

1.7: Deductive structure

**Objectives**

After studying this section, you will be able to

- Recognize that geometry is based on a deductive structure
- Identify undefined terms, postulates, and definitions
- Understand the characteristics of theorems and the ways in which they can be used in proofs

Geometry is based on a **deductive structure**—a system of thought in which conclusions are justified by means of previously assumed or proved statements. Every deductive structure contains the following four elements.

- Undefined terms
- Assumptions known as **postulates**
- Definitions
- Theorems and other conclusions

These are the only items that should be included in the axiomatic system notebook (ASN).

*Undefined terms* are undefined.

*Postulates* are not proven, but assumed true. They are not always reversible.

*Definitions* are reversible.

*Theorems* need to be proven. They are not always reversible.

**Symbolic Logic**

*Conditional Statement:* If, then.  $a \Rightarrow b$  is read “if a then b” or “a implies b”.  
It’s also called an *implication* or *hypothesis*.

*Converse*  $b \Rightarrow a$

*Inverse*  $\sim a \Rightarrow \sim b$

*Contrapositive*  $\sim b \Rightarrow \sim a$

**Truth tables** get a little more complicated when conjunctions and disjunctions of statements are included.

$\wedge$  is INTERSECTION,  $\vee$  is UNION  
 $\wedge$  is AND,  $\vee$  is OR

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

This is a brief introduction to Boolean Algebra, and is the logic used in search engines.

The **truth table** for an implication, or conditional statement looks like this:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

## 1.7 Homework

- 1 What four elements are found in any deductive structure?
  
- 2 Which of the following kinds of statements are always reversible?  
 a Definitions                      b Theorems                      c Postulates
  
- 3 Answer each question Yes or No.  
 a Do we prove theorems?                      b Do we prove definitions?
  
- 4 Tell whether each of the following statements is a theorem or a definition.  
 a If two angles are right angles, then they are congruent.      b If a ray bisects an angle, it divides the angle into two congruent angles.
  
- 5 a Write the converse of each of the following statements.  
 i If A, then B.  
 ii Rain  $\Rightarrow$  wet  
 iii If an angle is a  $45^\circ$  angle, then it is acute.  
 iv If a point is the midpoint of a segment, it divides the segment into two congruent segments.  
 b Discuss the truth of each of the converses in part a.

	Converse	T or F
i.		
ii.		
iii.		
iv.		

In problems 6 and 7, comment on the reasoning used.

- 6 The school colors are orange and black, so I'll wear my orange skirt to the game and everyone will notice me.
  
- 7 I've flipped this silver dollar five times and the toss has come up heads each time. Thus, the odds are greater than 50-50 that the toss will come up tails next time.

NAME

Ms. Kresovic

Adv Geo period

DATE

**Problem Set B**

In problems 8–12, study each of the arguments and state whether or not the conclusion is deducible. If it is not, comment on the error in the reasoning.

- 8 If a student at Niles High has room 303 as his or her homeroom, the student is a freshman. Joe Jacobs is a student at Niles High and has room 303 as his homeroom. Therefore, Joe Jacobs is a freshman.
  
- 9 If the three angles of a triangle are acute, then the triangle is acute. In triangle ABC, angle A and angle B are acute. Therefore, triangle ABC is acute.
  
- 10 All school buses stop at railroad crossings. A vehicle stopped at the Santa Fe railroad crossing. Therefore, that vehicle is a school bus.
  
- 11 All cloudy days are depressing. Therefore, since I was depressed on Thursday, Thursday was cloudy.
  
- 12 If two angles of a triangle are congruent, then the sides opposite them are congruent. In  $\triangle ABC$ ,  $\angle A \cong \angle B$ . Therefore, in  $\triangle ABC$ ,  $\overline{BC} \cong \overline{AC}$ .

13. **Where in USA?** Four children live in different US states. Find out where each of them lives.

	California	Texas	Illinois	New York
Ann				
Jon				
Mark				
Bill				

- 1. Bill's state does not have an "x" in it; neither does Ann's.
- 2. Mark's state is a single word.
- 3. Bill does not live near the west coast of the US.
- 4. Ann lives west of the Mississippi River.
- 5. Mark's state borders a Great Lake.

**14** The Bronx Zoo has a green lizard, a red crocodile, and a purple monkey. They are the only animals of their kind in existence. One violently windy Saturday, their name tags blew off, and their keeper's journal was torn to shreds. Inasmuch as they were to appear on television at 7:30 Sunday morning, the night watchman had to replace their name tags. He managed to piece together the following information from the mangled journal.

- 1 Wendy cannot get along with the lizard.
- 2 Katie playfully took a bite out of the monkey's ear one month ago.
- 3 Wendy never casts a red reflection in the mirror.
- 4 Jody has the personality of a crocodile, but she isn't one.

Match the animals with their names.

**15. Sports:** Four good friends have four different favorite sports. Read the clues to find out who prefers which sport.

	Kickball	Tennis	Football	Soccer
Robert				
Audrey				
Danny				
Jane				

1. Audrey likes neither soccer nor football.
2. Danny used to like football and kickball best, but he has changed his mind.
3. Neither of the boys here likes soccer best.

**16. River Crossing:** A man needs to cross a river in a canoe. With him, he has a bag of grain, a chicken, and a fox. He can only carry one of the three at a time. If he leaves the grain and the chicken, the chicken will eat the grain. If he takes the grain, the fox will eat the chicken. How does he successfully cross the river with his load?



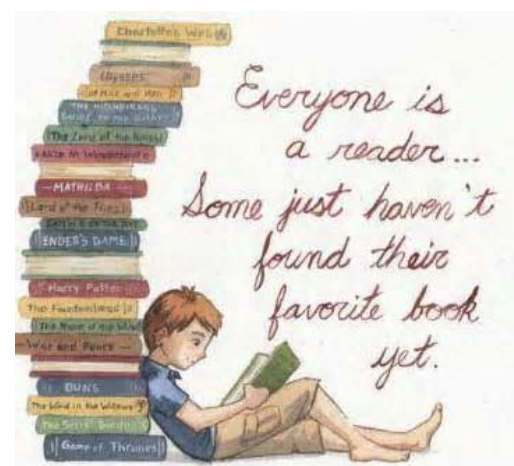
**Brain-buster Challenge (optional). Which book?** Six people all went to their library and borrowed just one novel. Can you find out the first name & surname of the borrower, find out what type of novel it is and who is the main character?

**First Names:** David, Elsie, Gladys, Graham, Joseph, Peter

**Surnames:** Atkins, Branch, Crabtree, Drake, Elfin, Fields

**Main Character:** Craig, Jennifer, Katy, Linda, Malcolm, Stewart

**Type of Book:** Cowboy, Romance, Thriller, Who Dunit, War, Adventure

- Graham, not with the surname Branch, is rather partial to Who Dunit Books and he borrowed one of these, but it didn't have the main character of Jennifer.
- Katy was the main character in a thriller; this book was not borrowed by Elsie or Mr. Fields.
- Joseph Crabtree detested War books and certainly didn't borrow one; the main character in Joseph's book was not Stewart.
- The adventure novel was borrowed by Mrs. Elfin; it didn't have the main character of Craig or Linda.
- Peter's novel, not the cowboy book, had either Jennifer or Malcolm as the main character, but the romance novel had the main character of Stewart or Linda.
- Malcolm was a character in the book borrowed by Mr Atkins.
- Craig was the main character in the book that Gladys borrowed.
- Mr. Atkins didn't borrow a Who Dunit and the person named Branch did not borrow a Cowboy book.
- The person named Fields did not borrow a book where Stewart was the main character.