AMDG

Name
Ms. Kresovic
Adv Geo Period
W 4 Sep 2013 — 1.5 Division of Segments and Angles

Objectives

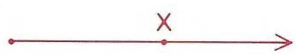
After studying this section, you will be able to

- Identify midpoints and bisectors of segments
- Identify trisection points and trisectors of segments
- Identify angle bisectors
- Identify angle trisectors

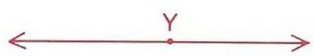
Midpoints and Bisectors of Segments

We shall often work with segments that are divided in half.

Definition A point (or segment, ray, or line) that divides a segment into two congruent segments ***bisects*** the segment. The bisection point is called the ***midpoint*** of the segment.



X is not a midpoint

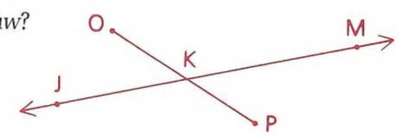


Y is not a midpoint

Why X a midpoint?
Why isn't Y a midpoint?
How many midpoints does $\overline{PQ}$ have?
How many bisectors could $\overline{PQ}$ have?

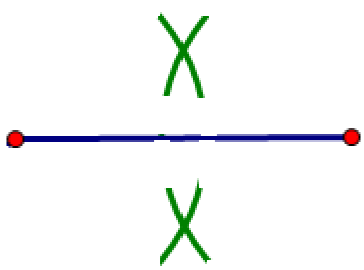
Example *If* $\overline{OK} \cong \overline{KP}$, *what conclusions can we draw?*

Conclusions:
K is the midpoint of $\overline{OP}$.
$\overleftrightarrow{JM}$ is a bisector of $\overline{OP}$.
Point K bisects $\overline{OP}$.



Constructing a segment bisector. The bisector will intersect the segment at the midpoint.

- Set the distance between the spike and the pencil to be greater than ½ the segment, but smaller than the segment itself. Do not adjust the compass after this.
- Place the spike at one segment endpoint. Mark an arc above and below the segment where you believe the midpoint is. Repeat from the other endpoint. Your construction should look similar to this:



- Connect the points found by the intersecting arcs. This line is the bisector. The point of intersection between the bisector and the segment is the midpoint.

Trisection Points and Trisecting a Segment

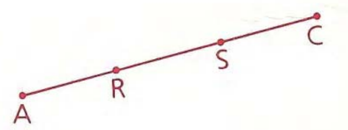
A segment divided into *three* congruent parts is said to be ***trisected***.

Definition Two points (or segments, rays, or lines) that divide a segment into three congruent segments ***trisect*** the segment. The two points at which the segment is divided are called the ***trisection points*** of the segment.

Again, only segments have trisection points; rays and lines do not have trisection points.

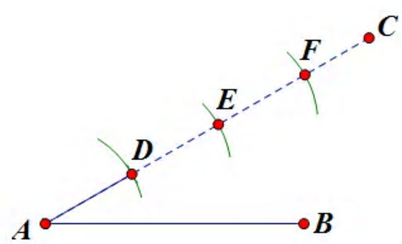
Example *If* $\overline{AR} \cong \overline{RS} \cong \overline{SC}$, *what conclusions can we draw?*

Conclusions:
R and S are trisection points of $\overline{AC}$.
$\overline{AC}$ is trisected by R and S.

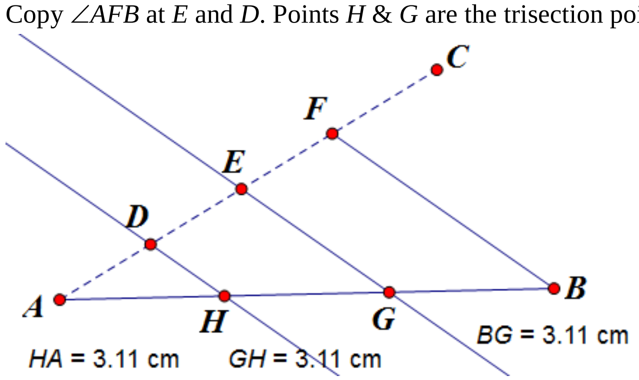


Constructing a segment trisector. The trisector will intersect the segment at the trisection points.

- Given a segment (like $\overline{AB}$ below) draw a random segment or ray (like $\overline{AC}$ below) from an endpoint of the given segment.
- Adjust the compass. Set the distance between the spike and the pencil to be less than 1/3 of the random segment (that is (like $\overline{AC}$ above). Do not adjust the compass after this.
- You are dividing $\overline{AB}$into 3 congruent parts. (There are many ways to divide a segment. I will use Euclid's method.) You will need 3 arcs on $\overline{AC}$ which locate 3 equidistant points. To find those points, place the spike of the compass at the vertex of the angle. Mark and arc on $\overline{AC}$. Label that point *D*. Place the spike at *D* and mark another arc on $\overline{AC}$. Label that point *E*. Move the spike to *E* and make another arc. Label that point *F*. Your construction should look similar to this:



- Draw $\overline{FB}$.
- Copy $\angle AFB$ at *E* and *D*. Points *H* & *G* are the trisection points:



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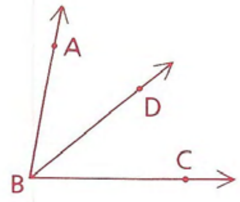
Angle Bisectors

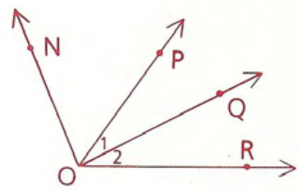
An angle, like a segment, can be bisected.

Definition A ray that divides an angle into two congruent angles ***bisects*** the angle. The dividing ray is called the ***bisector*** of the angle.

If $\angle ABD \cong \angle DBC$, then $\overrightarrow{BD}$ (not $\overrightarrow{DB}$) is the bisector of $\angle ABC$.

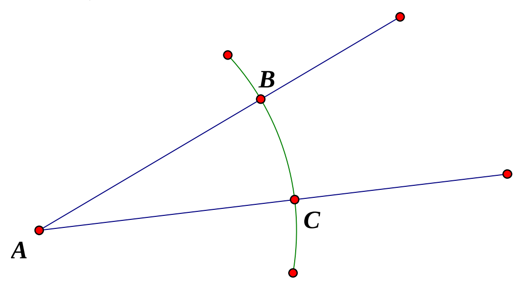


If $\angle NOP \cong \angle POR$ and $\overrightarrow{OQ}$ bisects $\angle POR$, then $\overrightarrow{OP}$ (not $\overrightarrow{PO}$) is the bisector of $\angle NOR$, and $\angle 1 \cong \angle 2$.

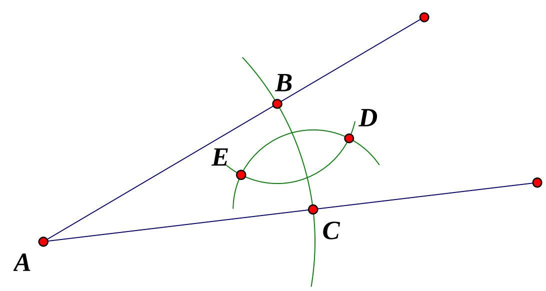


Constructing an angle bisector

- Adjust you compass to intersect both rays of the angle. Place the spike at *A* and draw the arc to find points *B* and *C*:



- You will probably need to adjust the compass distance now. You want to draw arcs that are equidistant with the spike at point *B* and point *C*. In other words, don't adjust the compass until both arcs are finished and you have found points *D* & *E*.



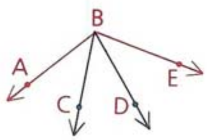
Draw a ray with endpoint *A* that contains *D*. If constructed correctly, *A*, *E*, and *D* are collinear. This is the angle bisector.

Angle Trisectors

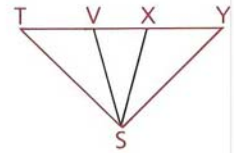
Two rays can divide an angle into three equal parts.

Definition Two rays that divide an angle into three congruent angles ***trisect*** the angle. The two dividing rays are called ***trisectors*** of the angle.

If $\angle ABC \cong \angle CBD \cong \angle DBE$, then $\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.



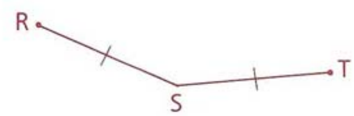
If $\overrightarrow{SV}$ and $\overrightarrow{SX}$ are trisectors of $\angle TSY$, then $\angle TSV \cong \angle VSX \cong \angle XSY$.



We will not construct angle trisectors.

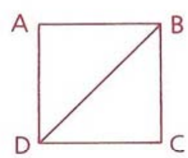
EXAMPLES

Problem 1 *The tick marks indicate that $\overline{RS} \cong \overline{ST}$. Is S the midpoint of $\overline{RT}$?*



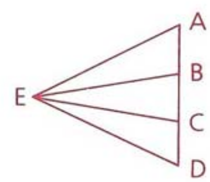
Answer No, the points are not collinear.

Problem 2 *If $\overrightarrow{BD}$ bisects $\angle ABC$, does $\overrightarrow{DB}$ bisect $\angle ADC$?*



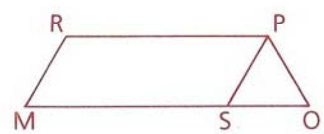
Answer No. We need more information.

Problem 3 *If B and C trisect $\overline{AD}$, do $\overrightarrow{EB}$ and $\overrightarrow{EC}$ trisect $\angle AED$?*



Answer No! It is true that $\overline{AB} \cong \overline{BC} \cong \overline{CD}$, but the fact that the *segment* has been trisected does not mean that the *angle* has been trisected.

Problem 4 *Given: $\overrightarrow{PS}$ bisects $\angle RPO$.*
Prove: $\angle RPS \cong \angle OPS$



Proof

Statements	Reasons
1 $\overrightarrow{PS}$ bisects $\angle RPO$.	1 Given
2 $\angle RPS \cong \angle OPS$	2 If a ray bisects an angle, it divides the angle into two congruent angles.

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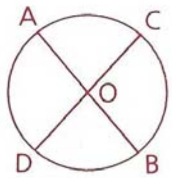
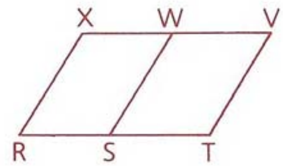
Homework

1 Name the congruent segments.

a O is the midpoint of $\overline{CD}$.

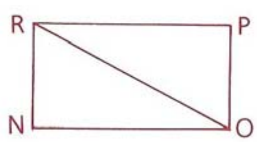


b $\overline{SW}$ bisects $\overline{XV}$.

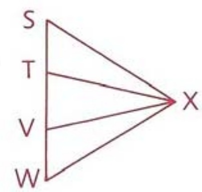


2 Name the congruent angles.

a $\overrightarrow{RO}$ bisects $\angle NRP$.

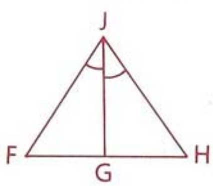


b $\overrightarrow{XT}$ and $\overrightarrow{XV}$ trisect $\angle SXW$.

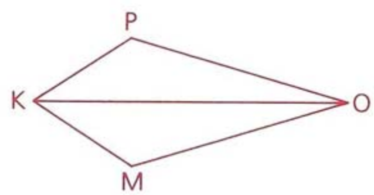


3 Name the angle bisector.

a



b $m\angle POK = m\angle MOK$



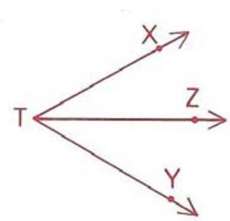
4 Find $\angle XTZ$ if $\overrightarrow{TZ}$ bisects $\angle XTY$ and $\angle XTY$ equals

a 60°

b 48°50′

c $36\frac{1}{2}°$

d 85°74′



5 B and C trisect $\overline{AD}$.

a Find the coordinates of B and C.

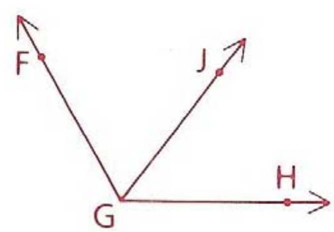
b Find AC.

6 Given: $OM = x + 8$,
$MP = 2x - 6$,
$OP = 44$

Is M the midpoint of $\overline{OP}$?

7 Given: $m\angle FGJ = 3x - 5$,
$m\angle JGH = x + 27$;
$\overrightarrow{GJ}$ bisects $\angle FGH$.

Find: $m\angle FGJ$



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☺ **8** B and C are trisection points of $\overline{AD}$, and AD = 12.

a Find AB.

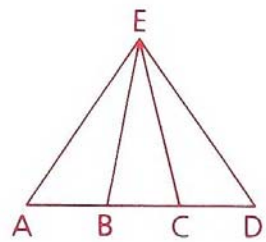
b Find AC.

c If AB = x + 3, solve for x.

d If AB = x + 3 and AE = 3x + 6, find AE.

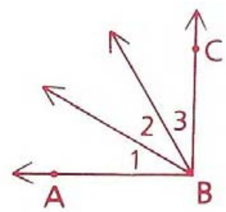
e What segment is C the midpoint of?

f Do $\overrightarrow{EB}$ and $\overrightarrow{EC}$ trisect $\angle AED$?



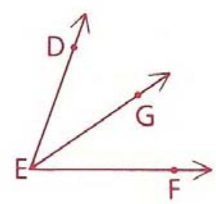
9 Given: $\angle ABC = 90°$,
$\angle 1 = (2x + 10)°$,
$\angle 2 = (x + 20)°$,
$\angle 3 = (3x)°$

Has $\angle ABC$ been trisected?



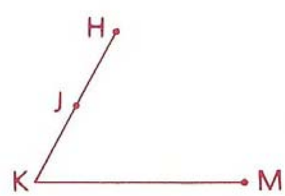
In problems 10 and 11, reason 2 in each proof is stated incorrectly. Supply the correct final reason for each problem.

10 Given: $\angle DEG \cong \angle FEG$
Prove: $\overrightarrow{EG}$ bisects $\angle DEF$.



Statements	Reasons
1 $\angle DEG \cong \angle FEG$	1 Given
2 $\overrightarrow{EG}$ bisects $\angle DEF$.	2 If a ray divides an angle into two angles, the ray bisects the angle. (*What is the correct reason?*)

11 Given: $\overline{KJ} \cong \overline{HJ}$
Prove: J is the midpoint of $\overline{HK}$.



Statements	Reasons
1 $\overline{KJ} \cong \overline{HJ}$	1 Given
2 J is the midpoint of $\overline{HK}$.	2 If a point is the midpoint of a segment, it divides the segment into two congruent segments. (*What is the correct reason?*)

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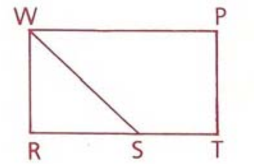
In problems 12–17, write a proof in two-column form.

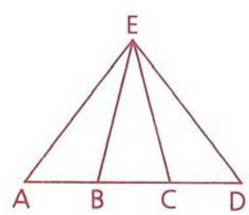
12 Given: $\overrightarrow{WS}$ bisects $\angle RWP$.
Prove: $\angle RWS \cong \angle PWS$



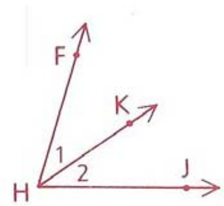
Statements	Reasons
1. $\overrightarrow{WS}$ bisects $\angle RWP$.	1. Given
2. $\angle RWS \cong \angle PWS$	2.

13 Given: $\overline{XY} \cong \overline{YZ}$
Prove: Y is the midpoint of $\overline{XZ}$.

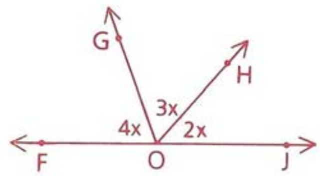
14 Given: $\angle AEB \cong \angle BEC \cong \angle CED$
Conclusion: $\overrightarrow{EB}$ and $\overrightarrow{EC}$ trisect $\angle AED$.



15 Given: $\angle 1 \cong \angle 2$
Conclusion: $\overrightarrow{HK}$ bisects $\angle FHJ$.

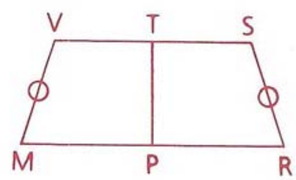


18 $\overrightarrow{OG}$ and $\overrightarrow{OH}$ divide straight angle FOJ into three angles whose measures are in the ratio 4:3:2. Find m$\angle$FOG.



19 Given: $\overleftrightarrow{TP}$ bisects $\overline{VS}$ and $\overline{MR}$.
$\overline{VM} \cong \overline{SR}$,
MP = 9, VT = 6,
perimeter of MRSV = 62

Find: VM



21 **a** Find the value of x.

b Is Q the midpoint of $\overline{PR}$?