSE

CONTRAPOSITIVE

b If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

CONVERSE

INVERSE

CONTRAPOSITIVE

9 What conclusion can be drawn from the following?

$\sim c \Rightarrow \sim f \qquad g \Rightarrow b \qquad p \Rightarrow f \qquad c \Rightarrow \sim b$